I TING SCHOOL MATHEMATICS

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INVESTIGATION DISCUSSION UTILIZATION

Investigating School Mathematics

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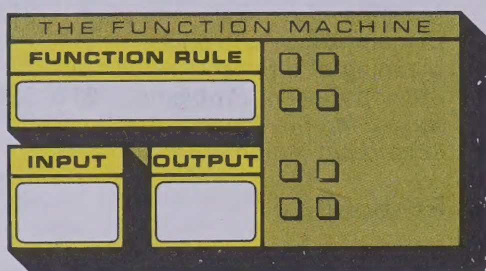
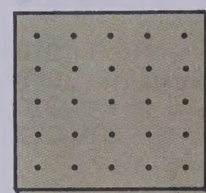
Investigating the Ideas

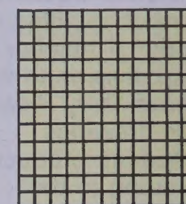
This is a sample lesson to help you understand how to use your book. In this part of a lesson you will find things to **investigate** and discover.

Can you find some Investigations where you will use objects like those shown below?

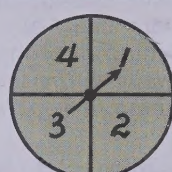




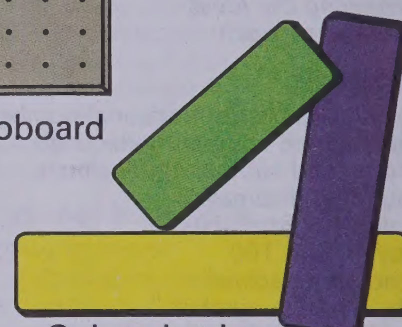
A geoboard



Graph paper



A spinner



Colored strips

Discussing the Ideas

In this part of a lesson you will **discuss the ideas** of the lesson with your classmates and teacher. You will share your ideas with others. You will be getting ready to use the ideas.

1. Look through your book. What other kinds of objects will you use for the "Investigating the Ideas" sections?

2. Sometimes a full page is devoted to "Discussing the Ideas." Can you find a page like this?

3. Find a page called "Keeping in Touch." What do you think this means?

In this part of a lesson you will be **using the ideas** that you investigated and discussed on the opposite page. You will work problems that will help you to improve your understanding of those ideas. Try the problems that follow.

1. What chapter might help you to improve your understanding of fractional numbers?

2. A What is the title of Chapter 10 of your book?
   B How many "Investigating the Ideas" sections does it have?

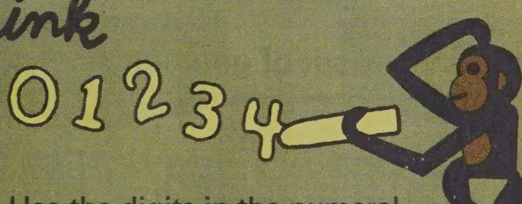
3. There is a special reference for you at the bottom of page 41. What is this special reference? Can you find another page with a reference like this?

4. On page 55 you are invited to explore one of the "Mathematical Activities." How many of these activities are there in this book?

5. Find the first page of the Appendix. How many different parts or sections does the Appendix have?

Problems in these boxes are special challenge problems for you. Be sure to try some of them. See if you can do this one.

think



Use the digits in the numeral for this year and the operations $+$, $-$, $\times$, and $\div$ to form as many of the whole numbers from 0 to 10 as you can.

Year: 1973

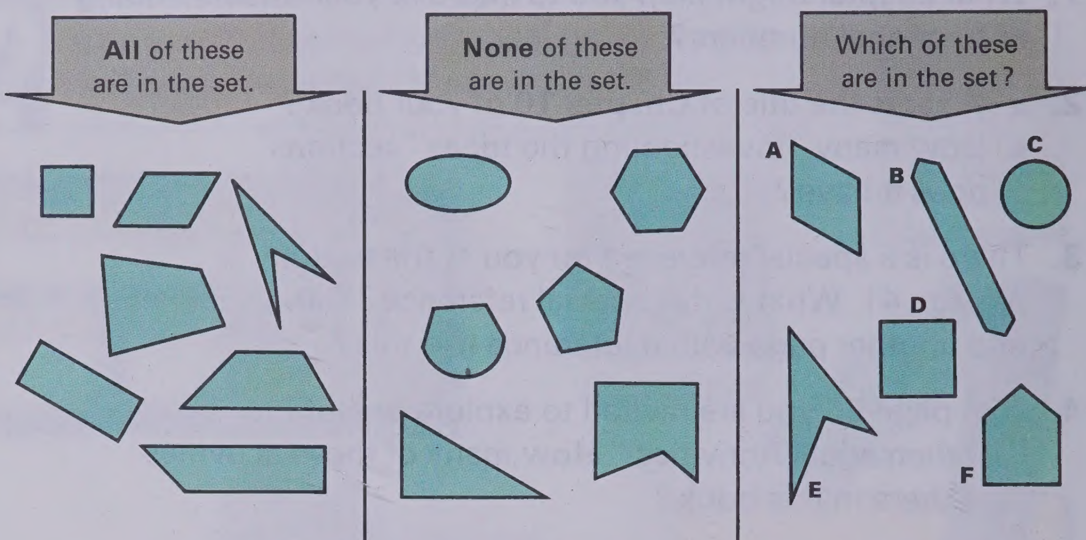
$(7 + 3) - (9 + 1) = 0$
$(3 \times 1) - (9 - 7) = 1$
$(9 + 3) \div (7 - 1) = 2$
$(9 + 7) - 13 = 3$
.
.
.

1 Sets, Logic, and Patterns

Can you name the set?

Investigating the Ideas

Here are some clues about a special set of geometric figures.



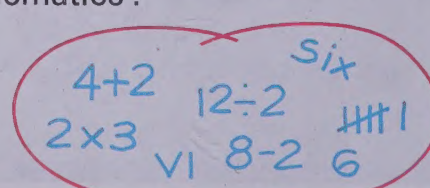
Can you answer the question and draw some more figures that are in the set?

Discussing the Ideas

1. How would you describe the special set of geometric figures in the Investigation?

2. What are some other sets of numbers or objects that you have studied in mathematics?

3. What is special about all the names or symbols in this set?



Answer the question and describe the set in each exercise.

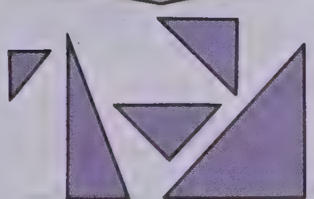
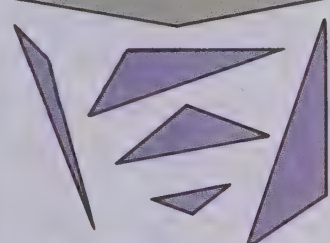
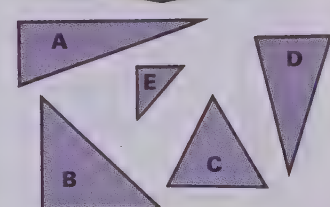
1.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
2, 4, 6, 16, 18, 20,	1, 3, 5 15, 17, 19	7, 8, 9, 10, 74 67, 128, 293

2.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
$2+2$ $8 \div 2$ 2×2 IV $3+1$ $4+0$ $6-2$	$4+1$ $5-2$ $3+0$ $4 \div 2$ $16 \div 8$ 4×0	$5-1$ $36 \div 9$ 1×4 $8 \div 4$ $4 \div 1$ $4-1$ VI

3.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
		

4.

All of these are in the set.	**None** of these are in the set.	Which of these are in the set?
$\frac{1}{2}$ $\frac{2}{4}$ $\frac{6}{12}$ $\frac{10}{20}$ $\frac{4}{8}$ $\frac{50}{100}$	$\frac{8}{4}$ $\frac{2}{5}$ $\frac{21}{41}$ $\frac{45}{85}$ $\frac{1}{3}$ $\frac{2}{1}$ $\frac{1}{20}$	$\frac{12}{6}$ $\frac{3}{6}$ $\frac{4}{9}$ $\frac{11}{21}$ $\frac{100}{200}$ $\frac{5}{10}$

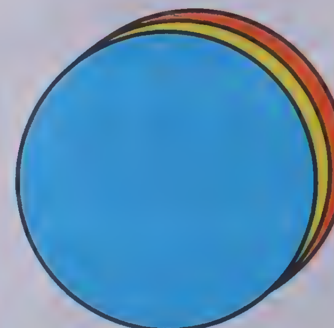
★ **5.** Make up some exercises like those above and give them to a classmate to solve.

Investigating the Ideas

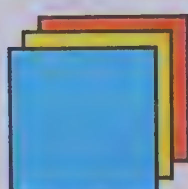
Trace, cut out, and color figures like these. Then mix them up.

3 large squares

3 large triangles

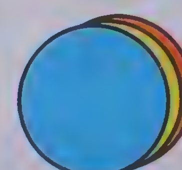


3 large circles



3 small squares

3 small triangles



3 small circles

? If someone secretly takes one of the figures, can you find out which one he took?

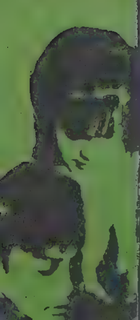
Discussing the Ideas

The figures you cut out are called **attribute pieces**.

1. Can you name three important ways in which one attribute piece might be different from another?

2. Use the attribute pieces to form the set of circles. How many more sets can you form and name?

3. Without looking, can you give the total number of pieces?

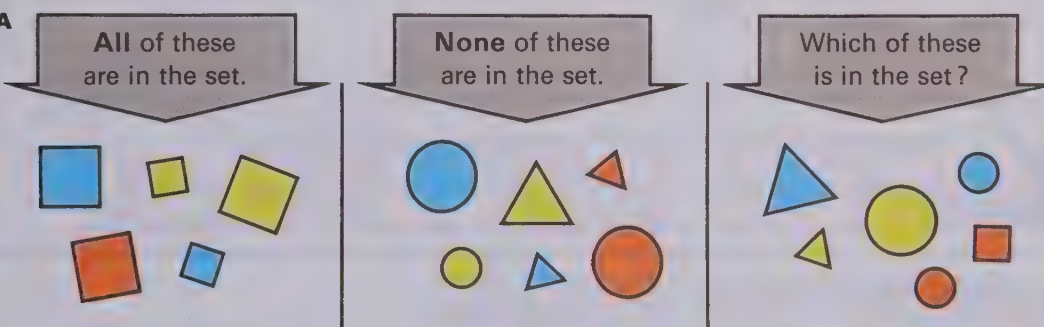


1. How many attribute pieces in each set?

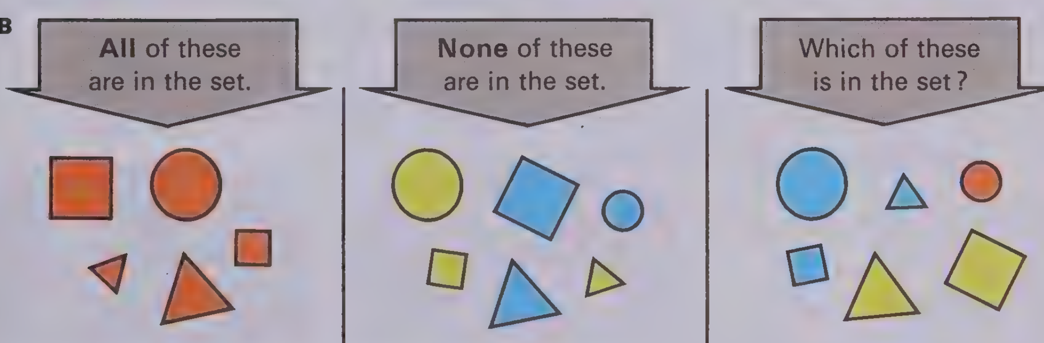
- A squares
- B triangles
- C circles
- D small pieces
- E large pieces
- F red pieces
- G yellow pieces
- H blue pieces
- I red squares
- J small triangles
- K small yellow pieces
- L large blue triangles
- M not-red pieces
- N not-circle pieces
- O small pieces with corners

2. Answer the question and describe the set of pieces.

A



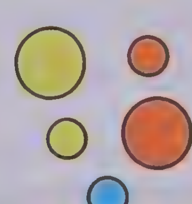
B



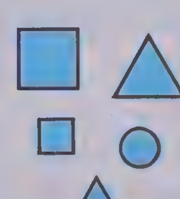
★ 3. One piece is missing from each set. Which piece is it?

A

B



C

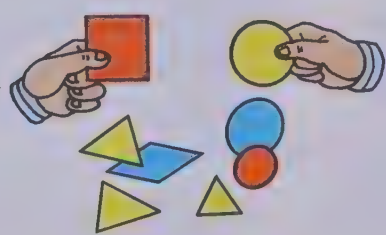


D

Let's explore similarities and differences.

Investigating the Ideas

Choose any pair of the attribute pieces you made on page 6, and keep a record of how they are different.



These figures are

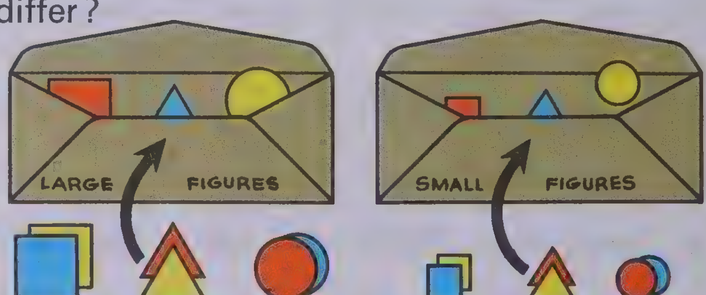
✓ different in COLOR
✓ different in SHAPE
___ different in SIZE (large and small)

? Can you find a pair of pieces that are different in only one way? in only two ways? in three ways?

Discussing the Ideas

1. The pieces you made may differ in **size** (large or small), **color** (red, blue, or yellow), or **shape** (triangle, square, or circle). Can you think of any other ways they might have been made to differ?

2. Suppose you put all pieces that are alike in a certain way into envelopes.

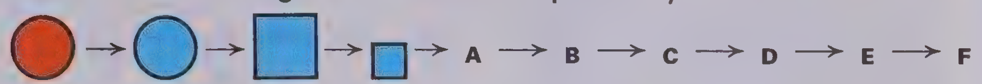


   A If you put all pieces of the same $\boxed{\text{size}}$ into separate envelopes, then there will be __?__ envelopes with __?__ pieces in each envelope.

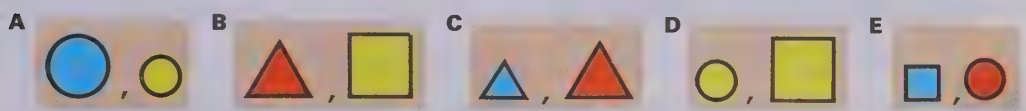
   B Can you fill in the blanks in the above sentence when **shape** is written in the red box? when **color** is written in the red box?

1. The pieces in each pair differ in just **one way**. Write "**size**," "**shape**," or "**color**" to show the way in which they differ.

2. Each piece differs in just **one way** from the piece before it. Continue the "string" and name the pieces you use.



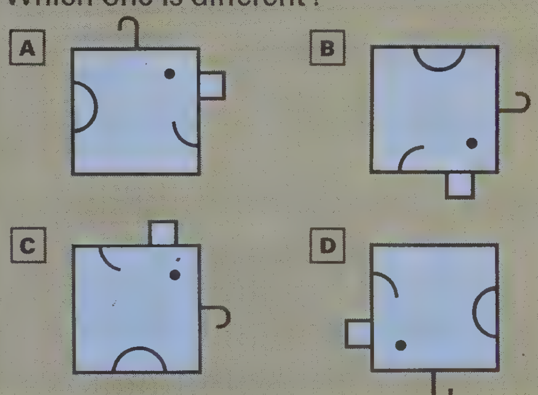
3. The pieces in each pair differ in just **two ways**. Tell in which ways they differ.



4. A Give 3 more pairs of pieces that differ in just one way.
   B Give 3 more pairs of pieces that differ in just two ways.
   C Give 3 pairs of pieces that differ in three ways.

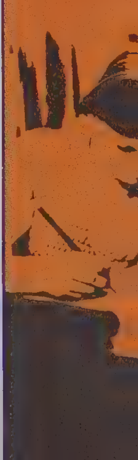
think

Which one is different?



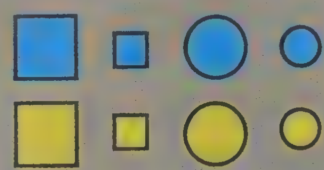
5. Make a string of 6 pieces, each of which differs from the next one in A 1 way. B 2 ways. C 3 ways.

★ 6. Use all your attribute pieces to make a string of pieces that differ in A 1 way. B 2 ways. C 3 ways.



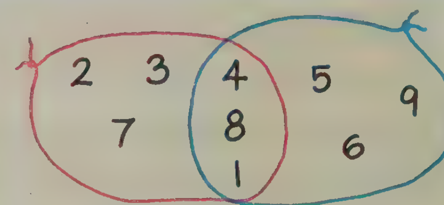
Investigating the Ideas

A



It is from this set.
It is **not** blue.
It is **not** square.
It is **not** small.
Which is it?

B



It is inside the red loop
and
it is inside the blue loop.
It is odd.
Which is it?

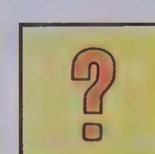
Can you use logical reasoning to answer the questions above, and then make up a set of clues like these to give to a classmate?

Discussing the Ideas

1. How did you figure out the answer to the question in A above?

2. Explain how the clues helped you find the number in B above.

3. One of the 18 attribute pieces is covered up by this yellow card. What can you conclude if someone who knows gives you these clues?



It is **not** a square and it is **not** a circle.

It is **not** large.

It is **not** yellow and it is **not** blue.

What piece is it?

1. Each yellow clue card has the picture of one of these attribute pieces on the other side. Which piece is it?

A

It is **not** small.
It is blue.
It is **not** a square
and **not** a circle.
Which is it?

B

It is small.
It is **not** yellow
and **not** blue.
It is **not** a circle
and **not** a triangle.
Which is it?

C

It is **not** large.
It is yellow.
It has **no** corners.
Which is it?

2. Each yellow clue card has one of these numerals written on the other side. Which number is it?

1 2 3 4 5 6 7 8 9 10

A

It is less than 5.
It is even.
It is **not** 4.
Which is it?

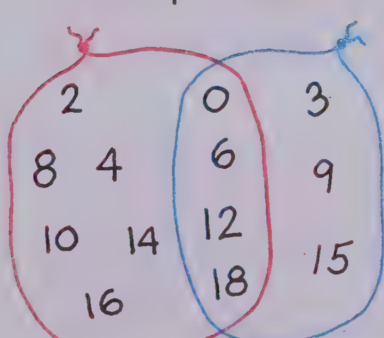
B

It is between
4 and 8.
It is **not** even.
It is **not** half
of 10.
Which is it?

C

It is odd.
It is more than 5.
3 will "go into" it.
Which is it?

3. Each clue card describes one of the numbers shown in the loops. Which number is it?



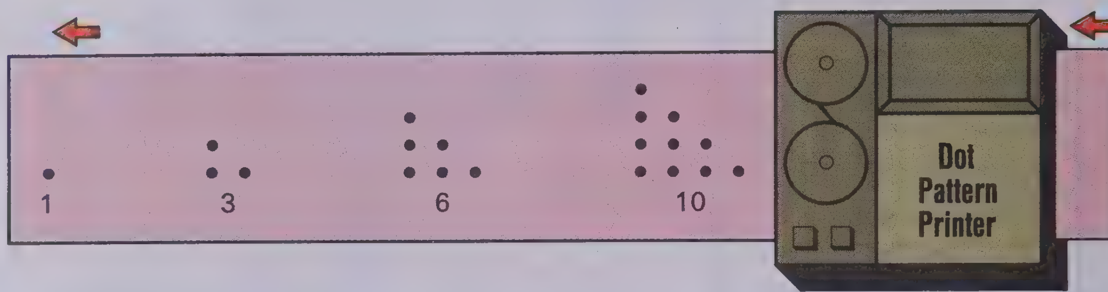
A

It is **not** inside
the red loop.
It is inside
the blue loop.
It is less than 8.
Which is it?

B

It is inside the
red loop **and**
it is inside the
blue loop.
It is between 8
and 15.
Which is it?

Investigating the Ideas



The **Dot Pattern Printer** printed the dot patterns above for the number sequence 1, 3, 6, 10, ▒, ▒, . . .

? Can you draw the next two patterns the "printer" will make and give the next two numbers in the sequence?

Discussing the Ideas

1. Why do you think the numbers in the sequence above are called triangular numbers? Can you give a triangular number greater than 30?

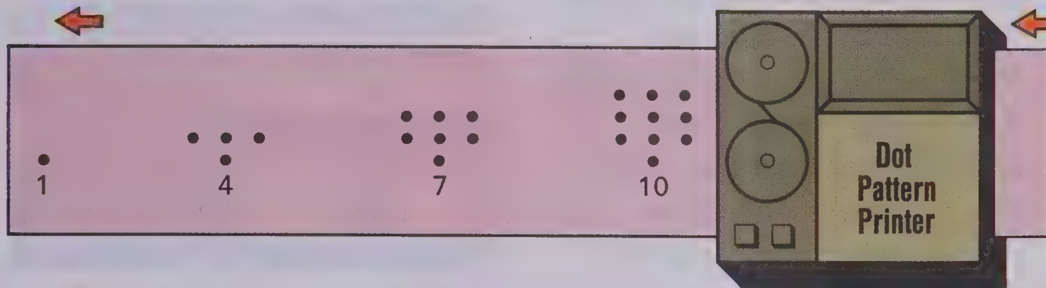
2. Here is an interesting pattern.
   - A What sum would equal the fifth triangular number? the sixth?
   - B Can you "see" these sums in the dot patterns for the triangular numbers?

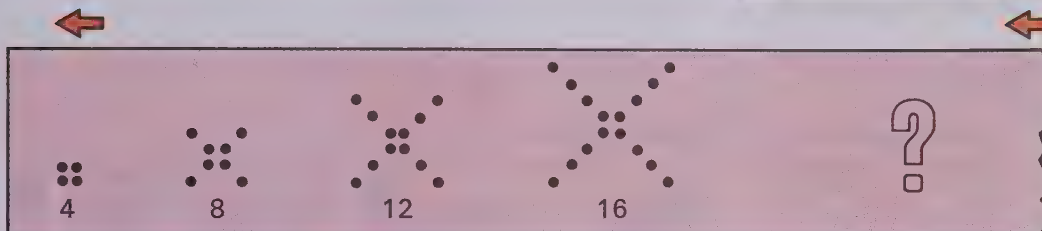
Triangular Numbers

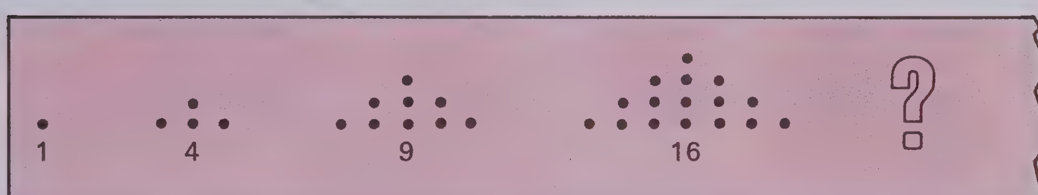
First:	**1**
Second:	**3** = 1 + 2
Third:	**6** = 1 + 2 + 3
Fourth:	**10** = 1 + 2 + 3 + 4

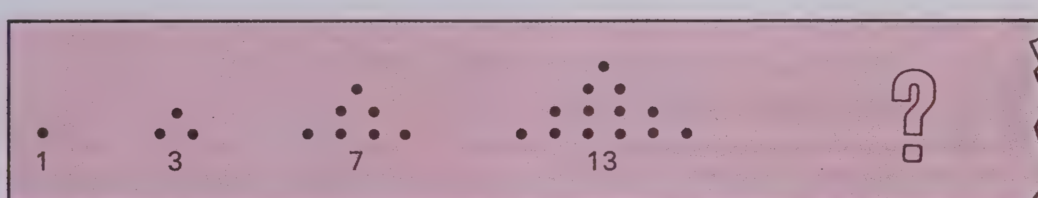
3. A Why do you think the numbers 8, 15, and 24 are called rectangular numbers?
   - B Can you name some other rectangular numbers?

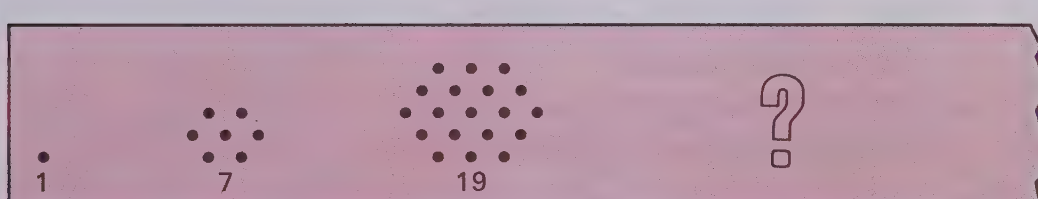
Draw the next dot pattern the "printer" will make and give the next two numbers in each sequence.

1. 

2. 

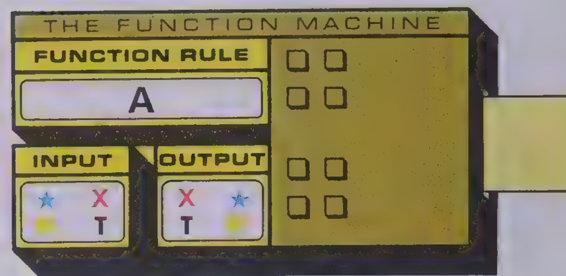
3. 

4. 

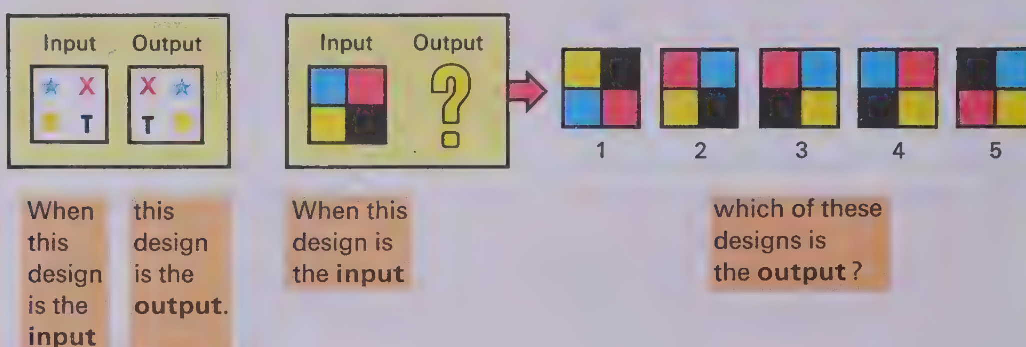
★ 5. 

Investigating the Ideas

When you put a design in as the input of the function machine, the machine changes it in a certain way. Each time the machine changes a design, it prints a yellow input-output card to show what happened.



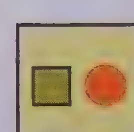
Rule **A**



? Can you make an input-output card by using Rule **A** and a design of your own?

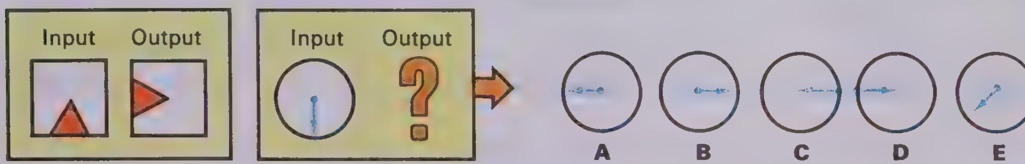
Discussing the Ideas

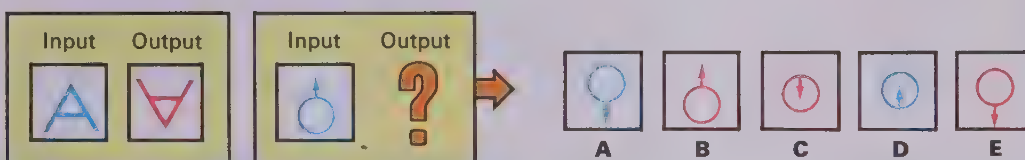
1. A How would the machine above change this design?
   B Can you describe the way the machine changes any figure?

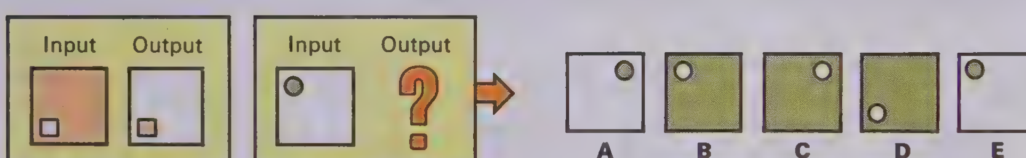


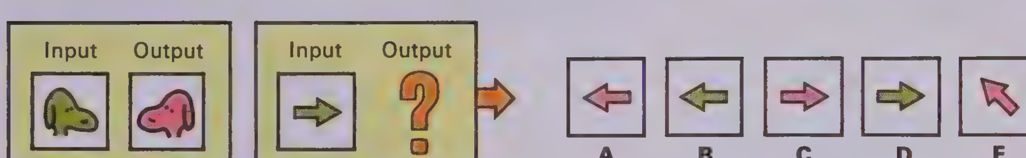
2. Can you make up another interesting rule for the machine?

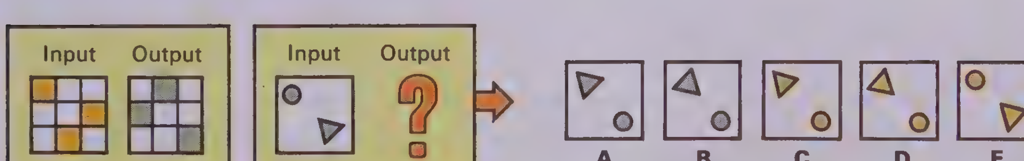
The first input-output card in each exercise shows what the machine does. Give the letter of the output for the second card.

1. 

2. 

3. 

4. 

5. 

Reviewing the Ideas

1. How many? (Use your attribute pieces from page 6.)
   - A blue squares
   - B large circles
   - C not yellow
   - D small and not triangular
   - E red and not square
   - F large and blue

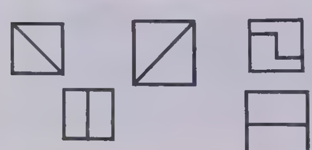
2. Name the attribute piece described for each part.

   A
   It is large.
   It is red.
   It is **not** a circle.
   It is **not** a triangle.

   B
   It is a square.
   It is **not** yellow.
   It is **not** large.
   It is **not** red.

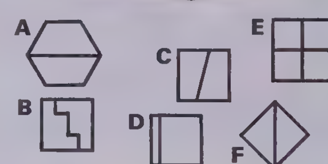
   C
   It is small.
   It is **not** red.
   It is **not** yellow.
   It is **not** a triangle.
   It is **not** a square.

3.
All of these are in the set.



None of these are in the set.

Which of these are in the set?



4.
All of these are in the set.

15 5 40 80 35 95 70 45 90

None of these are in the set.

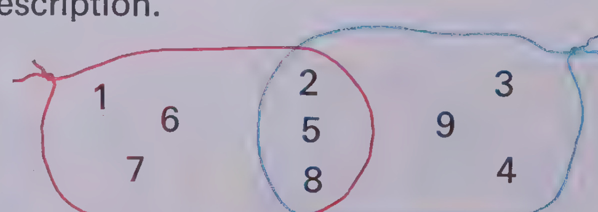
2 44 86 12 29 93 8 1 37 3 72

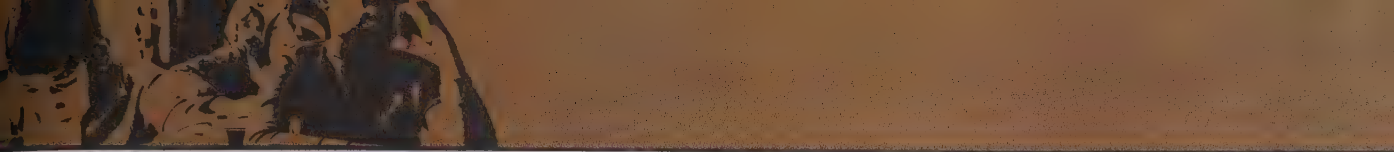
Which of these are in the set?

25 16 50 64 36 48 4 75 30 91

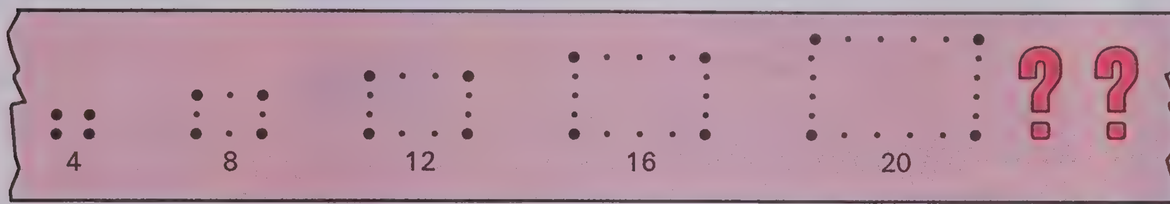
5. Give the number that fits the description.

It is inside both loops.
It is an even number.
It is greater than 4.



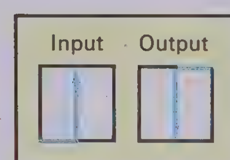


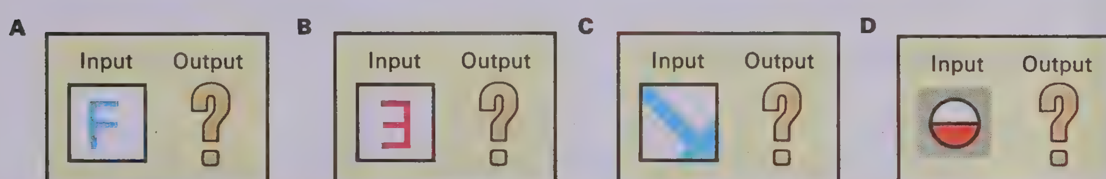
6. Draw the next dot pattern and give the next two numbers.



7. Name 5 pieces you would use to continue this string of "one-way" differences.

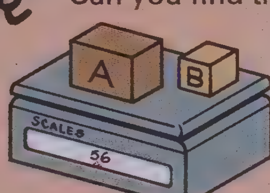
8. This card shows what a certain function machine does. Draw a picture of the output for each of the cards below.





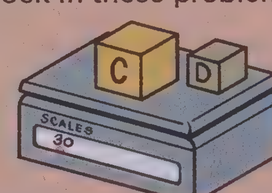
think Can you find the weight of each block in these problems?

1



Block **A** weighs 6 times as much as block **B**. Blocks **A** and **B** together weigh 56 grams.

2



Block **C** weighs 3 grams more than twice block **D**. Blocks **C** and **D** together weigh 30 grams.

3 Suppose block **E** and block **F** together weigh 85 grams and block **E** weighs 5 grams more than 3 times block **F**.

2 Numbers—Numerals

Let's explore symbols for numbers.

Investigating the Ideas

Cut out, mark, and color 3 slips of paper like these. →
Below are two numbers you can name by using 2 or 3 of your slips.

276
524
318

A | 276 | 318 |

B | 524 | 318 | 276 |

? There are 10 more numbers that you can name in this way. How many of them can you find? | Record your findings.

Discussing the Ideas

1. For the number named in example A, read, "two hundred seventy-six **thousand**, three hundred eighteen." Read the other number that these two slips can name.

2. For the number named in example B, read, "five hundred twenty-four **million**, three hundred eighteen **thousand**, two hundred seventy-six." Read this numeral: 318 524 276

3. Each group of 3 digits is called a **period**. The diagram below shows the **place value** of each digit. The "4" is in the thousands' place. What digit is in the ten millions' place?

Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
↓	↓	↓	↓	↓	↓	↓	↓	↓
3	1	8	5	2	4	2	7	6

1. In the numeral 534 896 201 tell what digit is in each of these places:

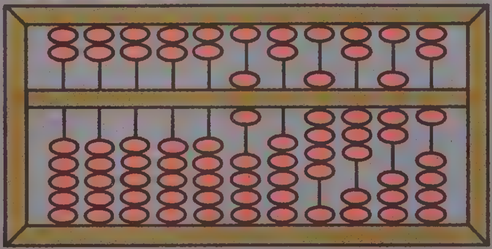
A ones'	D tens'	G hundreds'
B thousands'	E ten thousands'	H hundred thousands'
C millions'	F ten millions'	I hundred millions'

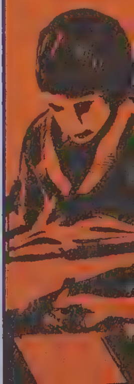
2. Starting at the right, the **first period** of the numeral 146 927 853 contains the digits 8, 5, and 3. Copy and complete these sentences about the numeral.
   A The **second period** contains the digits ___?___ and tells how many ___?___.
   B The **third period** contains the digits ___?___ and tells how many ___?___.

3. Copy the sentence. Give the missing words and numbers.
   A 687: The 8 in the ___?___ place means 8 × ▥.
   B 687: The 6 in the ___?___ place means 6 × ▥.
   C 395 687: The 5 in the ___?___ place means 5 × ▥.
   D 395 687: The 9 in the ___?___ place means 9 × ▥.
   E 395 687: The 3 in the ___?___ place means 3 × ▥.
   F 214 395 687: The 4 in the ___?___ place means 4 × ▥.
   G 214 395 687: The 1 in the ___?___ place means 1 × ▥.
   H 214 395 687: The 2 in the ___?___ place means 2 × ▥.
   I 214 395 687: The 7 in the ___?___ place means 7 × ▥.

think

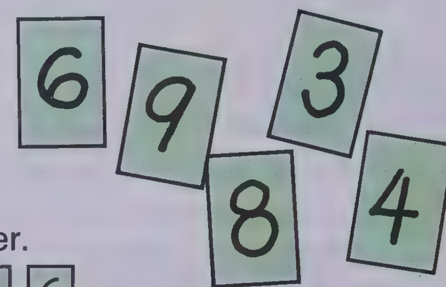
This Chinese abacus (called a **suan-pan**) shows the number 609 371. Can you draw an abacus that shows 732 854 903?





Investigating the Ideas

? Can you play the game LARGE, LARGER . . . LARGEST with a classmate?



Cut out five slips of paper and label them with different digits.

To play: **1.** Mix the slips of paper and place them in a row to name a 5-digit number.
Example: 4 8 3 9 6

2. Take turns **exchanging** the positions of any two digits so that after each exchange a larger number has been named.

3. The player whose exchange results in the largest possible number wins.

Discussing the Ideas

1. How can you tell when 5 different digits name the largest possible number?

2. What is the smallest number you could make with the 5 digits above?

3. In the example, the number is expressed in expanded notation.
Example:
$93\,864 = (9 \times 10\,000) + (3 \times 1000) + (8 \times 100) + (6 \times 10) + 4$
Can you use expanded notation to show these numbers?
A 9847 B 42 165 C 357 869 D 62 709

1. For each exercise, use expanded notation to represent each number.

A 436	C 6457	E 67 491	G 274 165
B 7243	D 5034	F 43 056	H 869 070

2. For "37 **is less than** 42" we write: $37 < 42$
For "56 **is greater than** 48" we write: $56 > 48$
Give the correct sign, $<$ or $>$, for each ●.

A 348 ● 328	D 7296 ● 7269	G 27 483 ● 31 120
B 694 ● 684	E 5409 ● 5390	H 56 257 ● 56 527
C 462 ● 439	F 6843 ● 68 430	I 83 765 ● 84 101

3. Give the correct sign ($>$, $<$, or $=$) for each ●.
A 285 ● $200 + 70 + 5$
B 368 ● $(3 \times 100) + (6 \times 10) + 8$
C 7659 ● $(7 \times 1000) + (6 \times 100) + (4 \times 10) + 9$
D 63 742 ● $(6 \times 10\,000) + (3 \times 1000) + 742$
E 59 406 ● $(6 \times 10\,000) + (0 \times 1000) + 406$
F 378 941 ● $(3 \times 100\,000) + (78 \times 1000) + 940$
G 8 406 000 ● $(8 \times 1\,000\,000) + (4 \times 100\,000) + (6 \times 10\,000)$

4. These exercises are about the number 342 817 695
A Rearrange the digits to name the largest number possible.
B Rearrange the digits to name the smallest number possible.
C Give the number 5 million greater.
D Give the number 42 million less.
★ E Give the number a half million less.
★ F What must be added to the number to reach 343 million?

think

I only have 6 digits,
And each one is the same.
My follower has seven.
Can you write
my name?

WHO AM I?

More practice, page A-1, Set 1

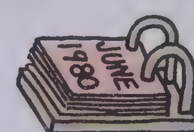
Investigating the Ideas

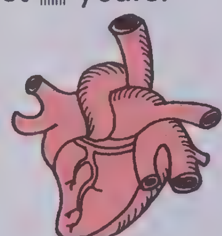
One billion is 1000 million.
We write: **1 000** 000 000

1. 182 classes, each containing 30 pupils, would weigh about one million kilograms. It would take ||||| such classes to weigh one billion kilograms.

2. One million centimetres is 10 kilometres. One billion centimetres is a little more than one-quarter of the way round the earth. One billion centimetres is ||||| kilometres.

3. One million minutes is almost 2 years. One billion minutes is almost ||||| years.





4. If your heart beats 70 times each minute, it will take about 10 days for it to beat one million times and about ||||| years for it to beat one billion times.

Can you estimate each of the missing numbers above?

Discussing the Ideas

1. Explain how you found your answers in the Investigation.

2. What number is:
   - A 1 billion more than 1 billion
   - B 2 billion more than 1 billion
   - C 1 billion more than 9 billion
   - D 5 billion more than 27 billion
   - E 1 million more than 1 billion
   - F 1 more than 1 billion
   - ★ G 1 less than 1 billion
   - ★ H one half of 1 billion

1. The diagram shows the names of some of the periods of a very large number.

Sextillions	Quintillions	Quadrillions	Trillions	Billions	Millions	Thousands	
4 7 6	5 8 7	3 1 9	6 0 8	7 4 2	8 7 1	0 2 3	9 6 4

   A What digits are in the quintillions' period?
   B What digits are in the sextillions' period?

2. The average distance from Earth to the sun is 149 565 801 km. The largest period for this numeral is millions.
   Give the largest period for each of the following numbers.
   A Sun's diameter: 1 392 021 905 metres
   B Estimated remaining life of the sun: 100 000 000 000 000 000 seconds
   C Sun's mass: 1 969 000 000 000 000 000 000 million tonnes
   D Recent estimate of Earth's age: 1 806 750 000 000 days
   E Earth's mass: 5 794 588 715 000 000 000 thousand tonnes

3. The nearest star beyond our sun is called **Proxima Centauri.** It is about 40 trillion kilometres away. Write this numeral.

4. Sirius is the brightest star in the heavens. It is 80 trillion km away, and its mass has been calculated to be 45 077 sextillion tonnes. Write each of these numerals.

5. Which has the greater mass, the Earth or the sun?

think

How much money would you lose if you traded a thousand million dollars for a million thousand dollars? Write the numeral for a million thousand.

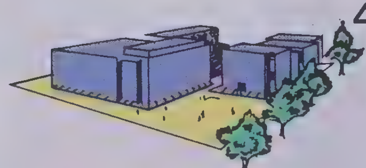
Discussing the Ideas

1. Study the chart below. Then give the missing words. In a recent census* the population of Canada was given as 21 830 772. When less accurate information is needed, we often use a number that is "close" or approximately equal to the exact population. We say that the population is:

	21 830 770	(rounded to the nearest ten)
or	21 830 800	(rounded to the nearest hundred)
or	21 831 000	(rounded to the nearest thousand)
or	21 830 000	(rounded to the nearest ten thousand)
or	21 800 000	(rounded to the nearest hundred thousand)
or	22 000 000	(rounded to the nearest _?_)
or	20 000 000	(rounded to the nearest _?_ _?_)

2. Can you give a rule for rounding to a given place?

3. Quebec and Ontario are the two largest provinces in Canada. The area of Quebec is 1 540 628 square kilometres. The area of Ontario is 1 068 546 square kilometres. Give each area rounded to the nearest ten thousand.



4. The Confederation building in Charlottetown cost \$5 600 050. Give the cost:
   A to the nearest ten thousand
   B to the nearest million

5. There were 79 830 600 telephones in North America in a recent year. Give this number:
   A to the nearest thousand   B to the nearest million

*June 1972

Canada's First Four Provinces

In 1867 when Queen Victoria signed the British North America Act, Canada became a nation. At that time there were only four provinces: Nova Scotia, New Brunswick, Upper Canada (Ontario) and Lower Canada (Quebec). The territory which today constitutes the northern sections of Ontario and Quebec belonged at that time to the Hudson Bay Company, so that the area of each of these provinces was much smaller than it is now.

Province	Population 1871	Population 1971
Nova Scotia	390 000	788 960
New Brunswick	275 000	634 557
Quebec	1 200 000	6 027 764
Ontario	1 650 000	7 703 106

1. Make a chart, as in the above example. Show the population figures rounded off to the nearest thousand.
2. **A** Rank the provinces according to the size of their populations in 1871.
   B Rank the provinces according to the size of their populations in 1971.
3. **A** Study the chart which you set up in Exercise 1. Which province has had the greatest increase in population between 1871 and 1971.
   B Which has had the least population increase between 1871 and 1971 ?
4. In 1971, which province had a population of 8 000 000, as rounded out to the nearest million ?
5. In 1871, which province had a population of 1 000 000, as rounded out to the nearest million ?
6. In the census of 1971, the territory with the smallest population was the Yukon, with 18 388 inhabitants, and Ontario had the largest population, with over seven million. If we round out the given figures to the nearest ten thousand, what is the difference between these two regions ?

Investigating the Ideas

Part of this chart was torn off. Can you figure out how many zeros are in the answers for parts C, D, and E?

A	10×10	$= 100$
B	$10 \times 10 \times 10$	$= 1000$
C	$10 \times 10 \times 10 \times 10 \times 10 \times 10$	$= 10000$
D	$\overbrace{10 \times 10 \times \ldots \times 10 \times 10}^{17 \text{ tens}}$	$= 10000$
E	$\overbrace{10 \times 10 \times \ldots \times 10 \times 10}^{100 \text{ tens}}$	$= 10000$

? Can you write a rule for finding the number of zeros in the answers for products like these?

Discussing the Ideas

1. The numbers indicated in red below are called exponents. Study the examples.

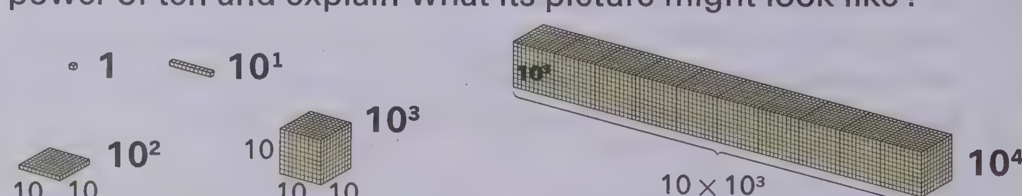
   For 10×10 we write 10^2. We read 10^2 as "10 to the 2nd power" or as "10 squared."

   For $10 \times 10 \times 10$ we write 10^3. We read 10^3 as "10 to the 3rd power" or as "ten cubed."

   Use exponents to write these products. Then read them.

   A $10 \times 10 \times 10 \times 10$ B $10 \times 10 \times 10 \times 10 \times 10 \times 10$

2. The numbers 10^2, 10^3, 10^4, and so on are called **powers of ten**. Ten is called the **base**. 10^1 means 10. Can you give the next power of ten and explain what its picture might look like?



1. Give the missing numbers.

Example: $10^2 = 100$

A $10^3 = \square$

B $10^4 = \square$

C $10^5 = \square$

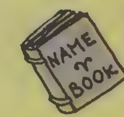
D $10^6 = \square$

E $10^7 = \square$

F $10^8 = \square$

G $10^9 = \square$

think



Suppose a book has 100 names listed in each column, with 5 columns per page. The book has 200 pages. How many books are needed to list all the people in Canada, if the population is 22 million?

2. Give the number for a. Then give the product for b.

A $5 \times 10^3 = 5 \times a = b$

B $12 \times 10^3 = 12 \times a = b$

C $37 \times 10^2 = 37 \times a = b$

D $3 \times 10^4 = 3 \times a = b$

3. Give the number for a. Then give the power of ten for b.

A $300 = 3 \times a = 3 \times b$

B $6000 = 6 \times a = 6 \times b$

C $2700 = 27 \times a = 27 \times b$

D $90\,000 = 9 \times a = 9 \times b$

4. Find the products.

A 4×10^3

B 8×10^2

C 13×10^3

D 48×10^2

E 4×10^4

F 65×10^3

G 57×10^2

H 39×10^3

I 8×10^5

J 7×10^6

5. Write each of the following as a product of a number between 1 and 10 and a power of ten.

Example: $700 = 7 \times 10^2$

A 900

B 8000

C 200

D 6000

E 70 000

F 90 000

G 20 000

H 600 000

6. Write each of the following as illustrated in the example.

Example: $362 = (3 \times 10^2) + (6 \times 10) + (2 \times 1)$

This is called expanded notation (exponent form).

A 457

B 384

C 4265

D 2348

E 2459

F 6782

G 5042

H 6703

I 8006

J 9000

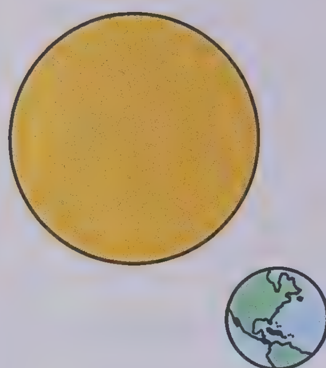
K 37 256

L 79 384

More practice, page A-1, Set 2

Investigating the Ideas

The 1975 population of the world is estimated at 4 000 000 000. For this large number we could write 4×10^9.



Facts about Earth	
Diameter of Earth	about 12 000 000 m
Orbital speed of Earth	nearly 110 000 km/h
Area of Earth	almost 520 000 000 square kilometres

? Can you name each of these large numbers as a product of a number between 1 and 10 and a power of ten?

Discussing the Ideas

1. When a number is named as a product of a number between 1 and 10 and a power of 10, the number is expressed in **scientific notation**. Express this number in scientific notation: 30 000 000 000 000

2. Why do you think scientists invented scientific notation?

3. Earth's mass is nearly 6×10^{21} tonnes. Can you name this number as an ordinary numeral?

1. Write the ordinary numeral for each exercise.

   A 10^7 C 10^9 E 5×10^3 G 7×10^6 I 4×10^{12}
   B 10^8 D 10^{10} F 8×10^2 H 6×10^4 J 8×10^{10}

2. Use scientific notation to name each of the following.

   A 5000 B 7000 C 70 000 D 20 000
   E 300 000 F 5 000 000 G 7 million
   H 80 million ★ I 6 sextillion ★ J 500 quadrillion

3. Use the chart and the diagram to help you understand and answer the questions.

Facts about the Universe
$\boxed{1}$ The diameter of our sun is about 1 392 000 km. It would take over a million Earths to make an object the size of the sun.
$\boxed{2}$ Earth is about 149 570 000 kilometres from the sun.
$\boxed{3}$ One of the two nearest stars, Alpha Centauri, is like our sun and is about 39 814 705 000 000 kilometres away.
$\boxed{4}$ There are probably 100 000 000 000 other stars in our galaxy, or group of stars.
$\boxed{5}$ As many as 500 000 000 000 other galaxies (groups of stars) have been detected by high-powered telescopes.

   A Give the sun's diameter to the nearest hundred thousand km.
   B Give the distance of Earth from the sun, rounded to the nearest 10 million kilometres.
   C Give the distance to Alpha Centauri, rounded to the nearest 10 trillion kilometres.

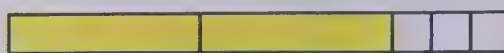
4. Name each answer to exercise 3 in scientific notation.

5. Use scientific notation to name the number of other galaxies.

Investigating the Ideas

Use 4 white and 4 yellow strips for this Investigation.

Here is a 13-centimetre train.



? When you use any number of the 8 strips listed above, how many trains of different lengths can you make and record like this?

RECORD

Length	Train
13 cm	2 fives and 3

Discussing the Ideas

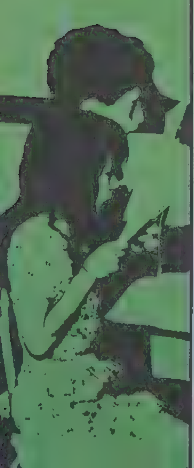
1. Study the chart below. Then explain how to write a **base-five** numeral for some of your trains in the Investigation.

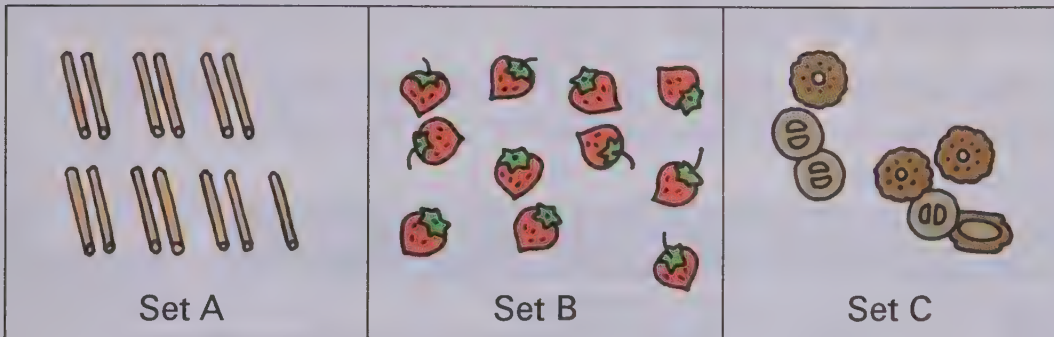
Length	Train	Base-five numeral
13 cm	2 fives and 3	$23_{(5)}$

2. Instead of using your strips, you can think of grouping objects. The figure shows thirteen sticks grouped by fives. Explain what you would get if you grouped by
   A fours. B sixes.

BASE FIVE

2 fives and 3
We write: $23_{(5)}$





1. For sets A, B, and C above, give the base-ten numeral that tells how many objects are in each set.

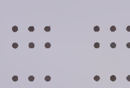
2. A How many groups of 6 sticks are in set A?
   B After all the possible groups of 6 are made, how many extra sticks are there?
   C Give the base-six numeral that tells how many sticks are in set A.

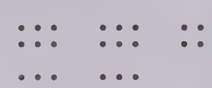
3. Give the base-six numeral that tells how many objects are in set B; in set C.

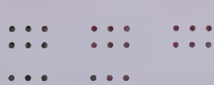
4. Since there is the same number of checkers whether we group by ten or six, we write $20_{(10)} = 32_{(6)}$. For each set, write an equation that uses the base-ten and base-six numeral for the number of dots. Exercise 4A is completed correctly.

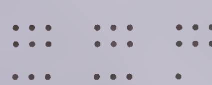
A 

$13_{(10)} = 21_{(6)}$

B 

C 

D 

E 

F 

★ 5. Copy and complete each equation.

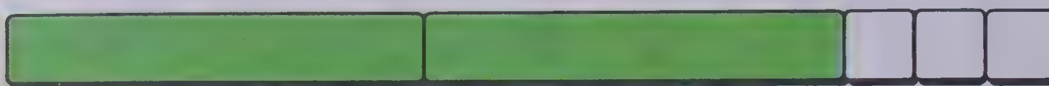
A $16_{(10)} = \_\_\_\__{(6)}$ C $\_\_\_\__{(10)} = 34_{(6)}$ E $30_{(10)} = \_\_\_\__{(6)}$ G $28_{(10)} = \_\_\_\__{(6)}$

B $\_\_\_\__{(10)} = 25_{(6)}$ D $35_{(10)} = \_\_\_\__{(6)}$ F $\_\_\_\__{(10)} = 5_{(6)}$ H $\_\_\_\__{(10)} = 15_{(6)}$

Can we compute in another base?

Investigating the Ideas

Below is a fifteen-centimetre train made with dark green and white strips. The base-six numeral for this train is $23_{(6)}$.
Build this train.



Can you use the dark green strips and five or fewer white strips to make another train that is four centimetres longer than this train?
Can you make one that is four centimetres shorter?

Discussing the Ideas

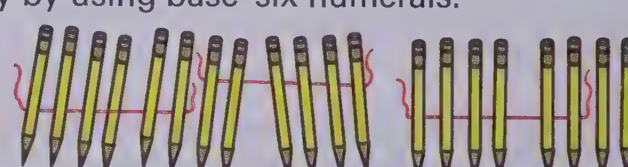
1. Explain how the Investigation can help you solve these equations by using only base-six numerals.

   A $23_{(6)} + 4_{(6)} = s$ B $23_{(6)} - 4_{(6)} = t$

2. Brad put 4 red pencils with 5 black pencils. Since he wanted to add by using base-six numerals, he put the pencils in groups of 6. Use the pencils to explain the equation.

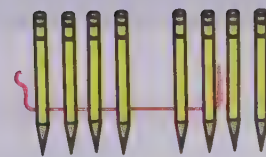
$5_{(6)} + 4_{(6)} = 13_{(6)}$

3. Brad decided to multiply by using base-six numerals. He selected 5 sets of 4 pencils and put them in groups of 6. Can you use the pencils to explain the equation?

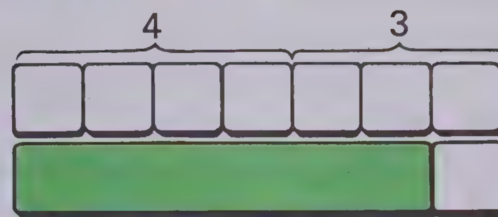


$5_{(6)} \times 4_{(6)} = 32_{(6)}$

1. You can use sets to help you find sums. Solve: $4_{(6)} + 4_{(6)} = n_{(6)}$



2. You can use your strips to help you find sums. $4_{(6)} + 3_{(6)} = m_{(6)}$



3. Find the sums. Use base-six numerals.

A $4_{(6)} + 4_{(6)}$	D $5_{(6)} + 1_{(6)}$	G $5_{(6)} + 5_{(6)}$	J $1_{(6)} + 5_{(6)}$
B $3_{(6)} + 3_{(6)}$	E $3_{(6)} + 5_{(6)}$	H $3_{(6)} + 4_{(6)}$	K $4_{(6)} + 0_{(6)}$
C $2_{(6)} + 4_{(6)}$	F $3_{(6)} + 2_{(6)}$	I $5_{(6)} + 2_{(6)}$	L $4_{(6)} + 5_{(6)}$

4. Copy this number line and label it with base-six numerals.

5. Find the sums and differences. Use base-six numerals.

A $\begin{array}{r} 13_{(6)} \\ +2_{(6)} \\ \hline \end{array}$	B $\begin{array}{r} 13_{(6)} \\ +3_{(6)} \\ \hline \end{array}$	C $\begin{array}{r} 12_{(6)} \\ -5_{(6)} \\ \hline \end{array}$	D $\begin{array}{r} 24_{(6)} \\ +1_{(6)} \\ \hline \end{array}$	E $\begin{array}{r} 11_{(6)} \\ -4_{(6)} \\ \hline \end{array}$	F $\begin{array}{r} 24_{(6)} \\ +3_{(6)} \\ \hline \end{array}$

6. Find the products. Use base-six numerals.

A $3_{(6)} \times 3_{(6)}$	D $4_{(6)} \times 2_{(6)}$	G $4_{(6)} \times 3_{(6)}$	J $4_{(6)} \times 4_{(6)}$
B $2_{(6)} \times 2_{(6)}$	E $2_{(6)} \times 4_{(6)}$	H $5_{(6)} \times 5_{(6)}$	K $5_{(6)} \times 1_{(6)}$
C $3_{(6)} \times 2_{(6)}$	F $3_{(6)} \times 4_{(6)}$	I $3_{(6)} \times 0_{(6)}$	L $2_{(6)} \times 3_{(6)}$

★ 7. Find the products. Use base-six numerals.

A $\begin{array}{r} 11_{(6)} \\ \times 4_{(6)} \\ \hline \end{array}$	B $\begin{array}{r} 12_{(6)} \\ \times 4_{(6)} \\ \hline \end{array}$	C $\begin{array}{r} 13_{(6)} \\ \times 3_{(6)} \\ \hline \end{array}$	D $\begin{array}{r} 21_{(6)} \\ \times 2_{(6)} \\ \hline \end{array}$	E $\begin{array}{r} 25_{(6)} \\ \times 2_{(6)} \\ \hline \end{array}$	F $\begin{array}{r} 30_{(6)} \\ \times 2_{(6)} \\ \hline \end{array}$

More practice, page A-2, Set 3

Reviewing the Ideas

1. In the numeral 624 135 078 what digit is in the
   A thousands' place? B millions' place?

2. Represent each number in expanded notation.
   A 56 B 532 C 8307 D 676 342

3. Give the correct sign ($<$, $>$, or $=$) for each ▒.
   A 6328 ▒ $6000 + (6 \times 100) + 28$ B 462 ▒ $400 + 500 + 2$
   C $654 + 325$ ▒ $644 + 325$ D $317 + 408$ ▒ $307 + 418$

4. In a recent year, the United States, with 5 894 536 kilometres of graded roads, had more roads than any other country. Give this information rounded to the nearest:
   A ten kilometres B hundred kilometres C thousand kilometres
   D ten thousand kilometres E million kilometres

5. Write the ordinary base-ten numeral for:
   A 10^3 B 10^4 C 10^5 D 10^6 E 8×10^5 F 34×10^3

6. Find the products.
   A 10×10^2 B $10^2 \times 10^2$ C $10^2 \times 10^3$

7. Write the number that is 10 more than one billion.

8. Write the largest possible 6-digit number that has no two digits alike.

9. Represent each number by using expanded notation (exponent form).
   A 463 B 6489 ★ C 8 740 684

★ 10. Round 3 568 394 to the nearest million. Then write it in scientific notation.

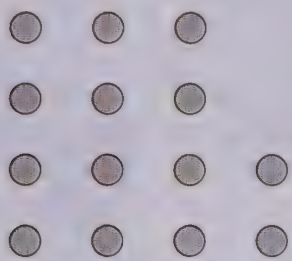
think

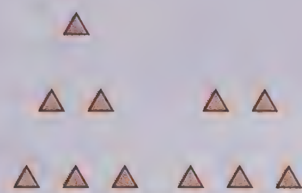
Egyptian numerals:
𓏺 = one 𓆼 = one thousand
𓎆 = ten 𓁨 = one million
𓍢 = one hundred

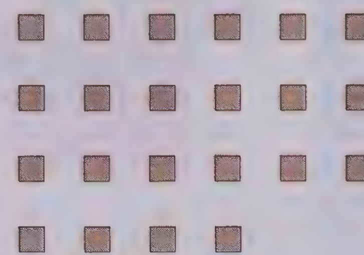
The Egyptians did not use place value. Give the base-ten numeral for each of these.

1. 𓆼 𓎆 𓏺 𓏺 3. 𓁨 𓁨 𓍢 𓍢
2. 𓁨 𓆼 𓍢 𓎆 𓏺 4. 𓁨 𓎆

11. For each set, write a base-six numeral for the number of objects in the set.

A 

B 

C 

12. Complete each sentence.
   - A 15 is 2 sixes and __ ? __.
   - B 21 is __ ? __ sixes and 3.
   - C 10 is 1 six and __ ? __.
   - D 35 is __ ? __ sixes and 5.
   - E 18 is 3 sixes and __ ? __.
   - F 29 is __ ? __ sixes and 5.

13. Write the base-six numeral for each base-ten numeral.
   A 15 B 21 C 10 D 35 E 18 F 29

14. Find the sums and differences. Use base-six numerals.
   - A $3_{(6)} + 4_{(6)}$
   - B $5_{(6)} + 5_{(6)}$
   - C $12_{(6)} + 14_{(6)}$
   - D $15_{(6)} + 23_{(6)}$
   - E $25_{(6)} + 22_{(6)}$
   - F $5_{(6)} - 2_{(6)}$
   - G $11_{(6)} - 4_{(6)}$
   - H $12_{(6)} - 5_{(6)}$
   - I $20_{(6)} - 13_{(6)}$
   - J $44_{(6)} - 25_{(6)}$

15. Find the products. Use base-six numerals.
   A $2_{(6)} \times 5_{(6)}$ B $3_{(6)} \times 2_{(6)}$ C $4_{(6)} \times 3_{(6)}$ D $5_{(6)} \times 3_{(6)}$

You are invited to explore

ACTIVITY CARD 1
Page 355

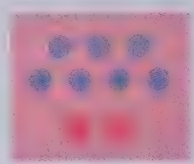
3 Whole Number Concepts

How much do you remember?

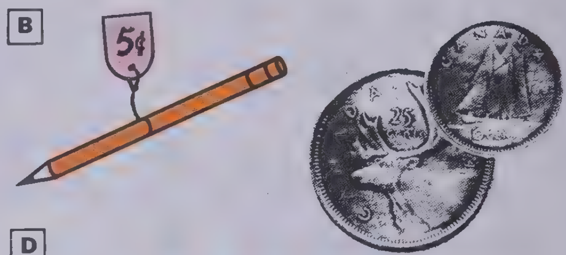
Investigating the Ideas

Each figure below suggests at least one equation.

A

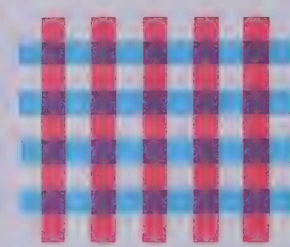


B

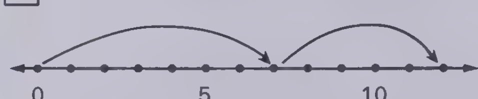


C

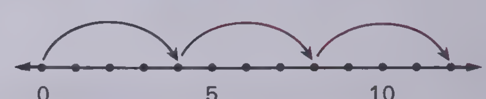
D



E



F

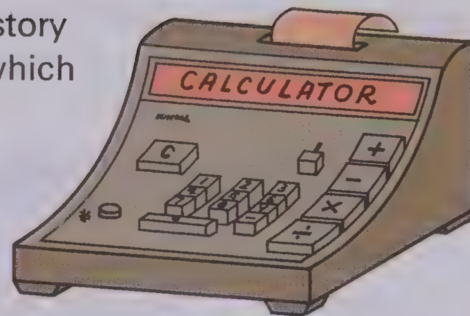


? Can you write one or more equations for each figure?

Discussing the Ideas

1. Can you tell which numbers are addends, which are sums, and which are differences in your equations?
2. Find the factors, products, and quotients in your equations.
3. Make up word problems for figures B and C.

No numbers are given in these short story problems. Use **A, S, M,** or **D** to tell which operation (**A**ddition, **S**ubtraction, **M**ultiplication, or **D**ivision) you would use to solve each problem. For a problem where two operations are needed, say subtraction and division, you should answer **S, D.**



1. Corn: |||| ears per plant. |||| plants. How many ears ?

2. Wheat: |||| dollars per bag. Oats: |||| dollars per bag. How much more per bag is the wheat than the oats ?

3. Bought a bike for ||||. Sold it for ||||. Sold it for how much less ?

4. |||| cookies in each box. |||| cookies in all. How many boxes of cookies ?

5. Triangle sides: |||| cm, |||| cm, and |||| cm. What is the perimeter ?

6. Rectangle: length |||| cm, width |||| cm. What is the area ?

7. Went |||| kilometres. Then went |||| more km. How far ?

8. Bought |||| metres of material for |||| cents. How much for one metre ?

9. |||| pads of paper. |||| sheets on each pad. Used |||| sheets. How many sheets left ?

10. Jim had |||| cents. Tom had |||| times as much as Jim. How much did they have together ?

★ **11.** Had |||| cents. Spent |||| cents. Then spent all the rest on |||| ping-pong balls. How much per ball ?

★ **12.** |||| litres and |||| millilitres of milk. How many 250 ml bottles of milk ?

Let's review the basic principles.

Discussing the Ideas

1. A Solve the equation.
$987 + 0 = n$
B The equation illustrates the zero principle for addition. State this principle in your own words.

2. A State the 1 principle for multiplication. How is it like the zero principle for addition?
B Give several examples.

3. A Solve the equation:
$863 + 92 = 92 + n$
B The equation illustrates the commutative principle for addition. State this principle in your own words.

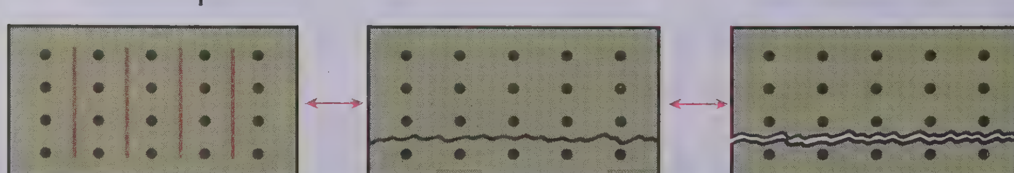
4. A What is the commutative principle for multiplication?
B Can you give examples?

5. A The equation below shows the associative principle for addition.
Solve the equation: $18 + (82 + 37) = (18 + 82) + n$
B Does this principle deal with the order of addends or the grouping of addends?

6. A Is there an associative principle for multiplication?
B Give a numerical example of the principle.

7. A Solve these equations.
$27 + 58 + 73 = 73 + 27 + n$ $\qquad$ $25 \times 38 \times \mathbf{r} = 38 \times 4 \times 25$
B Can you explain what principles can help you solve the equations without pencil and paper?

8. The figure illustrates the distributive principle
Solve the equation.



$5 \times 4 \quad = \quad 5 \times (3 + 1) \quad = \quad (5 \times 3) + (5 \times n)$

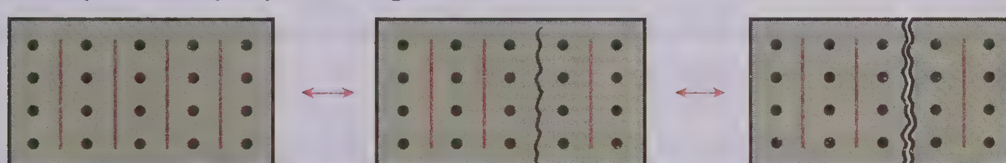
1. Show as many different ways of arranging the three addends 17, 25, and 39 in an addition problem as you can.
Examples: $(17 + 39) + 25$; $(39 + 25) + 17$

2. Copy and complete this multiplication table.
Try to avoid doing any computing.

×	37	84	97	59
59		4956		3481
97	3589		9409	5723
84		7056	8148	
37	1369	3108		2183

3. Solve each equation.
Then give the name of the principle involved.
A $76 + n = 76$
B $68 \times 1 = n$
C $53 \times 101 = 101 \times n$
D $4742 \times n = 4742$
E $796 + 487 = n + 796$

4. Study the display. Then give the number for n.



$5 \times 4 = (3 + 2) \times 4 = (3 \times 4) + (2 \times n)$

5. Solve the equations.
A $68 + 27 = n + 68$
B $(84 + 26) + 13 = 84 + (n + 13)$
C $74 \times n = 61 \times 74$
D $(n \times 32) \times 15 = 89 \times (32 \times 15)$

6. Solve the equations.
A $5 \times 12 = n \times (10 + 2)$
B $6 \times 74 = 6 \times (n + 4)$
C $8 \times 21 = 8 \times (20 + n)$
D $7 \times 56 = 7 \times (n + 6)$

think

How many ways can you show a sum of 8 by using two or more addends?

Investigating the Ideas

Martian Facts

+ ⋀ = ✳

Σ × ⊥ = ᗰ

Martian Arithmetic

A ⋀ + # = ?

B ✳ − ⋀ = ?

C ✳ − # = ?

D ⊥ × Σ = ?

E ᗰ ÷ ⊥ = ?

F ᗰ ÷ Σ = ?

Can you do the "Martian Arithmetic" if you know the Martian Facts?

Discussing the Ideas

1. Explain how you found the answer to part B.

2. Can you substitute some of our numerals for the Martian addition equation and then use them to rewrite parts A, B, and C?

3. Explain how you solved part E.

4. See if you can use the two "facts" given here to solve the problems below.

   7869 + 9758 = 17 627 963 × 879 = 846 477

   A 17 627 − 9 758

   B 879)846 477

   C 17 627 − 7 869

   D 963)846 477

1. Find the missing addend in the addition equation. Then copy the subtraction equation, and give the correct difference.

A $n + 8 = 14$ $14 - 8 = n$	C $c + 3 = 11$ $11 - 3 = c$	E $r + 9 = 17$ $17 - 9 = r$	G $x + 7 = 13$ $13 - 7 = x$
B $a + 9 = 15$ $15 - 9 = a$	D $b + 4 = 12$ $12 - 4 = b$	F $t + 8 = 16$ $16 - 8 = t$	H $s + 5 = 14$ $14 - 5 = s$

2. Find the missing factor in the multiplication equation. Then copy the division equation, and give the correct quotient.

A $n \times 5 = 30$ $30 \div 5 = n$	C $c \times 7 = 42$ $42 \div 7 = c$	E $r \times 9 = 63$ $63 \div 9 = r$	G $x \times 6 = 54$ $54 \div 6 = x$
B $a \times 4 = 28$ $28 \div 4 = a$	D $b \times 8 = 40$ $40 \div 8 = b$	F $t \times 4 = 32$ $32 \div 4 = t$	H $s \times 9 = 0$ $0 \div 9 = s$

3. Solve the equations.

A $6 + 7 = a$	I $14 = 9 + b$	Q $13 - d = 10$
B $35 \div 7 = k$	J $56 \div j = 7$	R $40 \div 8 = n$
C $15 = 10 + n$	K $s - 7 = 3$	S $(36 - 9) + 9 = n$
D $g \times 4 = 24$	L $8 \times i = 40$	T $(72 \div 9) \times 9 = c$
E $16 - 8 = f$	M $i = 8 + 7$	U $(75 - 46) + b = 75$
F $9 \times c = 18$	N $7 \times 6 = x$	V $(56 \div 8) \times b = 56$
G $3 + k = 12$	O $5 + x = 10$	W $(s - 68) + 68 = 75$
H $56 \div 7 = h$	P $72 \div y = 8$	X $(63 \div n) \times 9 = 63$

4. A Does the equation $\overset{F}{n} \times \overset{F}{0} = \overset{P}{7}$ have a solution?

B Does the equation $\overset{F}{n} \times \overset{F}{0} = \overset{P}{0}$ have **just one** solution?

C Since $\overset{P}{7} \div \overset{F}{0} = \overset{F}{n}$ has no solution and $\overset{P}{0} \div \overset{F}{0} = \overset{F}{n}$ has many solutions, we never divide by ▮.

think

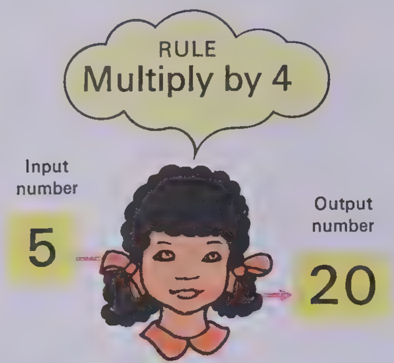
Joe's age is three times Susan's. Four years from now Joe will be twice as old as Susan. How old is Joe now?

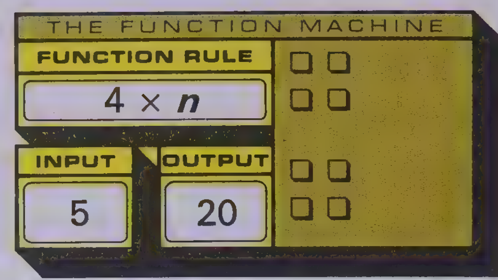
More practice, page A-2, Set 4

How does the function machine work?

Discussing the Ideas

This picture will remind you of how the function machine works.





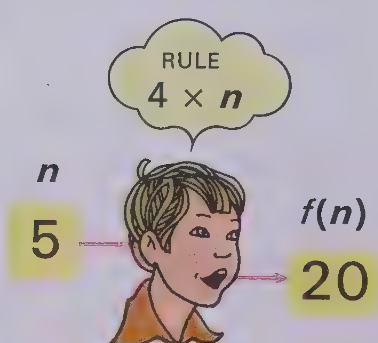
We describe this function rule more carefully by writing:

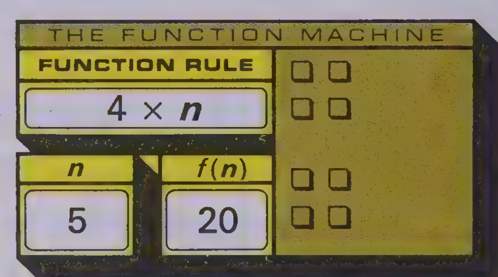
$$\text{output number} = 4 \times n$$

1. Give the output number for each of these input numbers (n).

A 6 B 8 C 3 D 1 E 5 F 0 G 10

We can use $f(n)$ (read, "f of n") for **output number.**
The example looks like this:



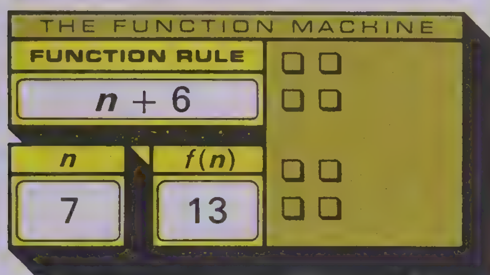


Now we describe the function rule by writing:

$$f(n) = 4 \times n$$

2. Give the output number for each input number in exercise 1 when the rule is changed to $f(n) = (3 \times n) + 5$.

Use the rule shown on the function machine and give the missing numbers in exercise 1. Then think about the function machine with the rule given in each of the other exercises, and give the missing numbers.



1. Function Rule: $n + 6$

	n	$f(n)$
A	7	
B	9	
C	5	

2. Function Rule: $n + 9$

	n	$f(n)$
A	5	
B	6	
C	7	
D	8	
E	9	

3. Function Rule: $n \times 5$

	n	$f(n)$
A	9	
B	8	
C	7	
D	6	
E	5	

4. Function Rule: $n + 8$

	n	$f(n)$
A	4	
B	2	
C		15
D		17
E		16

5. Function Rule: $n \times 6$

	n	$f(n)$
A	7	
B	8	
C		30
D		36
E		54

6. Function Rule: $(3 \times n) + 2$

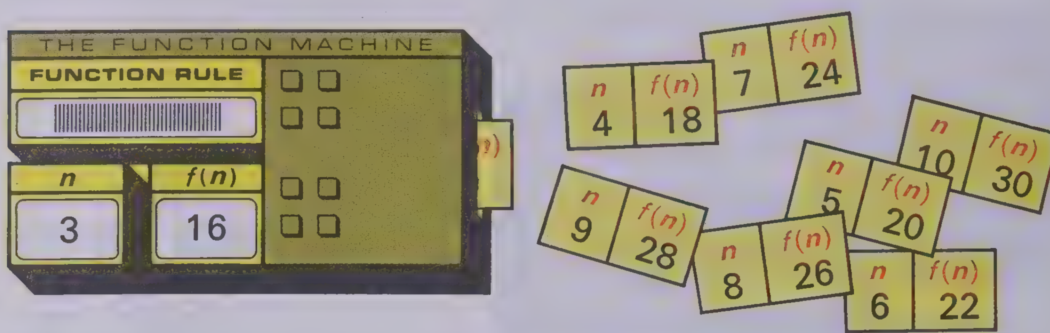
	n	$f(n)$
A	2	
B	5	
C	4	
D	3	
E	6	

7. Function Rule: $4 \times (n + 5)$

	n	$f(n)$
A	3	
B	0	
C	4	
★ D		24
★ E		28

Investigating the Ideas

This function machine prints a card for each input-output pair. Can you guess the rule for the set of cards shown?



?	Can you invent your own function rule and make a set of cards for it?	See if your classmates can guess your rule.

Discussing the Ideas

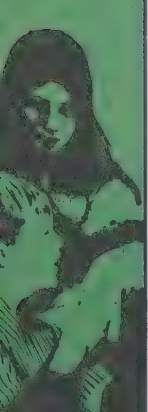
1. Use multiplication to find a rule for this set of cards.

n	$f(n)$
6	18
4	12
9	27
1	3
0	0
5	15

2. What addition rule could you use for the cards in exercise 1?

3. What multiplication rule would give the same set of cards as this rule? $f(\boldsymbol{n}) = \boldsymbol{n} + \boldsymbol{n} + \boldsymbol{n} + \boldsymbol{n} + \boldsymbol{n} + \boldsymbol{n}$

4. What addition rule would give the same set of cards as this rule? $f(\boldsymbol{n}) = 4 \times \boldsymbol{n}$



A function rule is given for each row of cards in exercises 1, 2, and 3. Give the missing numbers.

	A		B		C		D	
	n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
1. $f(n)=8\times n$	6	▒	▒	24	▒	40	0	▒
2. $f(n)=n\times 5$	10	▒	▒	0	▒	45	8	▒
3. $f(n)=n\times n$	6	▒	▒	25	7	▒	▒	100

Give the function rules for exercises 4, 5, and 6.

	A		B		C		D	
	n	$f(n)$	n	$f(n)$	n	$f(n)$	n	$f(n)$
4. $f(n)=$ ▒	7	14	9	18	0	0	50	100
5. $f(n)=$ ▒	6	60	7	70	1	10	0	0
6. $f(n)=$ ▒	3	12	6	21	9	30	0	3

7. What is the "addition rule" that corresponds to:

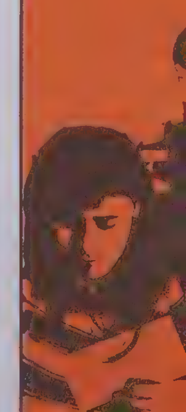
A $f(n)=5\times n$
B $f(n)=9\times n$
C $f(n)=6\times n$

8. What is the "multiplication rule" that corresponds to:

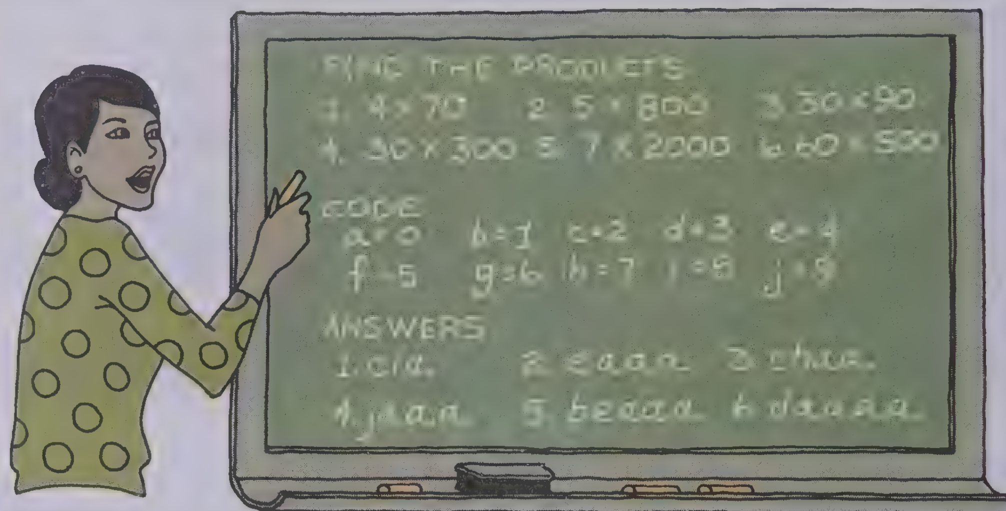
A $f(n)=n+n+n+n+n+n$
B $f(n)=n+n+n+n$
C $f(n)=n+n+n+n+n+n+n$

think

A pen cost a dollar more than an eraser. Together they cost one dollar and ten cents. How much was the eraser?



Investigating the Ideas



?	
Can you find the products above without missing more than one?	Grade your paper by using the code and answers.

Discussing the Ideas

1. Explain why you multiply by 100 in the flow chart below.

Problem 40 × 70 → First find 4 × 7. → Then multiply by 100. → The product is 2800.

$$40 \times 70 = 28 \times 100 = 2800$$

2. Give and explain flow chart steps for each of these products.
   A 20 × 800 B 6 × 4000

3. Use the flow chart to correct any mistakes you made in the Investigation.

1. Find the products.

   A 10×10 B 10×100 C 100×100

2. Give the number for *a*; then give the number for *b*.

   A $15 \times 10 = a \longrightarrow 3 \times 50 = b$
   B $42 \times 10 = a \longrightarrow 6 \times 70 = b$
   C $24 \times 10 = a \longrightarrow 4 \times 60 = b$
   D $18 \times 100 = a \longrightarrow 3 \times 600 = b$
   E $45 \times 100 = a \longrightarrow 5 \times 900 = b$
   F $28 \times 100 = a \longrightarrow 4 \times 700 = b$
   G $35 \times 100 = a \longrightarrow 50 \times 70 = b$
   H $48 \times 100 = a \longrightarrow 60 \times 80 = b$
   I $27 \times 100 = a \longrightarrow 30 \times 90 = b$
   J $36 \times 100 = a \longrightarrow 90 \times 40 = b$

3. Find the products.

   A 20×80
   B 60×40
   C 40×90
   D 70×20
   E 30×30
   F 50×60
   G 7×200
   H 5×600
   I 6×400
   J 20×300
   K 30×800
   L 30×500
   M 40×400
   N 70×300
   O 6×7000
   P 8×2000
   Q 80×9000
   R 50×8000

4. Read the function rule for these cards.
   Then give the missing numbers.

   $f(n) = n \times 30$

   A

n	$f(n)$
80	

   B

n	$f(n)$
	180

   C

n	$f(n)$
600	

5. Give a function rule
   for this set of cards.

n	$f(n)$
2	1400

n	$f(n)$
4	2800

n	$f(n)$
0	0

n	$f(n)$
1	700

n	$f(n)$
9	6300

n	$f(n)$
40	28 000

think

ROMAN NUMERALS

Give the products
in Roman numerals.

1. XX × L
2. XXX × XL
3. L × LX
4. CC × V
5. C × XL
6. D × III

1 = I
5 = V
10 = X
50 = L
100 = C
500 = D
1000 = M

Investigating the Ideas

Make 4 slips of paper with these numerals → 2 60 1800 30

and 3 slips with these signs. → ÷ = ×

? How many equations can you make with your slips? Record each equation you find by writing it on paper.

Example: 60 ÷ 30 = 2

Discussing the Ideas

1. Can you give two multiplication and two division equations by using the numbers 20, 70, and 1400?

2. Find the missing factors and quotients.
   - A Since $\boldsymbol{a} \times 70 = 2800$, we know that $2800 \div 70 = \boldsymbol{b}$.
   - B Since $\boldsymbol{a} \times 4 = 2400$, we know that $2400 \div 4 = \boldsymbol{b}$.
   - C Since $\boldsymbol{a} \times 300 = 2100$, we know that $2100 \div 300 = \boldsymbol{b}$.
   - D Since $\boldsymbol{a} \times 60 = 4200$, we know that $4200 \div 60 = \boldsymbol{b}$.

3. Finding the product $(7 \times 10^1) \times (6 \times 10^2)$ is much like finding the product 70×600. The exponents help you "keep track" of the zeros. Explain each example and give the missing exponent.

A
$$70 \times 600 = 42\,000$$
$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$
$$(7 \times 10^1) \times (6 \times 10^2) = 42 \times 10^{\square}$$

B
$$800 \times 3000 = 2\,400\,000$$
$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$
$$(8 \times 10^2) \times (3 \times 10^3) = 24 \times 10^{\square}$$

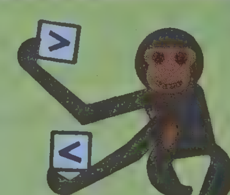
1. Find the quotients.
   - A Since $6 \times 40 = 240$, we know that $240 \div 6 = \boldsymbol{a}$.
   - B Since $5 \times 90 = 450$, we know that $450 \div 90 = \boldsymbol{r}$.
   - C Since $7 \times 800 = 5600$, we know that $5600 \div 7 = \boldsymbol{x}$.
   - D Since $9 \times 600 = 5400$, we know that $5400 \div 600 = \boldsymbol{m}$.
   - E Since $80 \times 30 = 2400$, we know that $2400 \div 80 = \boldsymbol{c}$.
   - F Since $30 \times 40 = 1200$, we know that $1200 \div 40 = \boldsymbol{n}$.
   - G Since $70 \times 20 = 1400$, we know that $1400 \div 20 = \boldsymbol{t}$.
   - H Since $60 \times 90 = 5400$, we know that $5400 \div 60 = \boldsymbol{q}$.

2. In each exercise, give the numbers for ***a*** and ***b***.
   - A $10 \times 10 = a$
     $10^1 \times 10^1 = 10^b$
   - B $10 \times 100 = a$
     $10^1 \times 10^2 = 10^b$
   - C $10 \times 1000 = a$
     $10^1 \times 10^3 = 10^b$
   - D $100 \times 100 = a$
     $10^2 \times 10^2 = 10^b$
   - E $50 \times 700 = a$
     $(5 \times 10^1) \times (7 \times 10^2) = b \times 10^3$
   - F $300 \times 40 = a$
     $(3 \times 10^2) \times (4 \times 10^1) = 12 \times 10^b$
   - G $30 \times 8000 = a$
     $(3 \times 10^1) \times (8 \times 10^3) = 24 \times 10^b$
   - H $700 \times 900 = a$
     $(7 \times 10^2) \times (9 \times 10^2) = 63 \times 10^b$

3. Find the products and quotients.
   - A 7×60
   - B $180 \div 20$
   - C 80×20
   - D $240 \div 40$
   - E $2400 \div 80$
   - F 70×70
   - G $3600 \div 60$
   - H $4200 \div 60$
   - I 30×90
   - J $3500 \div 50$
   - K $4500 \div 90$
   - L 5×900
   - M 80×30
   - N $1800 \div 30$

★ 4. Find the quotients.
   - A $27\,000 \div 90$
   - B $56\,000 \div 80$
   - C $320\,000 \div 40$
   - D $210\,000 \div 300$

think



Give the correct sign ($<$ or $>$) for each

1. 100×10^9 ___ 10^{12}
2. 10^{100} ___ 100×10^{10}
3. $10^{10} \times 10^{10}$ ___ 10^{100}
4. 10^{100} ___ 100^{10}

More practice, page A-3, Set 5

Let's use the distributive principle to help us find products.

Investigating the Ideas

Cindy and Carl made flow charts to show the steps in finding 3 × 47.

Cindy's flow chart:

Problem 3 × 47 → First find 3 × 40. → Then find 3 × 7. → Add the products. → Answer

Carl's flow chart:

Problem 3 × 47 → First find 3 × 7. → Then find 3 × 40. → Add the products. → Answer

Will Cindy's and Carl's flow charts give the correct answer to the problem? Find the answer.

? Can you make a flow chart that will show the steps for multiplying a 2-digit number by a 1-digit number that you choose?

Discussing the Ideas

1. Explain how Cindy and Carl used the distributive principle to find 3 × 47.

2. Find products ***a*** and ***b***. Then give the product for ***c***.

A $3 \times 50 = a$, $3 \times 4 = b$ → $3 \times 54 = c$

B $5 \times 30 = a$, $5 \times 4 = b$ → $5 \times 34 = c$

C $7 \times 20 = a$, $7 \times 4 = b$ → $7 \times 24 = c$

D $6 \times 30 = a$, $6 \times 5 = b$ → $6 \times 35 = c$

E $4 \times 70 = a$, $4 \times 3 = b$ → $4 \times 73 = c$

F $8 \times 40 = a$, $8 \times 2 = b$ → $8 \times 42 = c$

1. Find the numbers for *a* and *b*. Then find the product for *c*.

A $4 \times 20 = a$, $4 \times 1 = b$ $\longrightarrow$ $4 \times 21 = c$

B $5 \times 30 = a$, $5 \times 5 = b$ $\longrightarrow$ $5 \times 35 = c$

C $6 \times 40 = a$, $6 \times 3 = b$ $\longrightarrow$ $6 \times 43 = c$

D $7 \times 60 = a$, $7 \times 3 = b$ $\longrightarrow$ $7 \times 63 = c$

E $4 \times 90 = a$, $4 \times 7 = b$ $\longrightarrow$ $4 \times 97 = c$

F $5 \times 80 = a$, $5 \times 3 = b$ $\longrightarrow$ $5 \times 83 = c$

2. Find the products. Write only the answers.

A 3×21	E 7×21	I 7×32	M 3×18	Q 9×22
B 6×14	F 4×36	J 5×36	N 4×41	R 4×43
C 5×32	G 3×45	K 4×24	O 8×32	S 6×27
D 6×12	H 6×24	L 2×76	P 6×72	T 7×36

3. Find the product for *a*; then find the product for *b*. Write only the answers.

A $5 \times 32 = a \longrightarrow 50 \times 32 = b$

B $6 \times 24 = a \longrightarrow 60 \times 24 = b$

C $7 \times 32 = a \longrightarrow 7 \times 320 = b$

D $6 \times 72 = a \longrightarrow 60 \times 720 = b$

E $3 \times 45 = a \longrightarrow 30 \times 450 = b$

F $8 \times 32 = a \longrightarrow 800 \times 320 = b$

4. Find the products. Write only the answers.

A 40×36	E 20×76
B 70×21	F 70×36
C 50×36	G 40×41
D 30×21	H 90×22

think

Find the missing number pairs.

	Products				Differences
	24	6	× −	4	2
A	24		× −		5
B	40		× −		3
C	42		× −		1
D	35		× −		2
E	27		× −		6
F	54		× −		3

Investigating the Ideas

Study the problem, its flow chart, and its equation.

Problem: If 7 is multiplied by 8 and 18 is added to the product, the result is what number?

Flow Chart:

Input 7	→	Multiply the input by 8.	→	Add 18 to the product.	→	Output n
↓		↓		↓		↓
7		7×8		$(7 \times 8) + 18$		n

Equation: $(7 \times 8) + 18 = n$

? Can you write a problem like this one, then make a flow chart, and write an equation for it?

Discussing the Ideas

1. A What is the output number for the Investigation flow chart?
   B What is the solution to the equation?

2. Which flow chart below is correct for this problem?
   Problem: If 4 is subtracted from 12 and the difference is divided by 2, the result is what number?

   A: Input 12 → Divide by 2. → Subtract 4. → Output n

   B: Input 12 → Subtract 4. → Divide by 2. → Output n

   C: Input 12 → Divide by 4. → Subtract 2. → Output n

3. Explain how to use the correct flow chart to write an equation for the problem in exercise 2.

Write and solve an equation for each exercise.

1. What number is 8 more than the product of 6 and 2 ?
   Answer: $(6 \times 2) + 8 = n$
   $n = 20$

2. Beginning with 48, what is the result if you divide by 6 and then multiply by 7 ?

3. If 9 is multiplied by 7 and 6 is subtracted from the product, what is the result ?

4. The sum of 4 and 5 is multiplied by 8. What is the resulting number ?

5. The difference between 15 and 8 is multiplied by 9. What is the product ?

6. Find the product of 7 times the sum of 8 and 2.

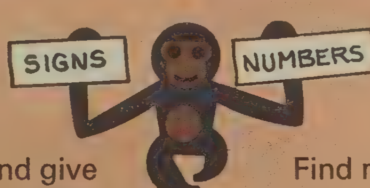
7. If you divide 54 by 6, then add 1, and multiply this sum by 8, what is the result ?

8. What is the quotient when you divide the sum of 68 and 4 by 9 ?

9. If you multiply 10 by itself, then subtract 10 from this product and divide the result by 10, what is the final number ?

★10. Find the number that is 7 less than the product of 8 times the sum of 6 and 3.

think



Copy each equation and give the correct sign for each ▒.

1. (6 ▒ 3) ▒ 7 = 25
2. 3 ▒ (7 ▒ 4) = 31
3. 5 ▒ (6 ▒ 4) = 10
4. (18 ▒ 3) ▒ 4 = 2

Find numbers for **a, b,** and **c** so that the equation is a true statement.

1. $a \times (b + c) = 56$
2. $(a \div b) - c = 2$
3. $a - (b \times c) = 0$

More practice, page A-3, Set 6

1. Solve the equations.

A $8 + 7 = a$	D $24 \div 6 = t$	G $17 - b = 9$	J $6 \times s = 48$
B $6 + n = 13$	E $15 - 7 = d$	H $8 \times 9 = m$	K $g + 7 = 14$
C $9 \times 3 = c$	F $r \times 6 = 42$	I $42 \div 7 = x$	L $48 \div 6 = q$

2. List the missing numbers for each table.

Function Rule: $n \times 40$

	n	$f(n)$
A	60	
B		1200
C	80	
D		360

Function Rule: $n \times 70$

	n	$f(n)$
E	10	
F	4	
G	30	
H		2800

Function Rule: $n \div 60$

	n	$f(n)$
I	1800	
J	4800	
K		20
L	6000	

3. Function rule: $f(n) = (7 \times n) + 8$. Give the number for:

A $f(3)$ B $f(6)$ C $f(20)$ D $f(50)$ E $f(10)$ F $f(80)$

4. Find the numbers for ***a***, ***b***, and ***c***.

A $5 \times 70 = a$, $5 \times 3 = b$ $\longrightarrow$ $5 \times 73 = c$

B $9 \times 40 = a$, $9 \times 2 = b$ $\longrightarrow$ $9 \times 42 = c$

5. Write and solve an equation for each part.

A If 17 is subtracted from 23 and the difference is multiplied by 5, what is the product?

B 14 is added to itself and the sum is divided by 4. What is the quotient?

think For what whole numbers is the following riddle true?

Add me to myself.
Multiply by 4.
When you divide by 8,
You'll have me once more.

WHO AM I?

1. Write each number in expanded notation.
   A 826 C 1032 E 34 576 G 469 506
   B 2745 D 52 280 F 80 365 H 9 037 260

2. Give the correct sign ($<$, $>$, or $=$) for each ■.
   A 78 263 ■ $70\,000 + (8 \times 1000) + (2 \times 10^2) + 60 + 3$
   B 5 293 648 ■ 5293×10^2
   C $387 + 256$ ■ $387 + 246$
   D $9203 + 6152$ ■ $9203 + 6252$
   E $6327 + 5461$ ■ $6337 + 5461$

3. Find the products.
   A $10^1 \times 10^2$ B $10^2 \times 10^2$ C $10^2 \times 10^3$ D $10^3 \times 10^1$

4. Write the ordinary base-ten numeral for each exercise.
   A 7×10^2 C 6×10^3 E 38×10^5 G 60×10^2 I 600×10^1
   B 3×10^4 D 25×10^1 F 43×10^4 H 160×10^3 J 750×10^4

5. Round these population figures (for a recent year) to the nearest million.
   A Canada: 21 681 000 B Mexico: 48 313 438

6. Give the missing numbers.

Power of ten	Ordinary base-ten numeral
10^4	A
B	1000
10^5	C
10^7	D
E	100 000 000

think

These counting sequences are in base six. Give the missing numerals.

1. 3, 4, ■, ■, ■, 12, 13
2. 34, 33, 32, ■, ■, ■, 24
3. 424, 425, ■, ■, ■, 433
4. 52, 53, ■, ■, ■, 101, 102

You are invited to explore

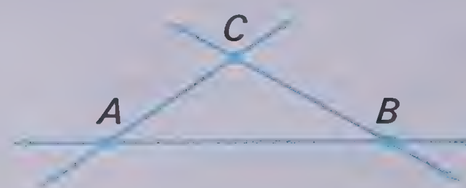
ACTIVITY CARD 2
Page 356

4 Geometry and Measurement I

Let's look at some basic geometric figures.

Investigating the Ideas

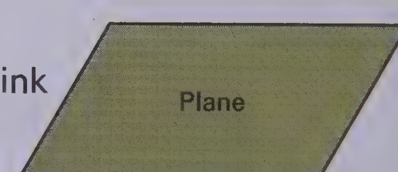
Study this drawing.



? How many different geometric figures can you find and name in the drawing?

Discussing the Ideas

1. To name line AB we write $\overleftrightarrow{AB}$. How many other lines did you find and name?
2. To name segment AC we write $\overline{AC}$. What other segments did you name in the drawing above?
3. A ray starts at a point on a line and contains all the points on one half of the line. Ray BC (symbol: $\overrightarrow{BC}$) is an example. How many other rays did you find?
4. A How many angles are shown in the drawing?
   B Explain which angle is meant by the symbol $\angle ACB$?
5. Did you name a triangle in the drawing? What is a triangle?
6. A plane is a flat surface that extends endlessly in each direction. Do you think that any three points that are not in a line can always be in one plane?



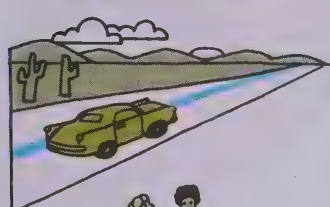
7. Describe some physical objects that remind you of points, lines, segments, rays, angles, triangles, and planes.

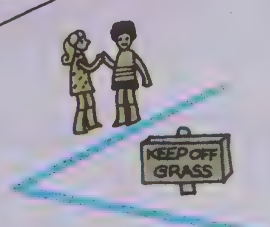
Match each picture with the name of the geometric figure it suggests, with the drawing of the figure, and with the symbol for the figure. For exercise 1, write C, H, Q.

We see

1. 

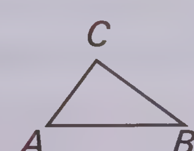
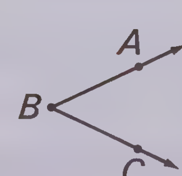
2. 

3. 

4. 

5. 

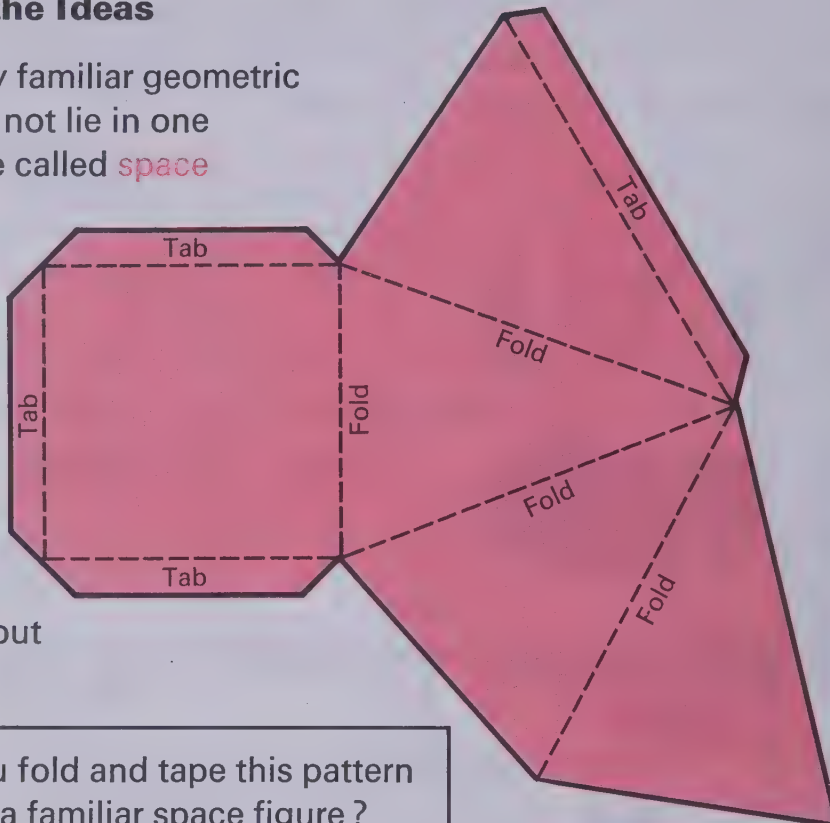
6. 

We think	We draw	We write
A line	G 	M A
B angle	H 	N $\overleftrightarrow{AB}$
C segment	I 	O $\overrightarrow{AB}$
D point	J 	P $\angle ABC$
E ray	K $\bullet A$	Q $\overline{AB}$
F triangle	L 	R $\triangle ABC$

7. Draw and label a figure for each symbol.
   A $\overline{PQ}$ B $\angle NEW$ C $\overrightarrow{ST}$ D $\overleftrightarrow{GQ}$ E $\triangle PQR$ F $\overrightarrow{BA}$

Investigating the Ideas

There are many familiar geometric figures that do not lie in one plane. They are called space figures.



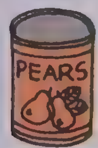
Trace and cut out this pattern.

? Can you fold and tape this pattern to form a familiar space figure?

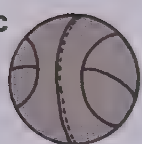
Discussing the Ideas

1. Does the space-figure model you made remind you of any physical objects you have seen? Do you know the name of the space figure?
2. How many faces, edges, and vertices does your space figure have?
3. The pattern above was made from plane figures. Do you know the names of these figures?
4. Can you make or describe a pattern that could be used to make a cube?

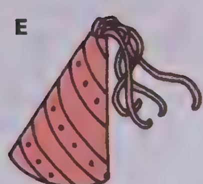
1. Match each physical object on the left with one of the geometric figures pictured on the right.

A


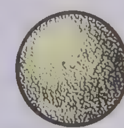
B


C


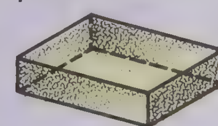
D

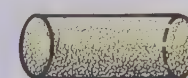

E


F

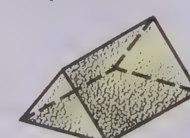

1 sphere


2 rectangular pyramid


3 rectangular prism


4 cylinder


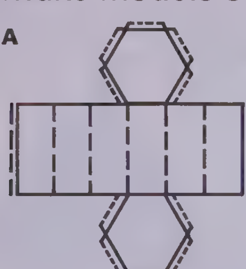
5 cone


6 triangular prism


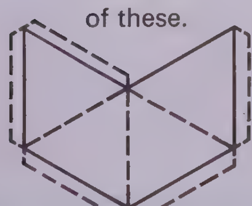
2. How many faces, edges, and vertices does each figure have?

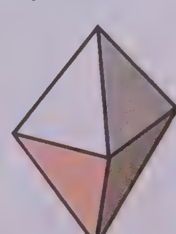
A rectangular prism B triangular prism

★ 3. Make models of these figures.

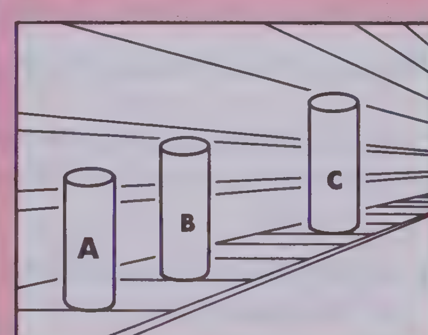
A



hexagonal prism

B Make two of these.



octahedron

think



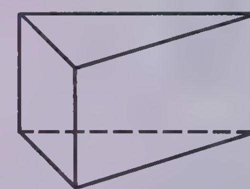
Which of the cylinders is tallest? After you have decided, check by measuring.

Investigating the Ideas

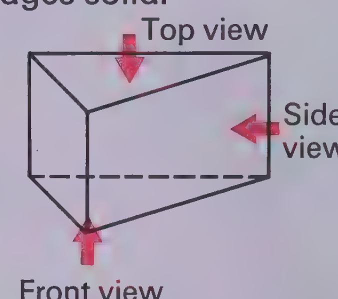
Follow these directions to draw a larger picture of a triangular prism.

Draw a triangle.

Draw three segments of the same length "straight down."



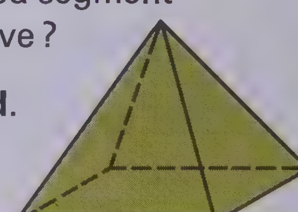
Make "hidden edges" dotted. Make other edges solid.



? Can you draw the front, side, and top views of a triangular prism?

Discussing the Ideas

1. On a flat surface we can draw a space figure that appears to be three-dimensional. Why is the one dotted segment drawn in the picture of a triangular prism above?



2. Here is a drawing of a **rectangular pyramid**.
   A Why is the pyramid called a rectangular pyramid?
   B How many hidden edges does this drawing of the pyramid have?

Front view Top view Side view

3. Here are the front, top, and side views of a figure. What figure do you think it is?

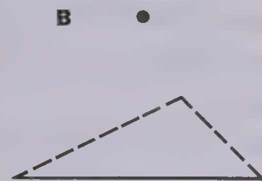
Using the Ideas

1. Trace and complete the drawing of each figure.
Be sure to show the hidden edges with dotted segments.

A

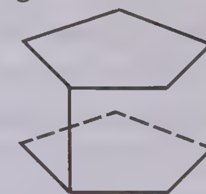
cube

B



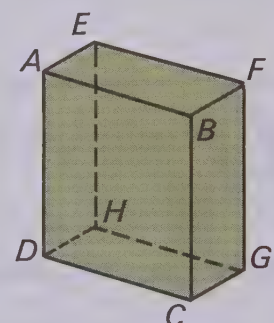
triangular pyramid

C

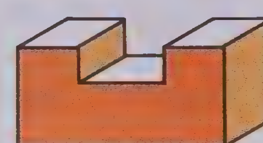


pentagonal prism

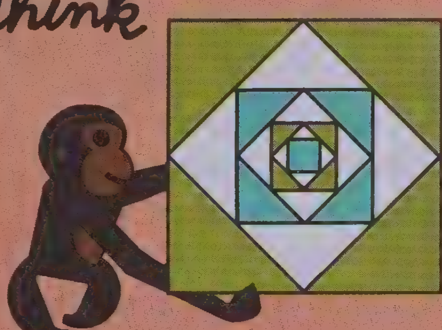
2. The vertices of this rectangular prism have been labelled.
   - **A** Name the edges that are "hidden."
   - **B** The front face is named *ABCD*. Name the faces of the prism that are "hidden."



3. This object was pictured in a draftsman's handbook. Can you draw front, top, and side views of the object?



think

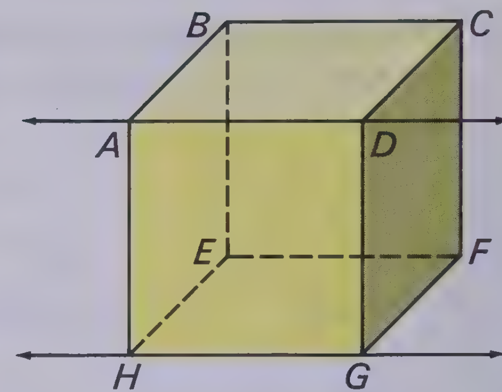


Here is a design made by drawing squares. Make and color an interesting design by using only squares.

★ 4. Choose an object, such as a birdhouse, a desk, or a table, and make a drawing of it. Then show the front view, top view, and side view.

★ 5. Find out what an octahedron is. Make a drawing of one.

Investigating the Ideas



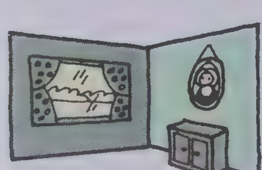
Lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{HG}$ can be contained in a single plane and do not meet. In the same figure find other pairs of lines that are in the same plane but do not meet.

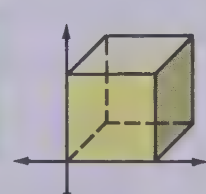
? How many of these pairs can you name?

Discussing the Ideas

1. Two lines which are in the same plane and do not meet, or intersect, are parallel lines. To show that $\overleftrightarrow{AD}$ is parallel to $\overleftrightarrow{HG}$, we write $\overleftrightarrow{AD} \parallel \overleftrightarrow{HG}$. Show how to write statements for other lines that are parallel to $\overleftrightarrow{AD}$ in the figure above.

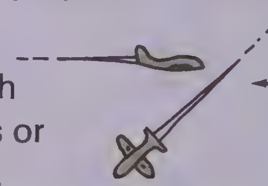
2. A long, straight section of railroad tracks may remind you of parallel lines. Name some other things that remind you of parallel lines.

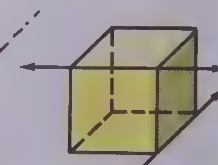




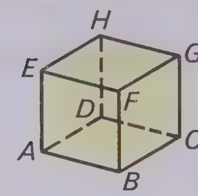
3. When two lines intersect and form a "square corner," they are perpendicular to each other. Name some things that remind you of perpendicular lines.

4. Lines in space that do not intersect and are not parallel are skew to each other. Show how to hold two rulers or two pencils to represent skew lines.

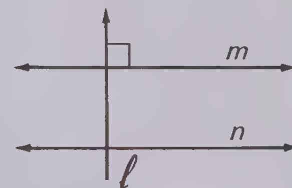




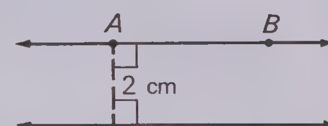
Using the Ideas



1. A Give all the edges that are parallel to $\overline{EH}$.
   B Give all the edges that are perpendicular to $\overline{AB}$.
   C Give the edges that are neither parallel nor perpendicular to $\overline{BC}$.



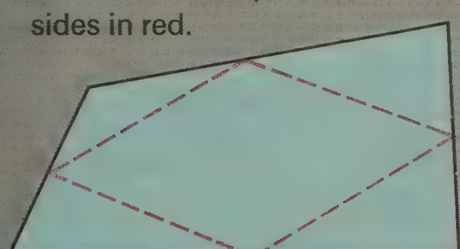
2. A Lines *m* and *n* are parallel to each other. Line *m* and line *ℓ* are perpendicular. What do you think about *n* and *ℓ*?
   B We use a segment that is perpendicular to two parallels to measure the distance between them. If we find that the lines are 2 cm apart at point *A*, what do you think we will find at any other point, such as *B*?



3. Lines *m* and *n* are straight. Are they parallel? Why or why not?

think

Draw any 4-sided figure and connect the midpoints of the sides in red.

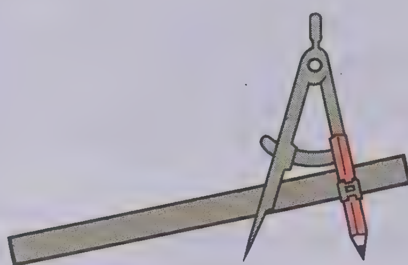


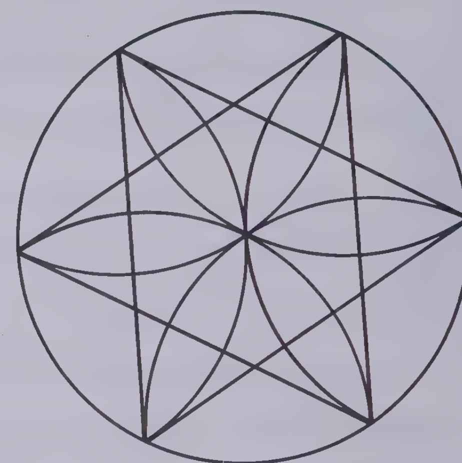
Do this with five 4-sided figures of different shapes. What can you discover about the red figures?

★ 4. Start with a line *AB* and use the idea in exercise 2B and the corner of your book to help you draw two parallel lines.

★ 5. Show how to draw an accurate picture of two parallel lines by using only your ruler.

Investigating the Ideas





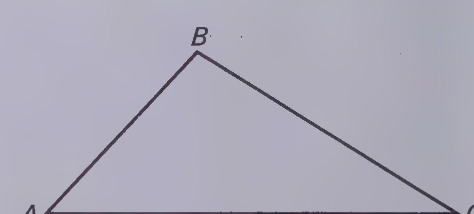
Can you make this design with only a compass and a ruler?

Discussing the Ideas

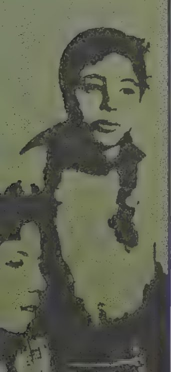
To **construct** a geometric figure we use only a compass and the edge of a ruler. No guessing or measuring is allowed.

1. How could you **construct** a segment that is just as long as $\overline{AB}$ on your paper?

2. A What does it mean to say that two angles are "the same size"?
   B How could you **construct** an angle the same size as $\angle ABC$ on your paper?



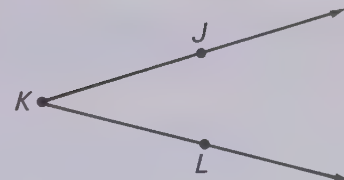
3. A What does it mean to say that two triangles are "the same size and shape"?
   B How could you **construct** a triangle the same size and shape as $\triangle ABC$ on your paper?



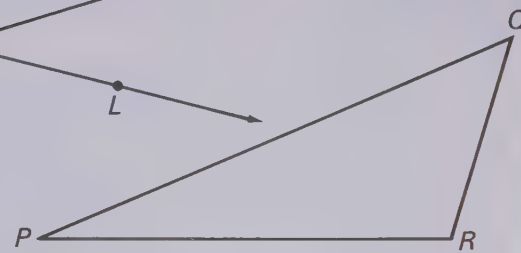
Using the Ideas

1. Construct a segment that is just as long as $\overline{EF}$.

2. Construct an angle that is the same size as $\angle JKL$.

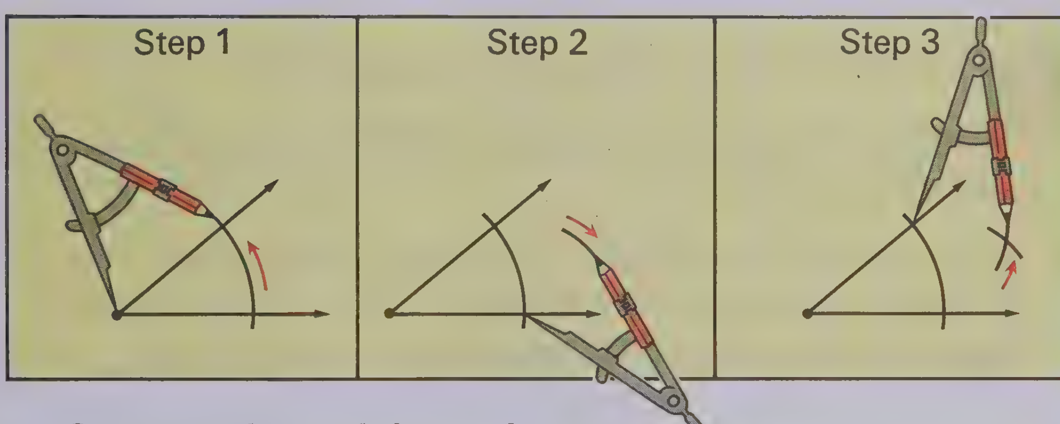


3. Construct a triangle that is the same size and shape as $\triangle PQR$.

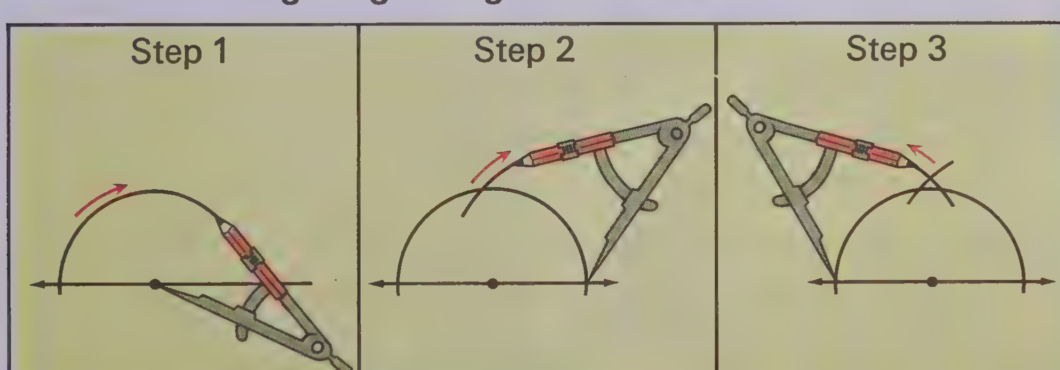


4. To "bisect" means to divide into two parts of the "same size." Study the first three steps and think about how to complete the construction in step 4. Then do each construction.

A Bisecting an angle

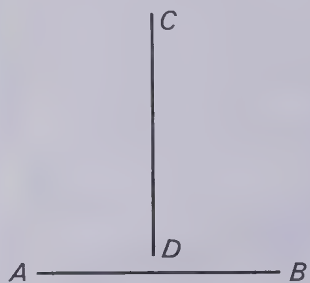


B Constructing a right angle

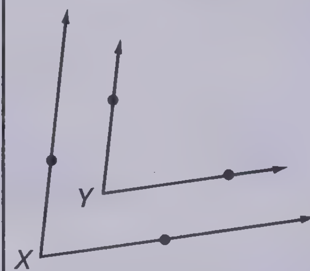


Investigating the Ideas

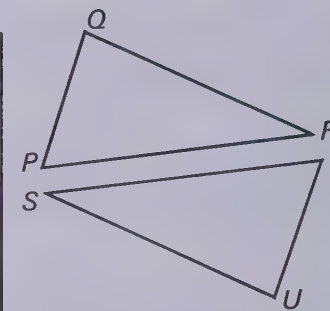
Answer each of these questions just by looking.



Is $\overline{AB}$ longer than, shorter than, or just as long as $\overline{CD}$?



Is $\angle X$ smaller than, larger than, or the same size as $\angle Y$?



Is $\triangle PQR$ the same size and shape as $\triangle UTS$?

? Can you find a way to check your answers?

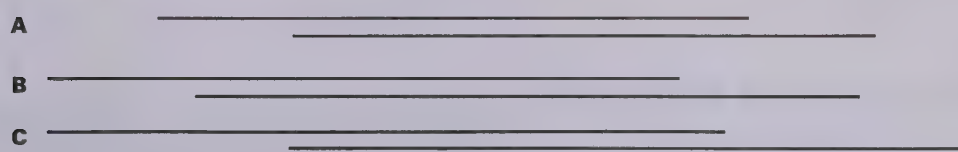
Discussing the Ideas

1. Two segments (like $\overline{AB}$ and $\overline{CD}$) are **congruent** if their end points are equally far apart. We write, "$\overline{AB} \cong \overline{CD}$," and read, "Segment AB is congruent to segment CD." How did you check to see if $\overline{AB}$ is congruent to $\overline{CD}$ above?

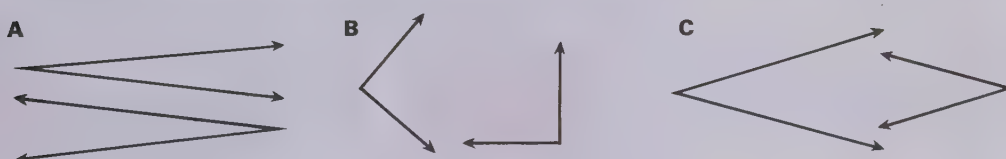
2. Two angles (like $\angle X$ and $\angle Y$) are **congruent** if "corresponding points" on their rays are equally far apart. We say, "Angle X is congruent to angle Y." We write, "$\angle X \cong \angle Y$." How could you use tracing paper or a compass to see if $\angle X$ above is congruent to $\angle Y$?

3. Two triangles (like $\triangle PQR$ and $\triangle UTS$) are congruent if the parts (angles and segments) of one can be matched with the parts of the other. How could you use tracing paper to see if $\triangle PQR \cong \triangle UTS$?

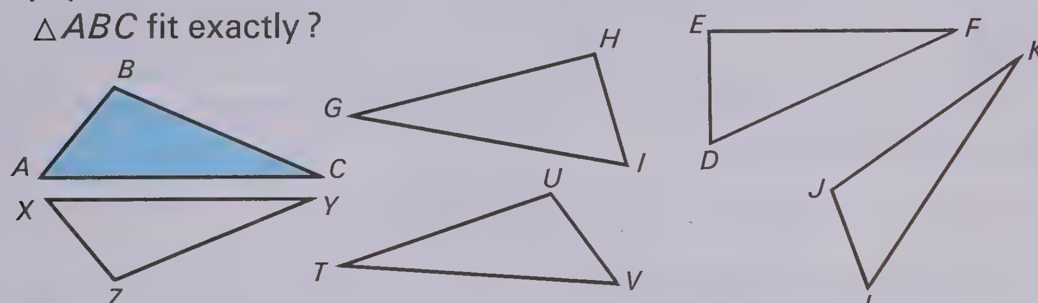
1. For each exercise, trace one segment and place it over the other segment. Tell whether the two segments are congruent.



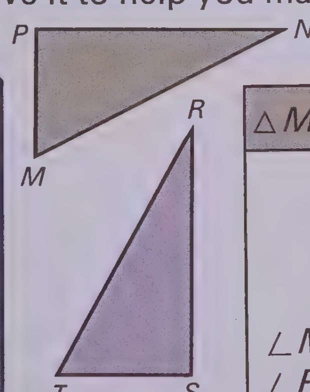
2. Trace each pair of angles to tell whether or not they are congruent.



3. Trace $\triangle ABC$. By sliding, turning, or flipping the tracing paper, on which of the other triangles can you make $\triangle ABC$ fit exactly?



4. Give the missing segments and angles in the table. You may want to trace $\triangle MPN$ and move it to help you match the segments and angles.



$\triangle MPN$	$\cong$	$\triangle TSR$
$\overline{MP}$	$\cong$	
	$\cong$	$\overline{SR}$
	$\cong$	$\overline{RT}$
	$\cong$	$\angle RTS$
$\angle MPN$	$\cong$	
$\angle PNM$	$\cong$	

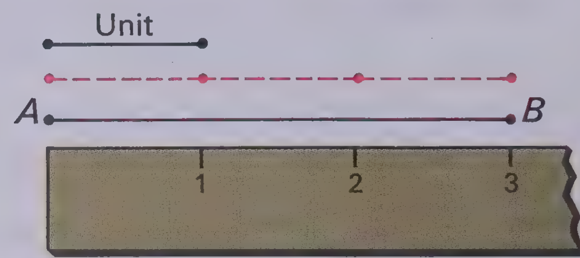
think

Arrange 7 dots so you can draw straight lines through 6 different sets of 3 dots.

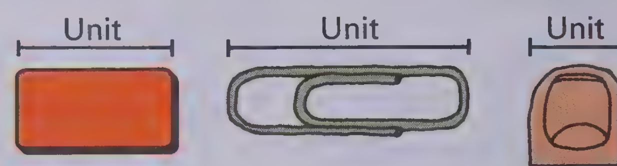
What are some units of length?

Investigating the Ideas

We count the **unit segments** to find the **length** of $\overline{AB}$. A **ruler** helps us to count units easily.



Choose a unit of length and make your own ruler.



?	
Can you use your ruler to find the lengths of some objects?	Make a record of the objects and their lengths.

Discussing the Ideas

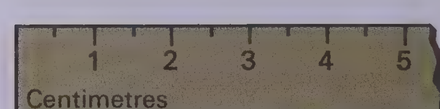
1. What is the length of $\overline{AB}$ shown above?

2. Barbara used the red-strip unit to measure $\overline{CD}$. Karen used the paper-clip unit and Elaine used the finger unit. Which girl got the largest number for the length of $\overline{CD}$? Who got the smallest number for the length?

3. The **centimetre** is a common unit of length.

A What is the width of your desk to the nearest centimetre?

B What is the length of your desk to the nearest centimetre?



4. Can you name some other common units of length?

1. Measure each of these segments to the nearest centimetre.

   A ______________ **B** ______________

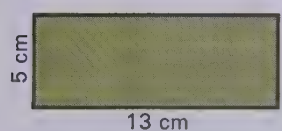
   C ______________

   D ______________

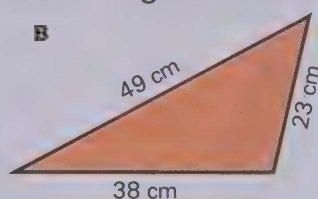
2. Find the length of each segment above to the nearest half centimetre.

3. The **perimeter** of a figure is the sum of the lengths of its sides. Find the perimeter of each figure below.

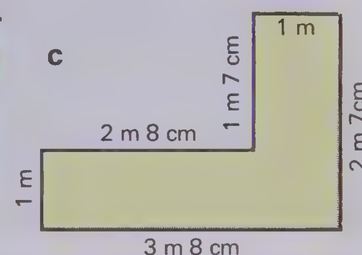
   A



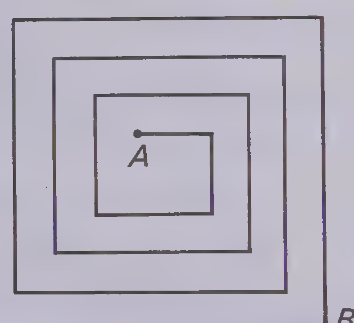
   B



   C

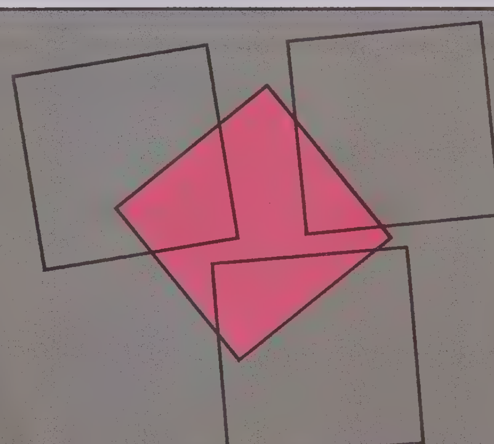


★ 4. Find the length of the path from A to B in centimetres.



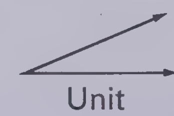
think

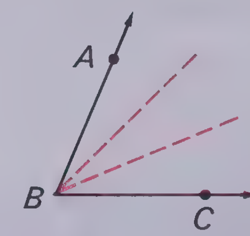
The figure shows three "clear" squares overlapping a red square of the same size. What is the largest number of "clear" squares that can overlap the red square if the "clear" squares cannot overlap?

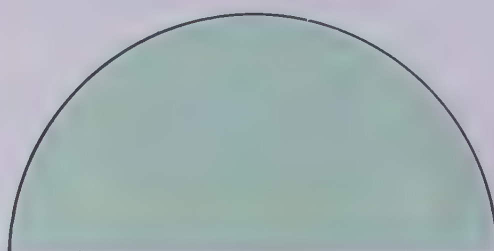


Investigating the Ideas

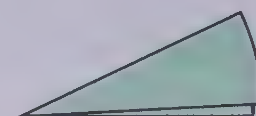
We count **unit angles** to measure an angle. Follow the instructions below to make your own **protractor** for measuring angles.



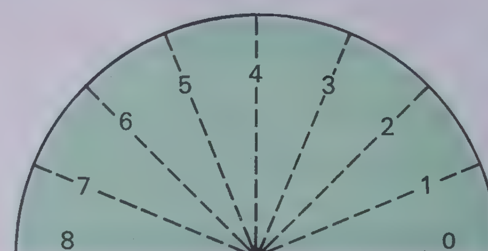




Cut out a semicircle from paper.



Fold the semicircle in half three times.

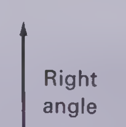


Number the fold lines for the unit angles on your protractor.

? Can you draw some angles and use your protractor to find their measures to the nearest unit?

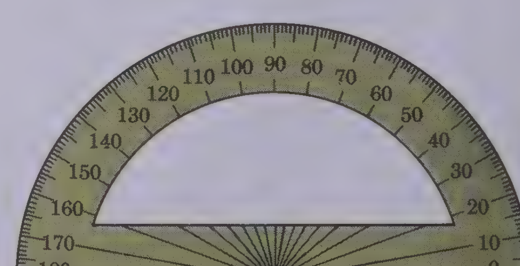
Discussing the Ideas

1. Explain how you would use your protractor to find the measure of this right angle.



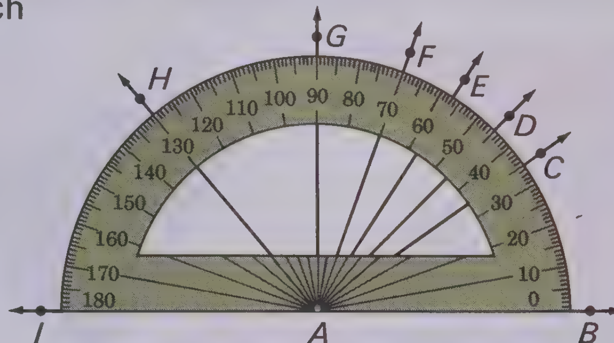
2. How could you use your protractor to draw an angle that measures 2 units? 5 units? 7 units?

3. A common unit of angle measure is called a **degree**. A protractor scaled in degrees is shown. What is the measure of a right angle in degrees?

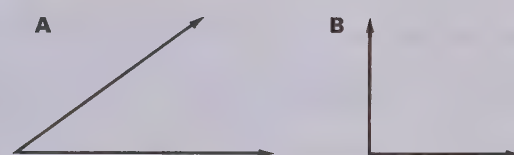


1. A protractor with degree units has been placed over some angles. $\angle BAC$ measures 35° (35 degrees). Give the measure of each angle below.

A	$\angle BAD$	G	$\angle DAG$
B	$\angle BAE$	H	$\angle DAH$
C	$\angle BAF$	I	$\angle FAG$
D	$\angle BAG$	J	$\angle GAH$
E	$\angle BAH$	K	$\angle GAI$
F	$\angle CAD$	L	$\angle FAI$



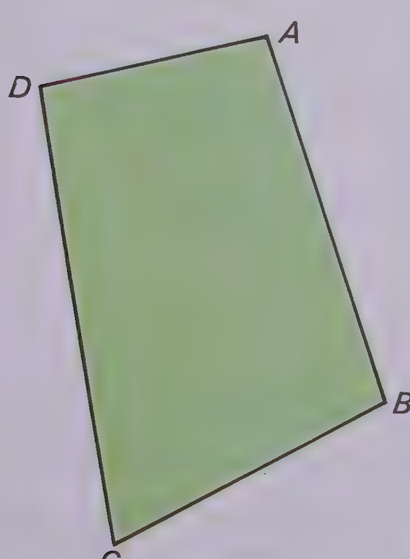
2. Use a protractor to find the degree measure of these angles.



3. Draw angles that have the following measures.

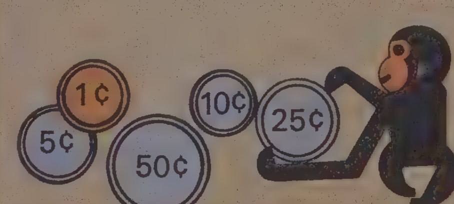
A 25° B 72° C 105° D 143° E 14°

★ **4.** Find the degree measure of each angle.
Find the length of each segment in centimetres.



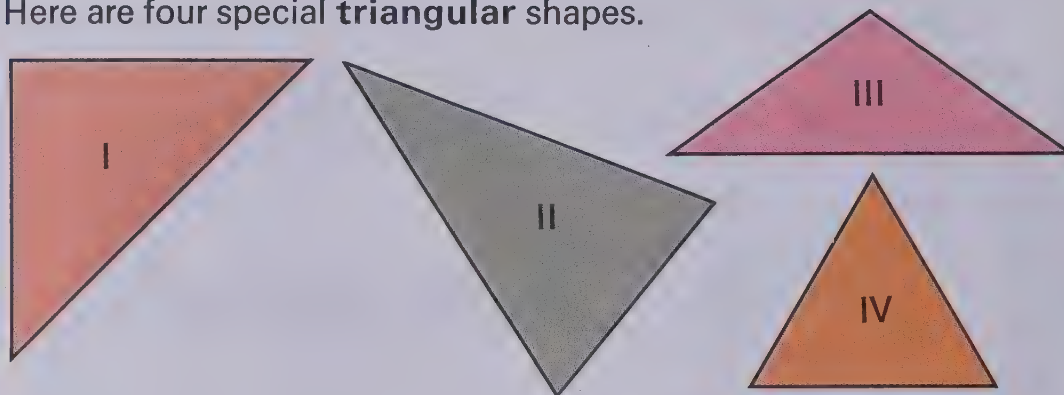
think

Jack has six coins. One third of his coins are dimes. The dimes are one fourth of the value of the coins. What coins does Jack have?



Investigating the Ideas

Here are four special **triangular** shapes.



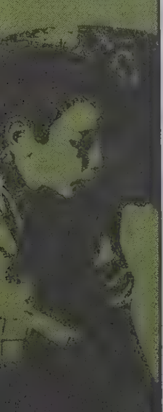
Here are ways to use these shapes to make other **polygons**.

Two I's → **square**	Two II's → **parallelogram**
Two III's → **rhombus**	Four I's → **rectangle**
One II, two III's → **pentagon**	Six IV's → **hexagon**

? Can you trace, cut out, and paste the triangular shapes to form the polygons?

Discussing the Ideas

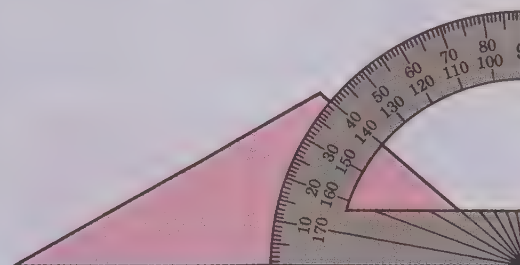
1. An **isosceles** triangle has at least two congruent sides. Which triangles above are isosceles?
2. An **equilateral** triangle has all sides congruent. Which triangle above is equilateral?
3. A **right triangle** has a 90° angle. Which triangle is a right triangle?
4. An **acute** angle is less than 90°. An **obtuse** angle is greater than 90°. Which triangles have acute angles? obtuse angles?
5. Describe each polygon you made. Can you think of other ways to use the triangles to make the polygons?



1. Use your compass to help you draw each of the following.
 A isosceles triangle B equilateral triangle C right triangle

2. Use your protractor to help you draw:
 A an acute angle B an obtuse angle C a right angle

3. A Draw a triangle large enough so that you can measure each angle easily. What is the sum of the measures of the three angles?

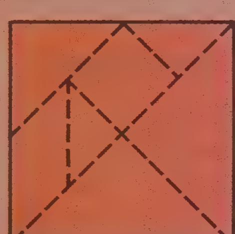


 B Try this again with a different triangle. What do you think might be true about the sum of the three angle measures for any triangle?

4. Draw a four-sided polygon (**quadrilateral**) on your paper. Make it large enough so that you can measure the angles easily. What is the sum of the four angle measures?

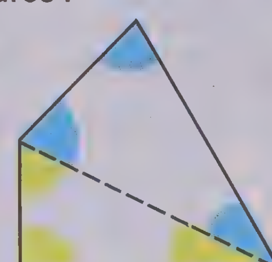
think

Trace this tangram square and cut it into the pieces shown.



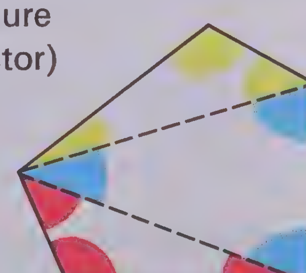
How many of the polygons named in the Investigation on page 72 can you make with all 7 tangram pieces?

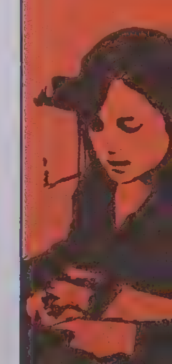
★ **5.** A What is the sum of the angles of each triangle?



 B What is the sum of the angles of the quadrilateral?

★ **6.** Use this figure (no protractor) to find the sum of the angles of this pentagon.

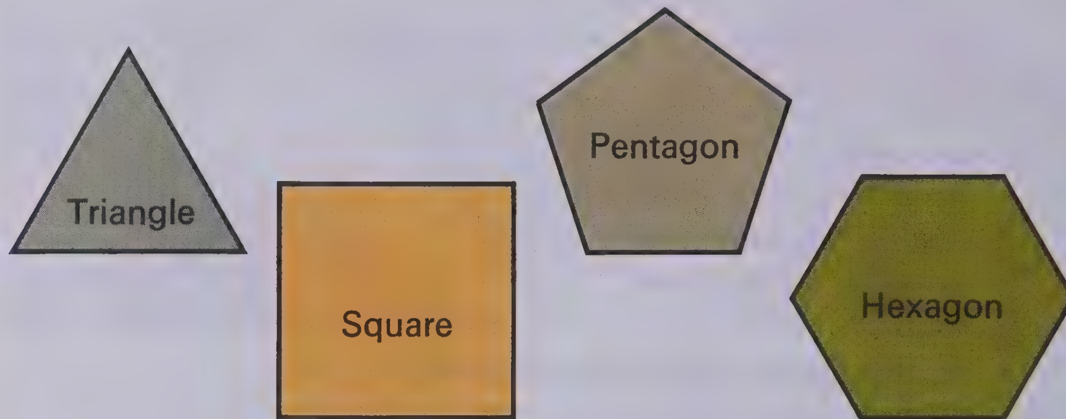




How many lines of symmetry does a polygon have?

Investigating the Ideas

Trace and cut out each of these regular polygon shapes.



? In how many ways can you fold each figure so one half fits exactly upon the other half?

Discussing the Ideas

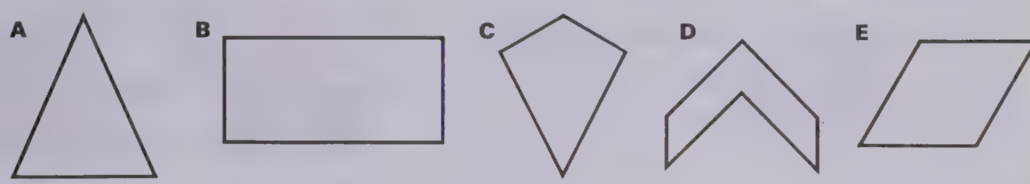
1. A figure has a **line of symmetry** if the fold on that line makes two halves of the figure match exactly. How many lines of symmetry does each figure above have?

2. A Can you name some other types of triangles and tell how many lines of symmetry each figure has?
   B Can you name some other quadrilaterals and tell how many lines of symmetry each one has?

3. A **regular polygon** has all sides congruent and all angles congruent. If you know how many sides a regular polygon has, can you tell how many lines of symmetry the polygon has?

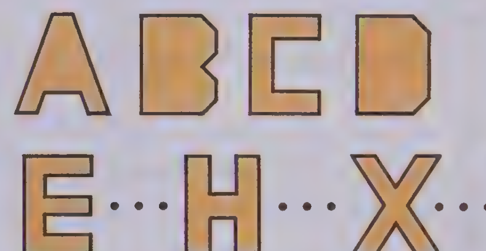
4. Figures with at least one line of symmetry are called **symmetrical**. Can you describe a familiar symmetrical figure?

1. How many lines of symmetry does each figure have?

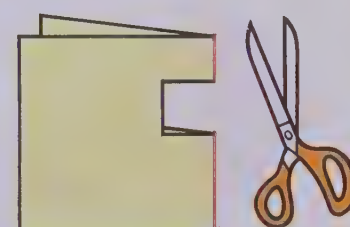


isosceles triangle — rectangle — "kite" — "chevron" — rhombus

2. Think about special polygons that are the outlines of the 26 letters of the alphabet. How many of these shapes have one or more lines of symmetry?

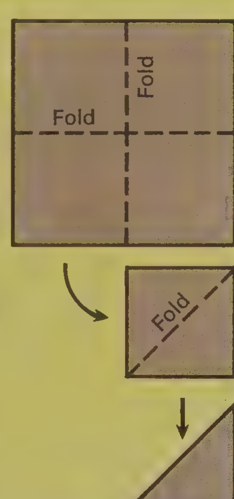


3. Suppose you fold a piece of paper and cut out the figures shown below. Draw and name the figure that is formed when you unfold each piece.



think

Cut out a square and fold it twice to form a small square. Fold the small square to form a triangle. Can you cut off one corner of the triangle so that it unfolds to form a square?



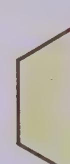
A

Cut out a square.

B

Cut out an isosceles right triangle.

C



Cut out an isosceles trapezoid.

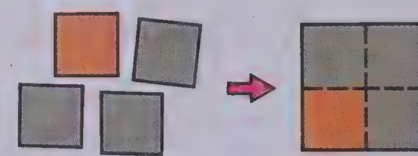
D

Cut out this quadrilateral.

More practice, page A-4, Set 8

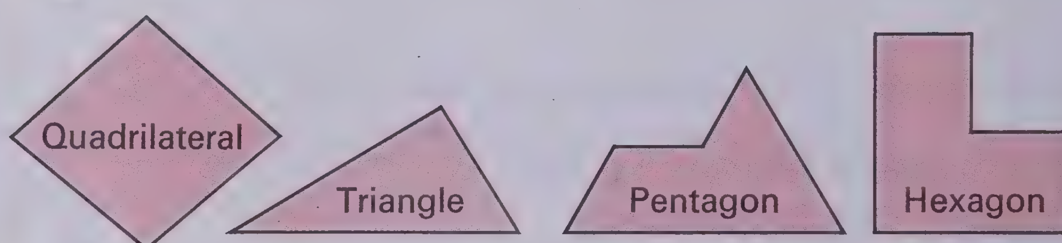
Investigating the Ideas

You can put four squares together to form a larger square of the same shape as the small ones.



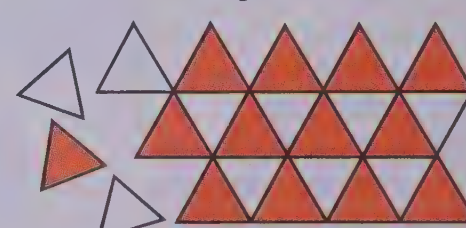
You can put four equilateral triangles together to form a larger equilateral triangle of the same shape as the small ones.

? Can you show a way to put four of each of these polygons together to form a larger polygon of the same shape?



Discussing the Ideas

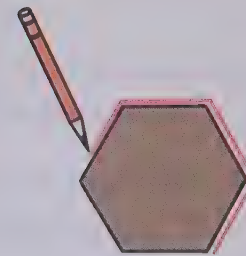
1. Two geometric figures that have the same shape are said to be **similar** to each other. Give examples of similar figures.



2. Here is a pattern formed by fitting together equilateral triangles.
   A Imagine that you had as many triangles as you needed. Could you completely cover any surface?
   B Patterns which can "cover any surface" are called **tessellations**. Explain why we can be sure that each polygon above could make a tessellation.

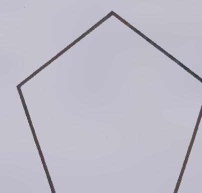
Using the Ideas

1. Make a cardboard cut-out of this regular hexagon. Then you can draw around it to make pictures of hexagons. Can you use regular hexagons to make a tessellation?



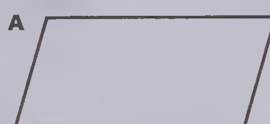
2. You have seen that you can make tessellations with at least three regular polygons. Which ones?

3. Complete exercise 1 by using this pentagon. Do you think tessellations can be made with any of the other regular polygons?

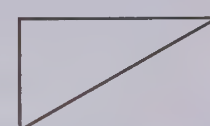


★ 4. Can you make and color two different tessellations with each of these figures?

A

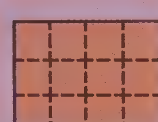


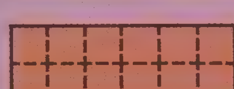
B



think

If you use twelve squares for each rectangle, you can form rectangles of three different shapes.



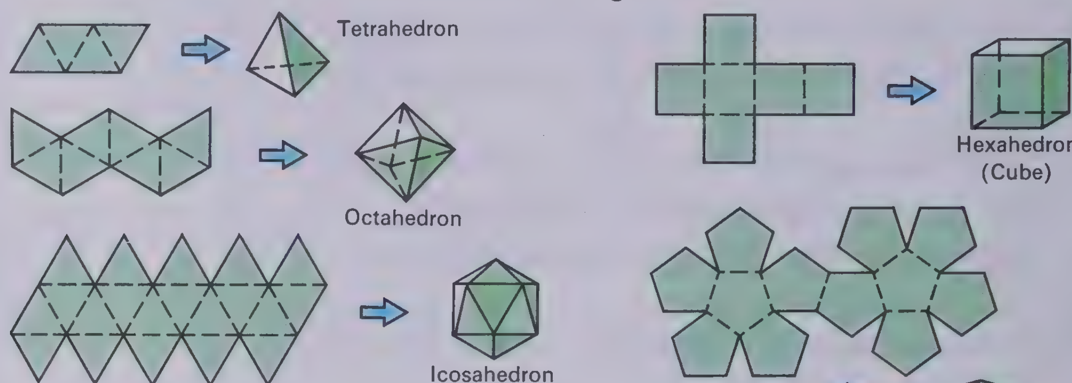


What is the smallest number of squares you can use to form rectangles of four different shapes?

● *Let's look at space figures made from regular polygons.*

Investigating the Ideas

The ancient Greeks discovered that there are only five space figures with faces that are congruent regular polygons. Here is how models of these solids might be made.



? Can you complete the table below for these five regular polyhedrons?

Name of the polyhedron	*Name of the face polygon*	*Number of faces*	*Number of vertices*	*Number of edges*

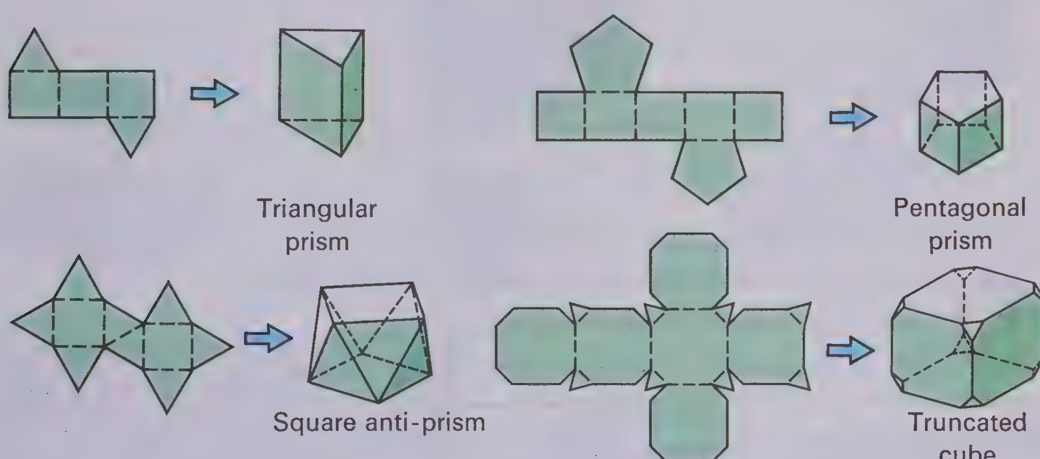
Discussing the Ideas

1. How can you easily find the number of faces that each of the five regular polyhedrons has?
2. If "poly-" means "many," "tetra-" means "four," "hexa-" means "six," and "octa-" means "eight," then what do you think "dodeca-" and "icosa-" mean?
3. For each polyhedron, find the sum of the number of faces and the number of vertices. How does this sum compare with the number of edges? Can you write a formula about what you have found?

Using the Ideas

1. A Draw a pattern that could be used to make a tetrahedron. Make it different from the pattern on page 78.
   B Draw another pattern for a cube.

2. Each face of a regular polyhedron is a regular polygon. Each face of a regular polyhedron is congruent to every other face. Here are some polyhedrons.
   A Why are they not regular polyhedrons?
   B How are they like regular polyhedrons?



Triangular prism

Pentagonal prism

Square anti-prism

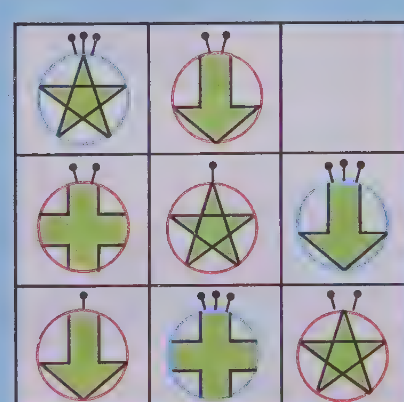
Truncated cube

3. A How many vertices, faces, and edges does each polyhedron in exercise 2 have?
   B If a polyhedron had 7 faces and 7 vertices, how many edges would it have?

★ 4. Make a model of at least one of the polyhedrons in exercise 2.

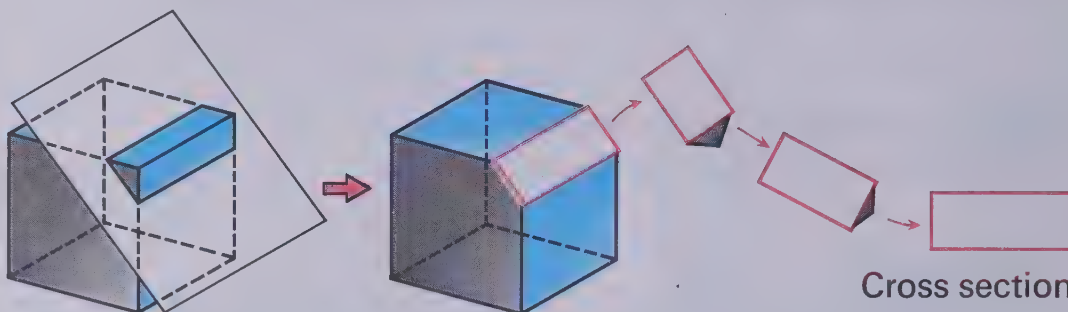
think

Draw the missing figure.



Investigating the Ideas

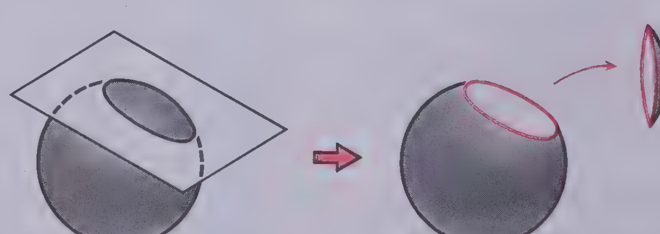
The figures show how we can cut a cube to get a cross section that is a rectangle.



? Can you draw some of the cross sections that you could make with other cuts?

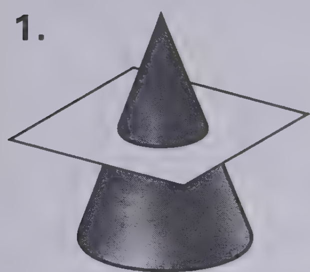
Discussing the Ideas

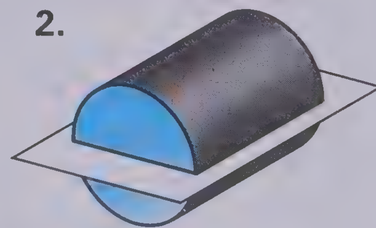
1. The plane geometric figure that is the intersection of a space figure and a plane is often called a cross section It may help you to think of "slicing" the figure and then looking at the edge of your cut. Can you think of another way to cut the above cube so that the cross section is a rectangle?

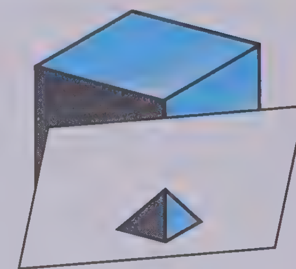


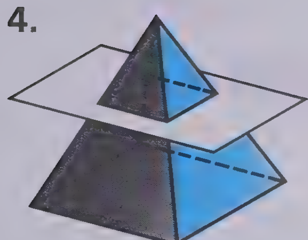
2. A Describe this cross section of the sphere.
   B Is there a way to cut a sphere so that the cross section is a different type of figure?

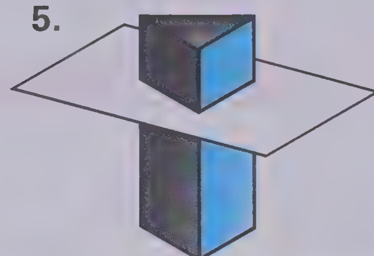
For each exercise, draw the cross section formed by the intersection of the plane and the space figure.

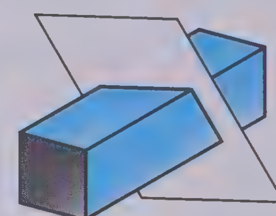
1. 

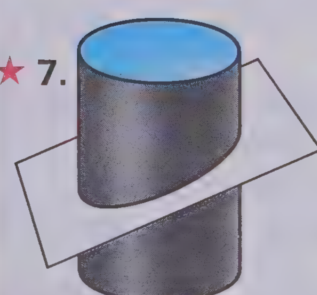
2. 

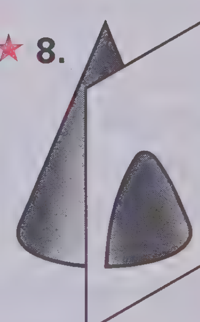
3. 

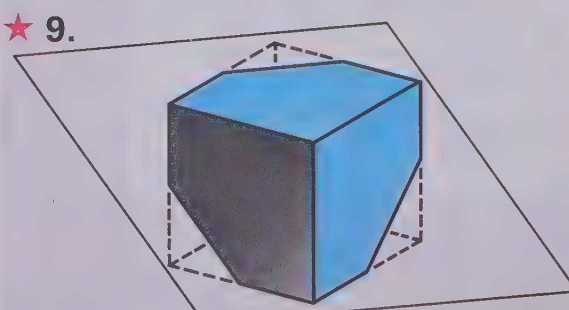
4. 

5. 

6. 

★ 7. 

★ 8. 

★ 9. 

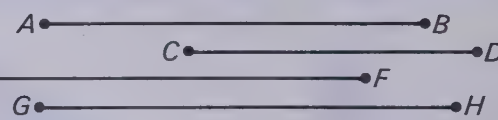
think

How many years old
is a person who
has lived one
million hours?

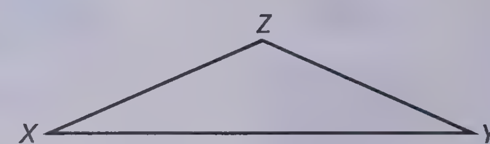
GUESS!
Check your guess.
How close did you come
to the correct answer?

Reviewing the Ideas

1. Which pair of segments are congruent?

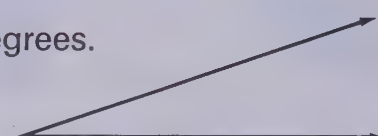


2. An isosceles triangle has a pair of congruent angles. $\triangle XYZ$ is isoceles.
   A Which two angles are congruent?
   B Which pair of sides are congruent?



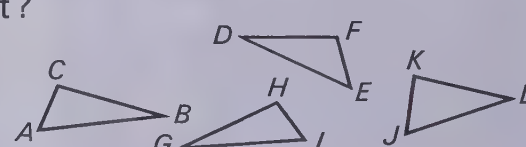
3. Find the length of the pencil
   A to the nearest centimetre.
   B to the nearest half centimetre.

4. A Find the measure of this angle in degrees.
   B Construct an angle congruent to this angle.

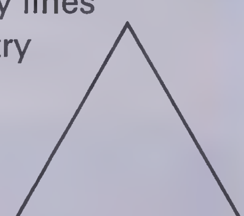


5. Is the sum of the angles of a triangle 90°, 100°, 180°, or 200°?

6. A Which two triangles are congruent?
   B Complete each statement.
   $\angle A \cong$ |||||| $\angle B \cong$ |||||| $\angle C \cong$ ||||||
   $\overline{AB} \cong$ |||||| $\overline{BC} \cong$ |||||| $\overline{AC} \cong$ ||||||



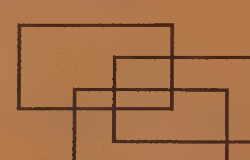
7. A How many lines of symmetry does an equilateral triangle have?



   B Can you tessellate a plane with an equilateral triangle?

think

Draw a figure like the one below. Now trace over the entire figure by drawing a path that does not cross itself or retrace itself.



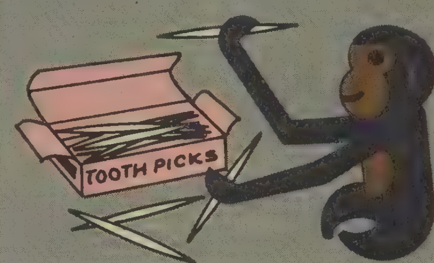
Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Place value
Principles

1. Give the number for n.
   - A $10\,000 = 10^n$
   - B $100\,000\,000 = 10^n$
   - C $1000 \times 100\,000 = 10^n$
   - D $4000 \times 60\,000 = 24 \times 10^n$
   - E $30\,000 \times 90\,000 = 27 \times 10^n$

think

Take 18 toothpicks (or small sticks) and arrange them to form 9 triangles of the same size.



2. Solve the equations.

A $a + 7 = 15$	G $42 \div 7 = d$
B $14 - n = 8$	H $f + 9 = 18$
C $5 \times 9 = r$	I $7 \times x = 49$
D $c \times 8 = 48$	J $8 + 9 = a$
E $s - 6 = 7$	K $q \div 3 = 7$
F $12 \div y = 6$	L $10 - b = 0$

3. Solve the equations.
   - A $5 \times 27 = (5 \times 20) + (5 \times n)$
   - B $6 \times 284 = (6 \times 200) + (6 \times n) + (6 \times 4)$

4. Find the products and quotients.

A 10×100	E 56×1000	I $480 \div 60$	M $2400 \div 8$
B 20×700	F 200×300	J $4800 \div 60$	N $3700 \div 10$
C 50×60	G 5000×200	K $4800 \div 600$	O $3700 \div 100$
D 800×40	H 760×20	L $480 \div 6$	P $2600 \div 200$

5. Solve the equation. $6 + 6 + 6 + 6 + 6 = n \times 6$

6. Solve the equation. $(6 \times 30) + (6 \times 7) = 6 \times n$

7. Solve:

A $\begin{array}{r} 347 \\ \times 4 \\ \hline \end{array}$	B $\begin{array}{r} 24 \\ \times 39 \\ \hline \end{array}$	C $\begin{array}{r} 3948 \\ +2765 \\ \hline \end{array}$	D $8\overline{)344}$	E $16\overline{)592}$	F $\begin{array}{r} 8403 \\ -1679 \\ \hline \end{array}$

You are invited to explore

ACTIVITY CARD 3
Page 356

5 Computing

How are your adding and subtracting skills?

Investigating the Ideas

Let's check your adding and subtracting skills.

1. $\begin{array}{r} 47 \\ +35 \\ \hline \end{array}$	**2.** $\begin{array}{r} 97 \\ +75 \\ \hline \end{array}$	**3.** $\begin{array}{r} 216 \\ +438 \\ \hline \end{array}$	**4.** $\begin{array}{r} 876 \\ +359 \\ \hline \end{array}$	**5.** $\begin{array}{r} 462 \\ 657 \\ +536 \\ \hline \end{array}$
6. $\begin{array}{r} 93 \\ -47 \\ \hline \end{array}$	**7.** $\begin{array}{r} 543 \\ -127 \\ \hline \end{array}$	**8.** $\begin{array}{r} 824 \\ -569 \\ \hline \end{array}$	**9.** $\begin{array}{r} 605 \\ -386 \\ \hline \end{array}$	**10.** $\begin{array}{r} 7003 \\ -3586 \\ \hline \end{array}$

? Can you find the sums and differences above without missing more than one?

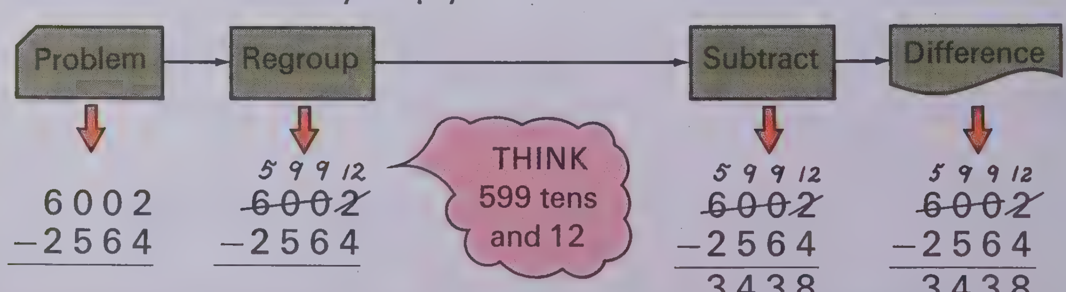
Use this code to grade your own paper:

a = 0 c = 2 e = 4 g = 6 i = 8
b = 1 d = 3 f = 5 h = 7 j = 9

Answers: **1.** ic **2.** bhc **3.** gfe **4.** bcdf **5.** bgff
6. eg **7.** ebg **8.** cff **9.** cbj **10.** debh

Discussing the Ideas

1. If you missed the subtraction problem with two zeros, this flow chart may help you correct it.



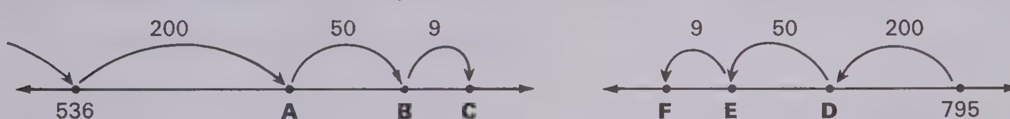
Study the chart and correct your mistakes.

2. Can you make a flow chart for Investigation problem 9?

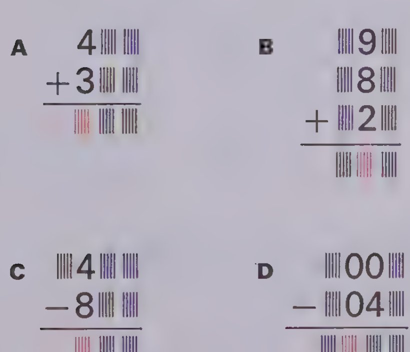
1. If you need practice in adding and subtracting, find these sums or differences.

A $\begin{array}{r}28\\+57\\\hline\end{array}$	B $\begin{array}{r}65\\+93\\\hline\end{array}$	C $\begin{array}{r}78\\+69\\\hline\end{array}$	D $\begin{array}{r}239\\+446\\\hline\end{array}$	E $\begin{array}{r}968\\+875\\\hline\end{array}$	F $\begin{array}{r}6472\\+3869\\\hline\end{array}$
G $\begin{array}{r}56\\75\\+81\\\hline\end{array}$	H $\begin{array}{r}934\\369\\+548\\\hline\end{array}$	I $\begin{array}{r}675\\87\\+948\\\hline\end{array}$	J $562 + 78 + 843 + 7651$	K $7659 + 8321 + 78 + 642$	
L $\begin{array}{r}86\\-49\\\hline\end{array}$	M $\begin{array}{r}80\\-52\\\hline\end{array}$	N $\begin{array}{r}96\\-39\\\hline\end{array}$	O $\begin{array}{r}732\\-415\\\hline\end{array}$	P $\begin{array}{r}653\\-287\\\hline\end{array}$	Q $\begin{array}{r}9247\\-1638\\\hline\end{array}$
R $\begin{array}{r}906\\-238\\\hline\end{array}$	S $\begin{array}{r}5037\\-1699\\\hline\end{array}$	T $\begin{array}{r}8007\\-5679\\\hline\end{array}$	U $562-371$	V $6003-706$	W $8076-1384$
X $7000-256$					

2. Give the number for the point above each letter.

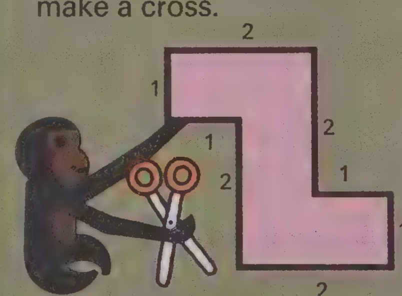


★ **3.** In each problem, some of the digits are covered with screens. In each exercise, give all possible digits that could be under the red screens.



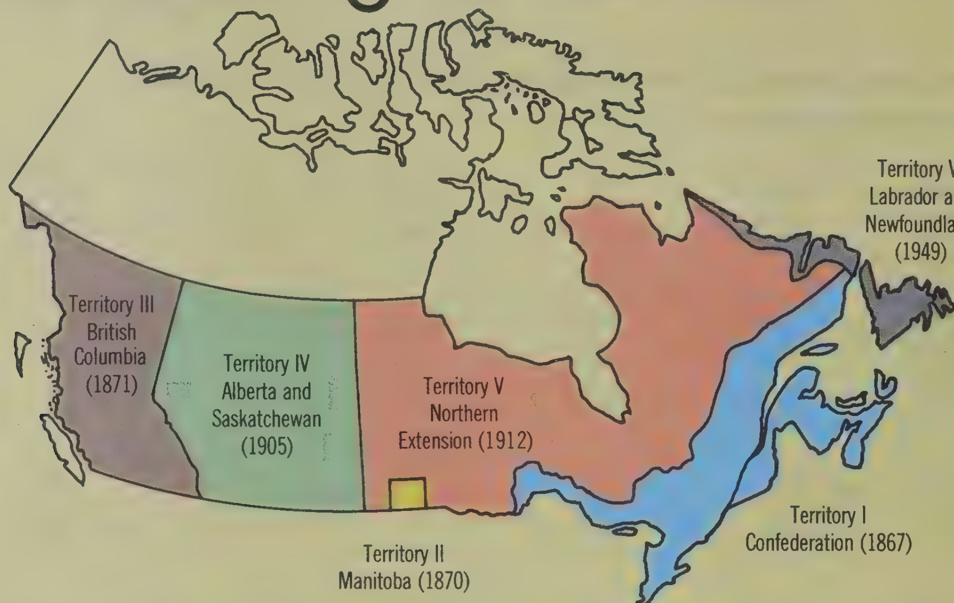
think

Trace and cut out four z-shaped figures like the one below. Show how to fit them together to make a cross.



More practice, page A-5, Set 9

The Making of Canada



1. When Prince Edward Island entered Confederation, Territory I had been established for only six years. In what year did Prince Edward Island enter Confederation ?
2. How long after Territory I did the last territory, Territory VI, enter Confederation ?
3. How long was it after 1873 before a new territory was added ?
4. The provinces which formed Territory 1 spread northward in 1912. How long had this area been a part of Confederation ?
5. In which territory is your province ? How long has it been a member of Confederation ?
6. In 1869, Canada bought the Northwest Territories. How long ago was that ?
7. In 1873, A. MacKenzie became prime minister of Canada. He was 51 years old. He lived to the age of 70. Give the year of his birth and the year of his death.

★ 8. The Canadian Pacific Railway Co. was established in 1880 and inaugurated its first transcontinental railway 19 years after the signing of the British North America Act. How long had the Canadian Pacific been established at that time ?

Vacation Trips

Route	Distance
From Cape Breton to Halifax	428 (km)
From the Bay of Fundy to Moncton	69
From 1000 Islands to Toronto	265
From Georgian Bay to Toronto	142
From Mt. Riding to Winnipeg	253
From Banff to Calgary	105
From Jasper to Edmonton	327

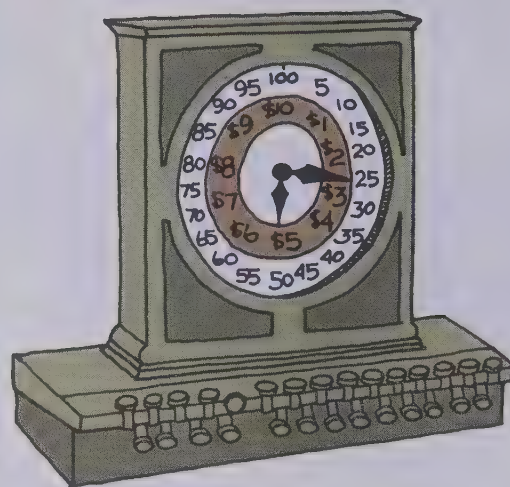
1. Use the map and the chart to give the distance
   - A From Regina to Jasper National Park by way of Edmonton ?
   - B From Toronto to Mt. Riding by way of Winnipeg ?
   - C From Montreal to the Cape Breton Highlands by way of Halifax ?
   - D From Ottawa to Banff by way of Calgary ?
   - E From Moncton to the Georgian Bay Islands by way of Toronto ?
   - F From Vancouver to Mt. Riding by way of Winnipeg ?
   - G From St. John to the Thousand Islands by way of Toronto ?

★ 2. Make use of the map to indicate the shortest distance
   - A From Vancouver to Georgian Bay.
   - B From St. John to Mt. Riding.
   - C From Winnipeg to the Bay of Fundy.

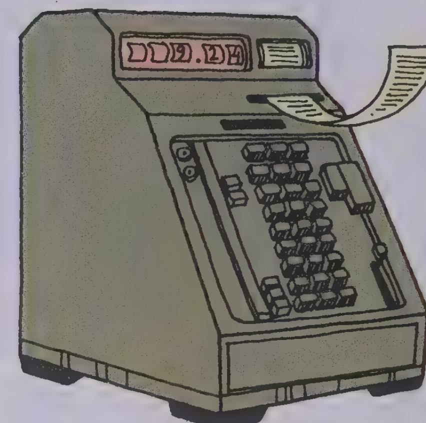
★ 3. Prepare a trip going from your home to one of the points on the map. Give the distance covered.

Can you find money totals and differences?

Investigating the Ideas



One of the first cash registers



A modern cash register

? Can you give three items whose prices would add up to the total shown on each cash register?

Discussing the Ideas

1. Explain how to read the total amount on the old cash register.

2. Modern cash registers show the amount for each purchase, the total amount, and how much change the buyer should receive.
   - A Suppose a cash register rings up these two purchases. Explain how to find the total it would show. $\square\square\boxed{7}.\boxed{8}\boxed{6}$ $\square\square\boxed{5}.\boxed{9}\boxed{2}$
   - B Explain how to find what the machine will show for your change if you pay with a 20-dollar bill.
   - C Explain how to find the amount of your purchase if this is the amount of change you get from a 10-dollar bill. Change: $\square\square\boxed{8}.\boxed{2}\boxed{1}$

1. Find the total amounts.

A	B	C	D	E
\$2.98	\$9.82	\$5.68	\$15.67	\$117.88
3.65	6.54	6.47	83.47	654.92
7.82	7.29	9.98	16.59	846.98

2. Find the differences in the amounts.

A	B	C	D	E
\$5.67	\$4.82	\$19.21	\$765.42	\$9.37
2.91	3.46	12.59	186.50	6.54

F	G	H	I	J
\$8.00	\$20.00	\$100.00	\$10.00	\$5.00
1.29	16.98	37.98	0.75	0.98

3. Use dollar notation to write each amount.

A 125¢	C 1298¢	E 5¢	G 50¢
B 700¢	D 95¢	F 10¢	H 25¢

Short Stories Money

1 Had 1 dollar 25 cents.
Spent 79 cents.
How much left?

2 Had \$12.53.
Earned 5 dollars and
a quarter. How much now?

3 Had 10 dollars. Spent 3 dollars.
Then spent \$2.79.
How much left?

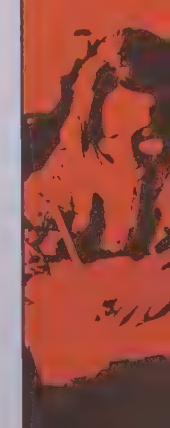
4 15 dollars in the bank.
Withdrew \$10.98.
How much left in the bank?

5 \$27.35 in the bank.
Deposited \$3.95.
How much in the bank now?

6 Had a quarter.
Earned 50 cents.
Spent 35 cents.
How much left?

7 Had a dollar.
Spent 49 cents.
Lost a quarter.
How much left?

8 Had 5 dollars. Spent \$2.98.
Earned \$1.25. How much now?



Investigating the Ideas

Let's check your multiplying skills.

1.	2.	3.	4.	5.
$\begin{array}{r} 65 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 96 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 287 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 709 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 5978 \\ \times 9 \\ \hline \end{array}$

? Can you find these products without missing any of them?

Use the code on page 84 to check your answers.
Answers: 1. bjf 2. hgi 3. bbei 4. ejgd 5. fdiac

Discussing the Ideas

1. Study the steps for multiplying 576 × 4 in the charts below. Then correct any mistakes you made in the Investigation.

$4 \times 6 = 24$ →	$4 \times 70 = 280$ →	$4 \times 500 = 2000$ →	Add
A $\begin{array}{r} 576 \\ \times 4 \\ \hline \mathbf{24} \end{array}$	$\begin{array}{r} 576 \\ \times 4 \\ \hline 24 \\ \mathbf{280} \end{array}$	$\begin{array}{r} 576 \\ \times 4 \\ \hline 24 \\ 280 \\ \mathbf{2000} \end{array}$	$\begin{array}{r} 576 \\ \times 4 \\ \hline 24 \\ 280 \\ 2000 \\ \hline 2304 \end{array}$
B $\begin{array}{r} 5\overset{2}{7}6 \\ \times 4 \\ \hline 4 \end{array}$	$\begin{array}{r} \overset{3}{5}\overset{2}{7}6 \\ \times 4 \\ \hline 04 \end{array}$	$\begin{array}{r} \overset{3}{5}\overset{2}{7}6 \\ \times 4 \\ \hline 2304 \end{array}$	You did the adding as you did each step.

2. Explain each step of the shortcut shown in chart B.

1. Find the products.

A $\begin{array}{r} 56 \\ \times 3 \\ \hline \end{array}$	B $\begin{array}{r} 72 \\ \times 6 \\ \hline \end{array}$	C $\begin{array}{r} 59 \\ \times 5 \\ \hline \end{array}$	D $\begin{array}{r} 62 \\ \times 8 \\ \hline \end{array}$	E $\begin{array}{r} 47 \\ \times 6 \\ \hline \end{array}$
F $\begin{array}{r} 159 \\ \times 9 \\ \hline \end{array}$	G $\begin{array}{r} 328 \\ \times 6 \\ \hline \end{array}$	H $\begin{array}{r} 527 \\ \times 2 \\ \hline \end{array}$	I $\begin{array}{r} 806 \\ \times 7 \\ \hline \end{array}$	J $\begin{array}{r} 564 \\ \times 4 \\ \hline \end{array}$
K $\begin{array}{r} 392 \\ \times 7 \\ \hline \end{array}$	L $\begin{array}{r} 564 \\ \times 8 \\ \hline \end{array}$	M $\begin{array}{r} 306 \\ \times 7 \\ \hline \end{array}$	N $\begin{array}{r} 530 \\ \times 9 \\ \hline \end{array}$	O $\begin{array}{r} 654 \\ \times 5 \\ \hline \end{array}$

P $\begin{array}{r} 1326 \\ \times 3 \\ \hline \end{array}$	Q $\begin{array}{r} 5804 \\ \times 6 \\ \hline \end{array}$	R $\begin{array}{r} 6001 \\ \times 5 \\ \hline \end{array}$	S $\begin{array}{r} 7643 \\ \times 8 \\ \hline \end{array}$

2. Find the product for ***a***. Then give the product for ***b***.

A $6 \times 84 = a \rightarrow 60 \times 84 = b$

B $7 \times 53 = a \rightarrow 70 \times 53 = b$

C $6 \times 364 = a \rightarrow 60 \times 364 = b$

D $7 \times 156 = a \rightarrow 70 \times 156 = b$

3. Find the products.

A $\begin{array}{r} 48 \\ \times 70 \\ \hline \end{array}$	B $\begin{array}{r} 63 \\ \times 80 \\ \hline \end{array}$	C $\begin{array}{r} 57 \\ \times 30 \\ \hline \end{array}$	D $\begin{array}{r} 69 \\ \times 50 \\ \hline \end{array}$	E $\begin{array}{r} 846 \\ \times 90 \\ \hline \end{array}$	F $\begin{array}{r} 562 \\ \times 30 \\ \hline \end{array}$

4. Find the products.

A 500×3

B 400×7

C 200×9

D 600×8

E 800×6

F 700×5

G 600×9

H 300×4

5. Estimate the products. Then find the products and check your estimates.

A $\begin{array}{r} 599 \\ \times 3 \\ \hline \end{array}$	B $\begin{array}{r} 807 \\ \times 6 \\ \hline \end{array}$	C $\begin{array}{r} 1087 \\ \times 4 \\ \hline \end{array}$

think

Find these products.

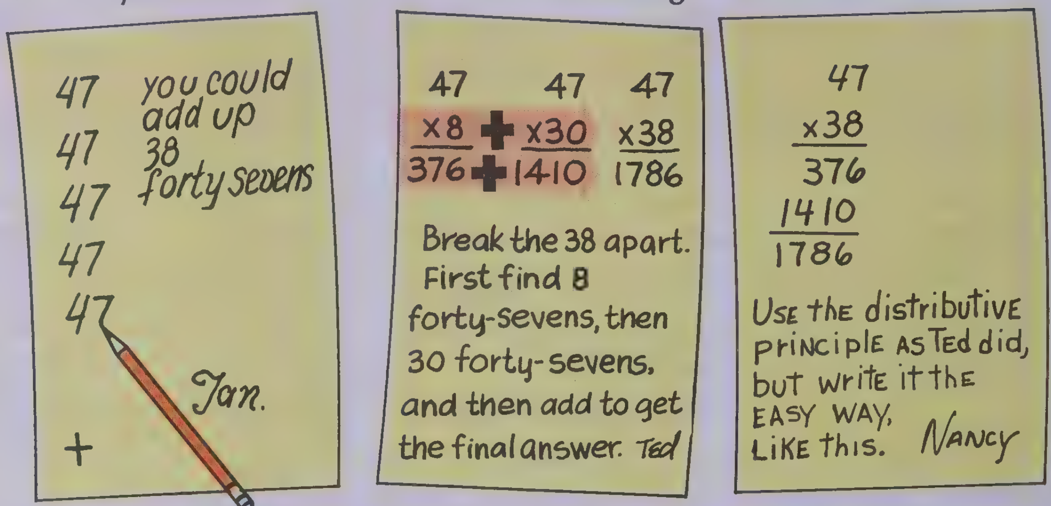
$\begin{array}{r} 142\,857 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 142\,857 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 142\,857 \\ \times 4 \\ \hline \end{array}$
$\begin{array}{r} 142\,857 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 142\,857 \\ \times 6 \\ \hline \end{array}$	

What pattern can you find in these products? Guess what this product will be. Find the product. Were you correct?

$\begin{array}{r} 142\,857 \\ \times 7 \\ \hline \end{array}$

Investigating the Ideas

How do you find 38 × 47 ? Three students gave these methods.



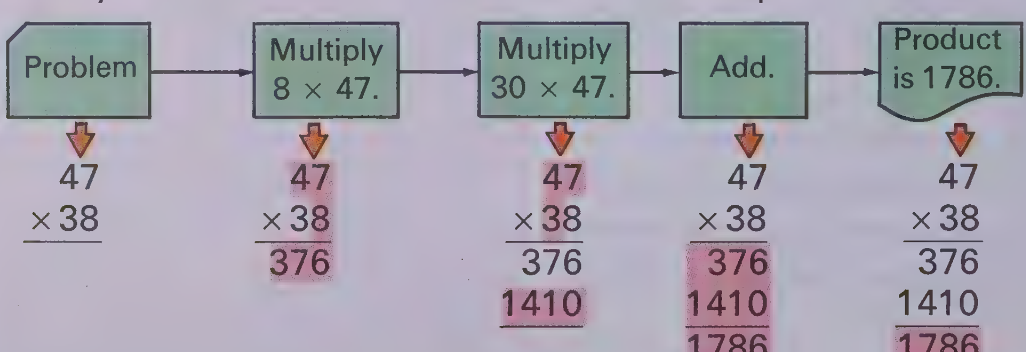
? Can you use one of these methods to find 26 × 34 ?

Discussing the Ideas

1. Why are Ted's and Nancy's methods more useful than Jan's?

2. A Explain how Ted's and Nancy's methods are alike.
   B How do these methods use the distributive principle?

3. Study the flow chart and use it to find another product.



1. Find the products.

A 36×48	B 59×27	C 78×35
D 26×39	E 342×87	F 647×98
G 826×48	H 392×64	I 822×41

2. For each exercise, give the products for ***a*** and ***b***.

A $6 \times 827 = 4962 \rightarrow$ $60 \times 827 = a$, $600 \times 827 = b$

B $4 \times 391 = 1564 \rightarrow$ $40 \times 391 = a$, $400 \times 391 = b$

C $7 \times 285 = 1995 \rightarrow$ $70 \times 285 = a$, $700 \times 285 = b$

3. Find the products.

A	347×2	347×60	347×300	347×362
B	641×6	641×20	641×400	641×426
C	917×4	917×70	917×500	917×574
D	186×7		186×800	186×807
E	342×5	342×30	342×900	342×935
F	253×8		253×300	253×308

4. Find the products.

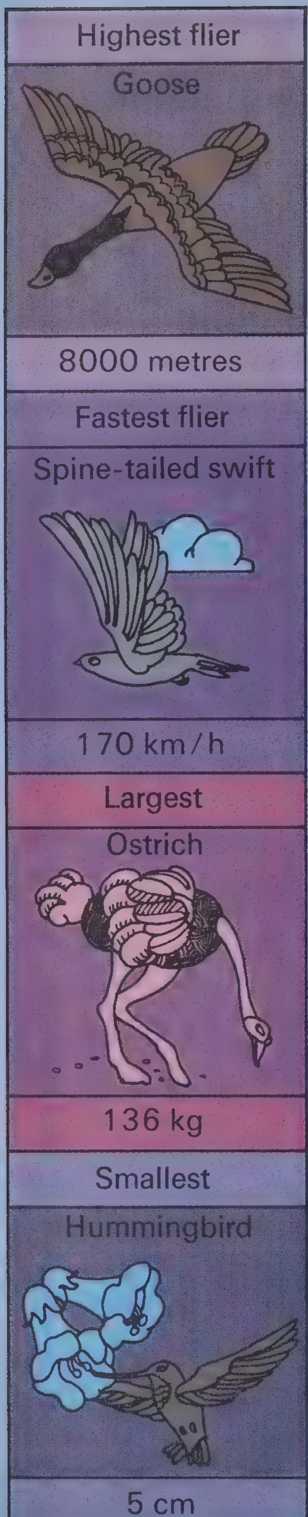
A 384×264	B 462×397	C 570×642
D 809×817	E 6125×256	F 928×407

think

Give the missing digits for this multiplication example.

$$\begin{array}{r} 932 \\ \times \;\square\square \\ \hline \square\square\square \\ \square\square\;\square\square\square \\ \hline 56\,\square\square\square \end{array}$$

More practice, page A-5, Set 10



BIRDS

1. Many modern jet aircraft fly as high as 11 000 metres. How much higher is this than the highest-flying goose ?

2. Some jet aircraft travel about 6 times as fast as the spine-tailed swift. How fast is this ?

3. How many 5-cm hummingbirds would have to be laid end-to-end to reach across the 3.6-metre wingspread of the albatross ?

4. One of the oldest men on record lived to be 113 years old. How much older is this than the "old" raven ?

5. How many grams does the ostrich weigh ?

6. Each year the arctic tern migrates 17 700 km each way from the Arctic to the Antarctic and back. How far would the tern travel in 6 years ?

---

*All records given are approximations and subject to dispute.

7. If an ostrich could run at his top speed for half an hour, how many metres would he travel ?

8. The largest flying bird weighs about 18 kg. It would take about 424 hummingbirds to weigh 1 kilogram. How many hummingbirds would it take to weigh 18 kilograms ?

9. The canvasback duck flies about 48 km / h faster than the ostrich can run. How many metres does the duck fly in an hour ?

10. An ostrich egg weighs about $1\frac{1}{2}$ kg. A hummingbird egg weighs about $1\frac{1}{2}$ milligrams. How many hummingbird eggs would it take to weigh as much as one ostrich egg ?

★ 11. The penguin is a bird that does not fly. Some penguins can swim as fast as 32 km / h. How many metres per second is this ?

★ 12. Sparrow eggs hatch in about 12 days. It takes 5 times that long for penguin eggs. Chicken eggs take 39 days less than penguin eggs. How much time does it take for penguin eggs and chicken eggs to hatch ?

Greatest wingspread

Albatross

3.6 metres

Fastest runner

Ostrich

65 km / h

Oldest

Raven

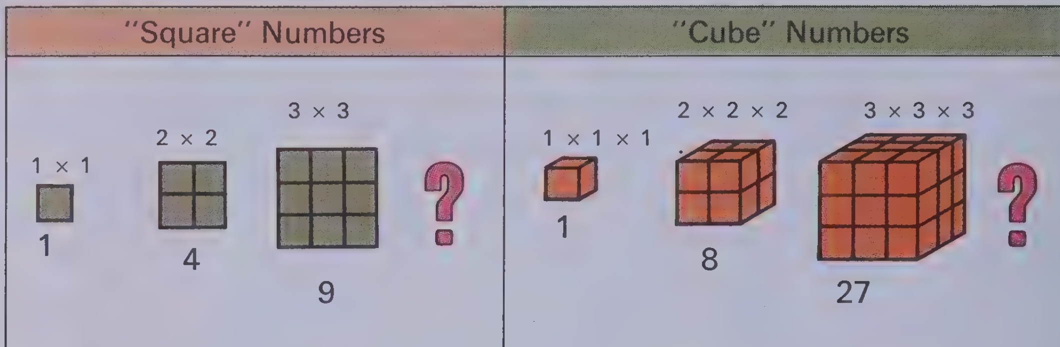
69 years

Greatest traveller

Tern

17 700 km

Investigating the Ideas



?	Can you find 3 more "square" numbers and 3 more "cube" numbers?	Draw pictures for them.

Discussing the Ideas

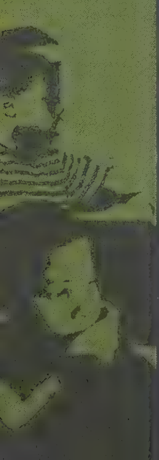
1. A How did you find the next three "square" numbers? "cube" numbers?

   B Why do you think it is said that 3 squared is 9, and 2 cubed is 8?

2. Study this chart. Then write and read some symbols like these.

For this symbol	we think	we write	we say
5^3	$5 \times 5 \times 5$	$5^3 = 125$	"5 cubed is 125."
4^2	4×4	$4^2 = 16$	"4 squared is 16."
3^4	$3 \times 3 \times 3 \times 3$	$3^4 = 81$	"3 to the fourth power is 81."

3. When we write 2^3, the "3" is an **exponent.** It tells how many times the **base** 2 is used as a factor. What are the bases and exponents for the examples in the chart above?



1. Give the missing words and numbers.
   - A 3^2 means that 3 is used as a factor __?__ times. $3^2 =$ ▒.
   - B 4^3 means that 4 is used as a factor __?__ times. $4^3 =$ ▒.
   - C 2^5 means that 2 is used as a factor __?__ times. $2^5 =$ ▒.
   - D 10^4 means that 10 is used as a factor __?__ times. $10^4 =$ ▒.

2. Use an exponent to write each of the following.
   - A 3×3 Answer: 3^2
   - B $2 \times 2 \times 2 \times 2 \times 2 \times 2$
   - C $5 \times 5 \times 5$
   - D $4 \times 4 \times 4 \times 4$
   - E $10 \times 10 \times 10 \times 10 \times 10$
   - F $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

3. Give the missing numbers. Example: $6^2 = 36$
   - A $7^2 =$ ▒
   - B $2^3 =$ ▒
   - C $4^3 =$ ▒
   - D $3^2 =$ ▒
   - E $3^4 =$ ▒
   - F $2^4 =$ ▒
   - G $5^2 =$ ▒
   - H $6^3 =$ ▒
   - I $7^1 =$ ▒
   - J $4^4 =$ ▒

4. Give the correct number for each sentence.
   - A If $a = 7$, then $a^2 =$ ▒.
   - B If $x = 3$, then $x^3 =$ ▒.
   - C If $m = 4$, then $m^2 =$ ▒.
   - D If $r = 2$, then $r^4 =$ ▒.

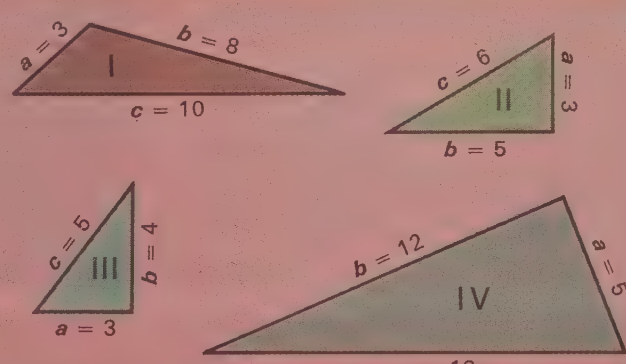
5. Mark **T** (true) or **F** (false).
   - A $5^2 = 2 \times 5$
   - B $4^5 = 4 \times 4 \times 4 \times 4 \times 4$
   - C $3^7 = 3 \times 7$
   - D $5^3 = 125$

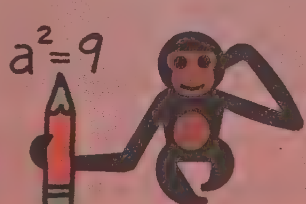
★ 6. Is 2^{15} more or less than 15 000? Guess. Then check your guess.

think

For which triangles is this equation true?

$$a^2 + b^2 = c^2$$





1. Find the quotients.

A $24 \div 4$	C $54 \div 9$	E $5400 \div 9$	G $6300 \div 7$	I $420 \div 6$
B $240 \div 4$	D $540 \div 9$	F $63 \div 7$	H $42 \div 6$	J $4200 \div 6$

2. Find the largest possible whole number for n.

A $n \times 40 < 243$	E $n \times 30 < 220$	I $n \times 70 < 400$
B $n \times 50 < 360$	F $n \times 60 < 375$	J $n \times 30 < 100$
C $n \times 80 < 331$	G $n \times 40 < 375$	K $n \times 40 < 300$
D $n \times 20 < 161$	H $n \times 90 < 730$	L $n \times 60 < 500$

3. Find the largest multiple of 10 for n.

A $n \times 20 < 1150$	D $n \times 30 < 1280$	G $n \times 80 < 3700$
B $n \times 60 < 2300$	E $n \times 40 < 3300$	H $n \times 90 < 5800$
C $n \times 50 < 2400$	F $n \times 50 < 2700$	I $n \times 70 < 6100$

4. Give the number for n.

A $3682 = (n \times 100) + 82$
B $5823 = (n \times 10) + 3$
C $9176 = (n \times 100) + 76$

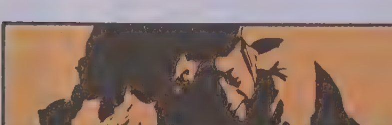
5. For each of the following equations, tell whether the solution is a 1-, 2-, or 3-digit number.

A $n \times 35 = 805$	F $n \times 26 = 182$
B $n \times 35 = 8050$	G $n \times 26 = 1820$
C $n \times 47 = 94$	H $n \times 76 = 684$
D $n \times 47 = 940$	I $n \times 76 = 6840$
E $n \times 47 = 9400$	J $n \times 34 = 2720$

think

Each equation below has two solutions from the set: $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Solve the equations.

1. $(n \times n) + 10 = 7 \times n$
   Answer: $n = 2$ or $n = 5$
2. $(n \times n) + 12 = 7 \times n$
3. $(n \times n) + 21 = 10 \times n$
4. $(n \times n) + 35 = 12 \times n$
5. $(n \times n) + 54 = 15 \times n$
6. $(n \times n) = 6 \times n$



You are invited to explore

ACTIVITY CARD 4
Page 357

TEMPERATURE

1 Iron melts at 1535°C. How much hotter is this than the melting point of ice?

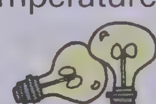
2 The melting point of lead is 1187°C cooler than that of iron. What is this melting point?

3 The melting point of gold is 472°C less than the melting point of iron. Give this temperature.

4 The melting point of paraffin is 39°C greater than average room temperature. What is it?

6 The boiling point of alcohol is 77.8°C. How much less is this than the boiling point of water?

5 Tungsten is used in light bulbs since it has a very high melting point. This melting point is about 153 times the average room temperature. Find it.



7 Metals have a boiling point just as they have a melting point. Iron boils at a temperature that is 1465°C hotter than its melting point. What is the boiling point of iron? (See exercise 1.)

★ **8** Mercury is liquid at room temperature. It has a melting point of $^{-}38.8$°C. How much does this differ from the melting point of ice?

★ **9** Mercury boils at 356.9°C. What is the difference between the melting and boiling points of mercury?

Investigating the Ideas

Let's check your dividing skills when the divisor has 1 digit.

1. $9\overline{)63}$ **2.** $4\overline{)96}$ **3.** $6\overline{)378}$ **4.** $9\overline{)7659}$ **5.** $8\overline{)5209}$

? Can you find the quotients above without missing more than one?

Use the code on page 84 to check your answers.

Answers **1.** h **2.** ce **3.** gd **4.** ifb **5.** gfb, R b

Discussing the Ideas

1. This example shows that to find the **quotient** (726), you have to find the greatest number of times the **divisor** (6) can be subtracted from the **dividend** (4359).

A How many sixes were subtracted the first time? How many in all?

B What was the remainder?

C Can you describe another way this quotient might have been found?

$$
\begin{array}{rrl}
 & 726 & \\
6) & 4359 & \\
 & -4200 & \leftarrow 700 \times 6 \\
 & 159 & \\
 & -120 & \leftarrow 20 \times 6 \\
 & 39 & \\
 & -36 & \leftarrow 6 \times 6 \\
 & 3 &
\end{array}
$$

2. This flow chart shows a way (called "short division") to find the quotient when the divisor is less than 10. Explain how to use this method to find one of the quotients above.

Problem 4356 ÷ 6 → Divide the hundreds. 43 ÷ 6 = 7, R1 → Divide the tens. 15 ÷ 6 = 2, R3 → Divide the ones. 36 ÷ 6 = 6 → The quotient is 726.

$6\overline{)4356}$ $\quad 6\overline{)4\,3^{1}5\,6}$ with 7 above $\quad 6\overline{)4\,3^{1}5^{3}6}$ with 7 2 above $\quad 6\overline{)4\,3^{1}5^{3}6}$ with 7 2 6 above $\quad \begin{array}{r}726\\6\overline{)4356}\end{array}$

Study this example. Then find the quotients and remainders below. Check your work.

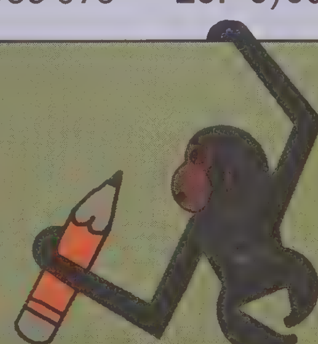
Step 1	Step 2
Think about **hundreds:** $16 \div 8 = 2$ $\begin{array}{r} 2 \\ 8\overline{)1649} \end{array}$	Think about **tens:** There are 0 eights in 4. The remainder is 4. $\begin{array}{r} 20 \\ 8\overline{)1649} \end{array}$
Step 3	**Check**
Think about **ones:** $49 \div 8$ Quotient, 6 Remainder, 1 $\begin{array}{r} 206 \text{ R1} \\ 8\overline{)1649} \end{array}$	$\begin{array}{r} 206 \\ \times 8 \\ \hline 1648 \end{array} \rightarrow \begin{array}{r} 1648 \\ +1 \\ \hline 1649 \end{array}$

1. $2\overline{)568}$	**2.** $5\overline{)755}$	**3.** $4\overline{)2964}$	**4.** $3\overline{)6389}$
5. $7\overline{)6321}$	**6.** $8\overline{)5642}$	**7.** $9\overline{)6584}$	**8.** $6\overline{)4224}$
9. $4\overline{)3478}$	**10.** $8\overline{)6666}$	**11.** $7\overline{)432}$	**12.** $5\overline{)5305}$
13. $6\overline{)39\,872}$	**14.** $9\overline{)65\,363}$	**15.** $3\overline{)18\,654}$	**16.** $4\overline{)607}$
17. $7\overline{)9305}$	**18.** $8\overline{)42\,603}$	**19.** $9\overline{)55\,079}$	**20.** $6\overline{)60\,704}$

think

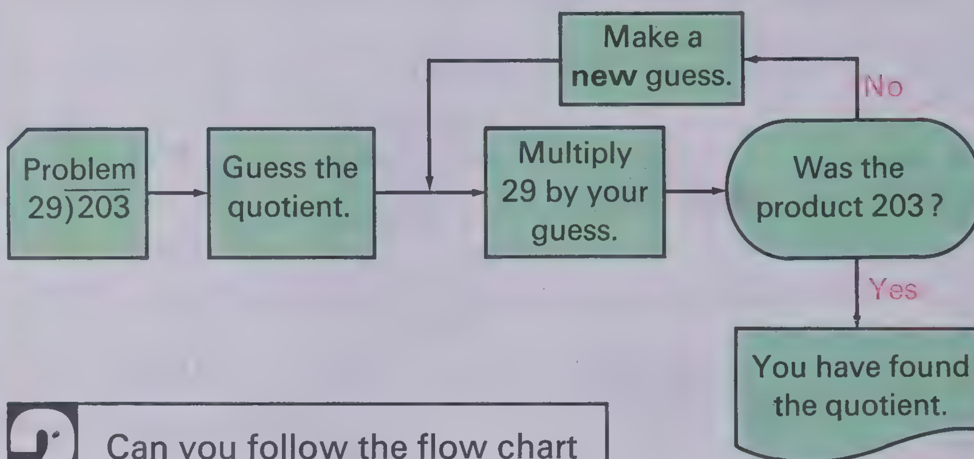
Some of the digits of this example are covered. Copy this example and give the missing digits.

$$\begin{array}{r} 43 \\ \times\ \blacksquare\blacksquare \\ \hline \blacksquare\blacksquare\blacksquare \\ \blacksquare\blacksquare\blacksquare \\ \hline \blacksquare\blacksquare\blacksquare 4 \end{array}$$



Let's explore divisors less than 100.

Investigating the Ideas



? Can you follow the flow chart and find the quotient ?

Discussing the Ideas

$n \times 30 \leq 243$
? is the largest number for n.

1. How many estimates did you make before you found the quotient above ?

2. Find the largest number for n in the diagram. Explain why 30 was used in this estimation. What is the remainder ?

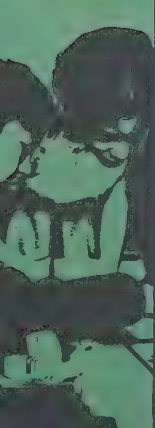
$$\begin{array}{r} ? \\ 29\overline{)\ 243} \\ -232 \\ \hline 11 \end{array} \quad ?$$

3. $28\overline{)126}$ $27\overline{)126}$ $26\overline{)126}$ $25\overline{)126}$

   $31\overline{)267}$ $32\overline{)267}$ $33\overline{)267}$ $34\overline{)267}$

   Since **30** is "close" to each of the divisors above, we usually use 30 to help estimate quotients in problems such as these. Explain how 30 is used to estimate the quotients above. Find each quotient and remainder. Explain your work.

4. State a general rule for choosing multiples of 10 to use in estimating quotients.



1. Find the number for n. Then use this estimate to help you find the quotient.

A $n \times 40 \leq 294$
$38\overline{)294}$

B $n \times 60 \leq 372$
$64\overline{)372}$

C $n \times 30 \leq 208$
$26\overline{)208}$

2. Find the quotients and remainders. Check your work.

A $40\overline{)234}$	B $35\overline{)247}$	C $15\overline{)124}$	D $39\overline{)317}$
E $29\overline{)178}$	F $77\overline{)658}$	G $76\overline{)692}$	H $45\overline{)300}$
I $17\overline{)153}$	J $52\overline{)467}$	K $39\overline{)210}$	L $44\overline{)246}$
M $13\overline{)85}$	N $38\overline{)293}$	O $23\overline{)107}$	P $85\overline{)637}$
Q $50\overline{)371}$	R $60\overline{)500}$	S $94\overline{)758}$	T $12\overline{)113}$

3. Find the divisor for each exercise. The quotient is given.

A $\begin{array}{r} 72 \\ \square\overline{)648} \end{array}$ B $\begin{array}{r} 94 \\ \square\overline{)564} \end{array}$ C $\begin{array}{r} 42 \\ \square\overline{)294} \end{array}$ D $\begin{array}{r} 59 \\ \square\overline{)177} \end{array}$ E $\begin{array}{r} 504 \\ \square\overline{)504} \end{array}$

★ **4.** For each exercise, find one number that can serve as both quotient and divisor.

A $\begin{array}{r} \square\square \\ \square\square\overline{)100} \end{array}$ B $\begin{array}{r} \square\square \\ \square\square\overline{)144} \end{array}$

C $\begin{array}{r} \square\square \\ \square\square\overline{)400} \end{array}$ D $\begin{array}{r} \square\square \\ \square\square\overline{)900} \end{array}$

E $\begin{array}{r} \square\square \\ \square\square\overline{)225} \end{array}$ F $\begin{array}{r} \square\square \\ \square\square\overline{)625} \end{array}$

think

Since it takes the planet Mercury 88 of our days to travel once around the sun, a "year" on Mercury is 88 of our days. If a person were 10 years old, give his age in "Mercury years." Use 365 days for an "Earth year."

More practice, page A-6, Set 11

Investigating the Ideas

$a \times 40 \le 2752$	$b \times 40 \le 172$	
$43\overline{)2752}$	$43\overline{)172}$	→ $\begin{array}{r} ?\,? \\ 43\overline{)2752} \end{array}$

Find the largest multiple of 10 for ***a***.
Then find the largest single-digit number for ***b***.

? Can you use this information and find the quotient for $2752 \div 43$?	Record your work.

Discussing the Ideas

1. How did you use the information above to find the quotient?

2. If you become skillful in estimating quotients, you will be able to find quotients more easily. Study the example. To find the quotient for $3990 \div 58$, think

   A What is the largest multiple of 10 that will make the sentence true?

   B Explain how the number you found in part A is used in step 1 of the example.

   C To find $510 \div 58$ and complete the dividing, think

   D What is the largest number that will make the sentence true?

   E Explain how the number you found in part D is used in step 2 to complete the dividing.

   F What are the quotient and remainder? Check your work.

$? \times 60 \le 3990$

Step 1

$$\begin{array}{r} 58\overline{)\ 3990} \\ -3480 \\ \hline 510 \end{array} \quad 60$$

$? \times 60 \le 510$

Step 2

$$\begin{array}{r} 68 \\ 58\overline{)\ 3990} \\ -3480 \\ \hline 510 \\ -464 \\ \hline 46 \end{array} \quad \begin{array}{l} \\ \\ 60 \\ \\ \\ 8 \\ \\ \end{array}$$

1. When you estimate quotients, you will find that your estimate does not always give the correct number. Often your estimate will be too large, and you will have to choose a smaller number and try again. Other times your estimate will be too small, and you will have to subtract an extra time. Each of these cases is illustrated in the examples below. In each exercise, find the largest multiple of 10 for n. Then use this estimate and find the quotient.

A	B	C
$n \times 50 \le 1395$	$n \times 10 \le 936$	$n \times 30 \le 900$
$45\overline{)1395}$	$13\overline{)936}$	$27\overline{)900}$

2. Find the quotients and remainders. Check your work.

A $19\overline{)1670}$	B $35\overline{)2586}$	C $15\overline{)483}$	D $77\overline{)2567}$
E $52\overline{)835}$	F $17\overline{)554}$	G $43\overline{)3000}$	H $56\overline{)3875}$
I $31\overline{)2863}$	J $13\overline{)876}$	K $16\overline{)1253}$	L $98\overline{)1000}$
M $79\overline{)894}$	N $12\overline{)158}$		
O $85\overline{)6550}$	P $18\overline{)907}$		

3. Find the quotients and remainders.

A $5\overline{)4000}$	B $30\overline{)500}$
C $7\overline{)5836}$	D $9\overline{)80\,000}$
E $62\overline{)494}$	F $8\overline{)30\,000}$
G $80\overline{)700}$	H $11\overline{)558}$
I $3\overline{)5967}$	J $16\overline{)349}$

think

A rock dropped from the top of a tall building will fall about:
4.9×1^2 m in 1 sec
4.9×2^2 m in 2 sec
4.9×3^2 m in 3 sec
$4.9 \times n^2$ m in n sec
(use any number for n.)
How many seconds (to the nearest second) would it take a rock to fall from the top of a building 176.4 metres tall?

More practice, page A-6, Set 12

Investigating the Ideas

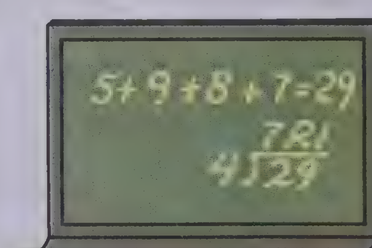
A $4 + 8 + 9 = 21$	C $5 + 12 + 6 + 13 = 36$
B $5 + 9 + 10 = 24$	D $20 + 24 + 32 + 36 = 112$

In example A, when 7 is substituted for each addend, the sum is the same as before.

? Can you find a number that will substitute for each addend and give the same sum for each of the other examples?

Discussing the Ideas

1. Since 7 will substitute for each addend and give the same sum in example A, we say that 7 is the **average** (or **arithmetic mean**) of 4, 8, and 9. How did you find the average of 20, 24, 32, and 36?
2. Give the missing words. Then give an example to explain what the following sentence means. To find the arithmetic mean of a set of numbers, divide the __?__ of the numbers by the __?__ of addends.
3. The chalkboard shows Tom's work in finding the arithmetic mean of 4 numbers.



   A If you substitute 7 for each addend, will the sum be the same?

   B Can any other whole number be substituted for each addend to give a sum closer to 29? We say that the arithmetic mean of 5, 9, 8, and 7 is 7 (to the nearest whole number).
4. Explain how to find the average of 9, 4, 7, 8, and 5 (to the nearest whole number).

Sports Stories

1. Jack played 9 holes of golf.
   A What was his total score?
   B What was his average score for each hole?

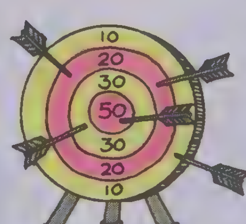
Score Card					
Hole	1	2	3	4	5
Score	4	4	6	3	5
Hole	6	7	8	9	Total
Score	7	6	6	4	

2. Nancy and Carol each bowled three games.
   A Find Nancy's total score.
   B Find Nancy's average score.
   C Find Carol's total score.
   D Find Carol's average score.

NAME	Score			
	1	2	3	Total
Nancy	108	131	118	
Carol	114	121	98	

3. Joe read that a player on the Montreal Expos baseball team averaged one hit for every three times at bat. If he was at bat 294 times, how many hits did he make?

4. What is the average score for the arrows on the archery target?



5. A Find the average score for the home team.

Basketball Scores						
Home	49	50	61	44	68	52
Opponent	52	44	46	45	42	51

   B Find the opponents' average score to the nearest whole point.

Solving Story Problems

The sea contains millions of cubic kilometres of water. This water contains billions of plants and animals and tons of minerals. The sea is one of the great natural resources for water, food, power, and valuable minerals.

Ocean	Approximate Area (in square kilometres)
Pacific	178 154 000
Atlantic	81 631 000
Indian	73 414 000
Antarctic	14 838 000
Arctic	14 084 000

1. How many square kilometres larger is the Antarctic Ocean than the Arctic Ocean ?

2. The total area of Earth's surface is 509 903 500 square kilometres. Use the table above to find the total surface area covered by oceans. About how many square kilometres of Earth's surface is covered by land ?

3. Sailors used to measure the depth of the ocean by dropping a weighted rope overboard. As they pulled it up, they measured by using their outstretched arms as a unit.
   The unit was about 1.8 metres long.
   A The deepest part of the ocean is 10.8 kilometres. How many units ?
   B The sea is 4.14 km deep at the North Pole. How many units ?

Depth (in metres)	Temperature
180	16°C
360	10°C
540	7.2°C
720	5.6°C
900	4.4°C
1080	3.9°C
1260	3.3°C
1440	2.8°C
1520	2.2°C

4. A device called a bathythermograph is used to record underwater temperatures. The chart shows temperatures recorded every 180 metres in a certain spot in the ocean. At greater depths the temperature usually stays about the same, from 0° to 1.7°C. Give the average of the temperatures in the table.

5. A cubic metre of sea water contains about 20.7 kilograms of salt. Salt is often purchased in 750-gram containers. How many of these containers could be filled with the salt from a cubic metre of sea water ?

6. Waves as high as 34.2 metres have been reported on the "high seas." If each floor of a building is 4.2 m tall, the wave would be as tall as a building with how many floors ?

7. A cubic kilometre of sea water contains about 12.6 kilograms of gold.

A How many cubic kilometres of water would be needed to have 5292 kilograms of gold ?

★ B 1 kg of gold will make a thin wire about 3500 km long. The distance around the earth is 40 068 km. With the gold from 1 cubic kilometre of sea water, how many kilometres of wire would you have left after you wrapped it around the earth once ?

Investigating the Ideas

A $39\overline{)225}$ **B** $39\overline{)307}$ **C** $39\overline{)346}$

Find each of these single-digit quotients and the remainders.

> **?** Can you use the quotients you found above to help you find the quotient for $39\overline{)22576}$?

Discussing the Ideas

1. How did you use the one-digit quotients to find the larger quotient in the Investigation?

2. You can find any quotient, no matter how large, by finding the one-digit quotients for a series of simpler problems. Study this example.

A Dividing hundreds

$$\begin{array}{r} 2 \\ 53\overline{)13038} \\ -106 \\ \hline 24 \end{array}$$

B Dividing tens

$$\begin{array}{r} 24 \\ 53\overline{)13038} \\ -106 \\ \hline 243 \\ -212 \\ \hline 31 \end{array}$$

C Dividing ones

$$\begin{array}{r} 246 \\ 53\overline{)13038} \\ -106 \\ \hline 243 \\ -212 \\ \hline 318 \\ -318 \\ \hline \end{array}$$

A Which three "simpler" division problems were solved in order to find the quotient 13 038 ÷ 53 ?

B Why are the steps in the example labelled "dividing hundreds," "dividing tens," and "dividing ones" ?

3. It is important to be able to find one-digit quotients quickly. Explain how the heavy black numerals can help you estimate the quotient.

$$\begin{array}{r} 60 \\ 59\overline{)436} \end{array}$$

1. Use the heavy black numerals to help you estimate the quotients.

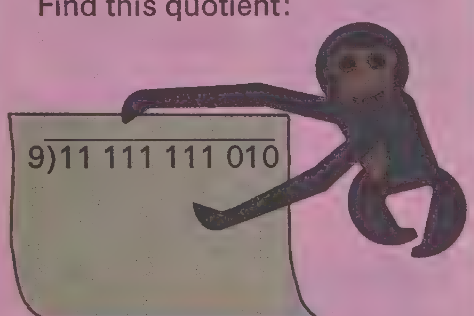
A 12)49	B 59)267 60	C 71)573	D 18)173 20
E 65)435 70	F 34)197	G 46)358 50	H 83)746

2. Find the quotients.

A 61)26 535	B 18)2232	C 49)25 780	D 93)22 150
E 17)14 235	F 85)9642	G 34)21 537	H 56)30 764
I 58)25 734	J 23)13 156	K 72)17 784	L 89)64 547
M 35)34 721	N 14)13 482	O 47)35 967	P 53)20 511

think

Find this quotient:



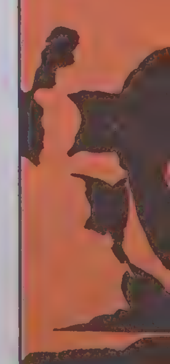
What interesting pattern do you see?

3. Find these special quotients. Why are they special?

A 32)1024	B 15)225
C 25)625	D 49)2401

4. Find these quotients. What pattern do you see?

A 32)736	B 74)3478
C 51)765	D 59)5605



More practice, page A-7, Set 13

Discussing the Ideas

1. A Explain how to estimate the number of **hundreds** in the quotient.
   B Explain the work shown in step 1.
   C Explain how you can tell that there are 0 **tens** in the quotient.
   D Explain how to estimate the number of **ones** in the quotient.
   E Explain step 3.
   F What is the quotient? What is the remainder?
   G Explain how to check your work.

2. A Explain how to estimate the number of **hundreds** in the quotient.
   B Explain step 1.
   C Explain how to estimate the number of **tens** in the quotient.
   D Explain step 2.
   E Explain how you can tell that there are 0 **ones** in the quotient.
   F Give the quotient and remainder. Check the work.

1

Step 1 Dividing hundreds

```
        2
68 ) 14096
    -136
       4
```

Step 2 Dividing tens

```
        20
68 ) 14096
    -136
       49
```

Step 3 Dividing ones

```
       207
68 ) 14096
    -136
       496
      -476
        20
```

2

Step 1 Dividing hundreds

```
       3
14 ) 4911
    -42
      7
```

Step 2 Dividing tens

```
       35
14 ) 4911
    -42
      71
     -70
       1
```

Step 3 Dividing ones

```
       350
14 ) 4911
    -42
      71
     -70
       11
```

1. Find the quotients and remainders. Check your work.

A 68)20 740	B 32)23 685	C 45)13 500	D 17)8615
E 18)3726	F 31)8990	G 56)3000	H 64)52 300
I 5)80 000	J 19)3847	K 55)35 862	L 17)5246

2. Study the example. Then complete the exercises.

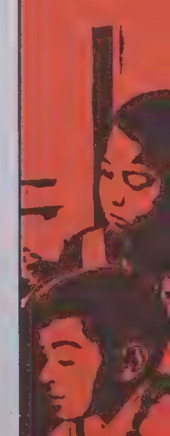
Dividing thousands	Dividing hundreds	Dividing tens	Dividing ones
$\begin{array}{r} 2 \\ 37\overline{)89355} \\ 74 \\ \hline 15 \end{array}$	$\begin{array}{r} 24 \\ 37\overline{)89355} \\ 74 \\ \hline 153 \\ 148 \\ \hline 5 \end{array}$	$\begin{array}{r} 241 \\ 37\overline{)89355} \\ 74 \\ \hline 153 \\ 148 \\ \hline 55 \\ 37 \\ \hline 18 \end{array}$	$\begin{array}{r} 2415 \\ 37\overline{)89355} \\ 74 \\ \hline 153 \\ 148 \\ \hline 55 \\ 37 \\ \hline 185 \\ 185 \\ \hline \end{array}$

A 29)64 932	B 61)129 684	C 18)40 000	D 42)326 572
E 38)74 987	F 23)176 238	G 57)64 593	H 75)246 483

3. Study the examples. Then complete the exercises.

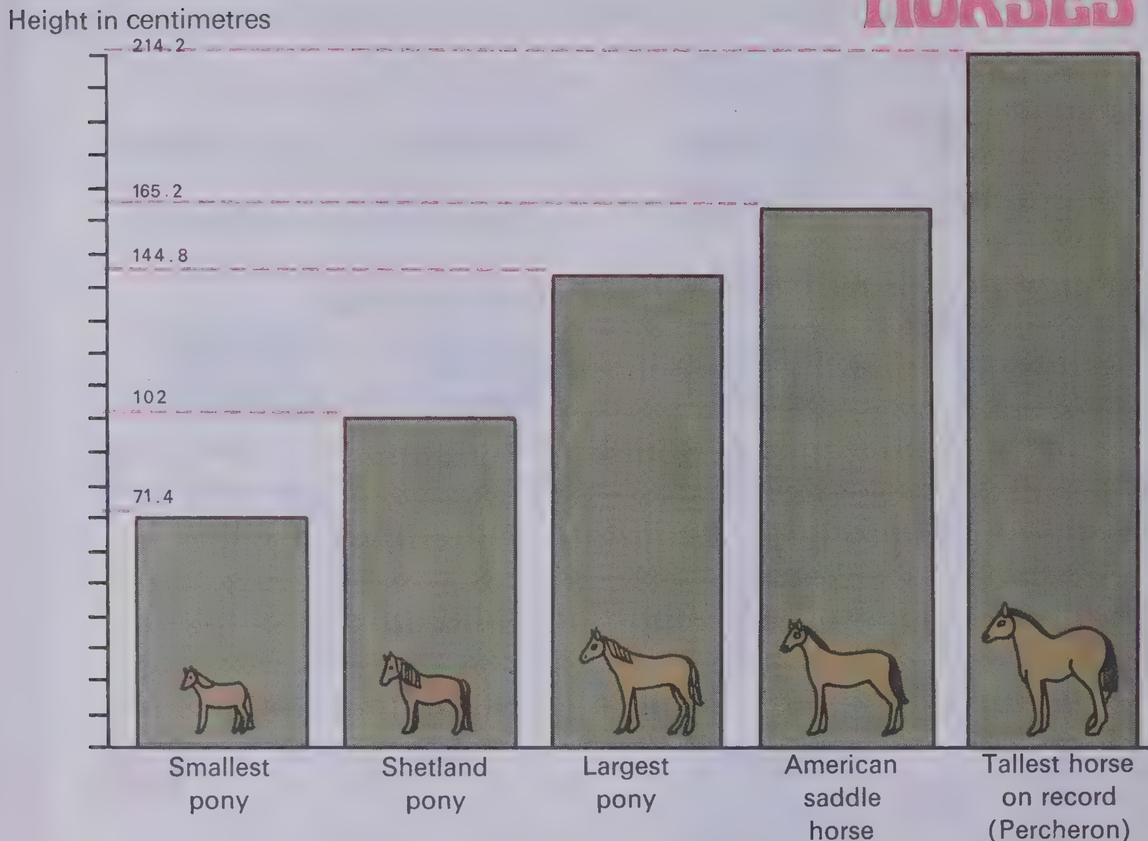
A	B	C
$\begin{array}{r} 6 \\ 431\overline{)\ 2586} \\ -2586 \\ \hline \end{array}$	500 $\begin{array}{r} 5 \\ 486\overline{)\ 2670} \\ -2430 \\ \hline 240 \end{array}$	500 $\begin{array}{r} 3 \\ 450\overline{)\ 1655} \\ -1350 \\ \hline 305 \end{array}$

A 200)1800	B 600)3000	C 205)1562	D 595)2534
E 410)1640	F 823)7407	G 675)4350	H 621)1972
★ I 423)10 152	★ J 678)36 612	★ K 237)17 785	



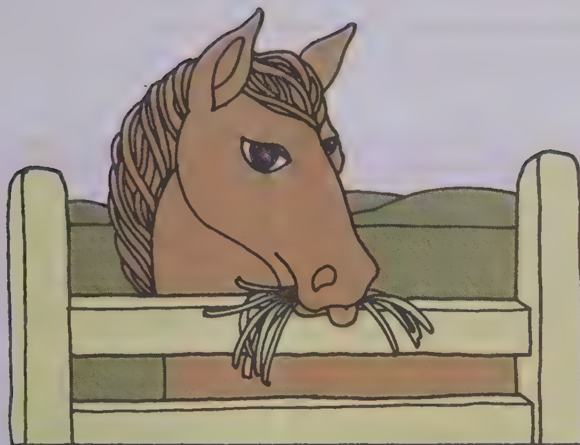
More practice, page A-7, Set 14

HORSES



Horses are measured in metres or centimetres.
A horse's height is measured from the ground to the highest point of the ridge between its shoulder bones.

1. Give the height (in cm) of each horse mentioned above.
2. Give the height (in metres) of each horse shown above.
3. Give the average height (in cm) of the horses mentioned above. Use the numbers you found in exercise 1.
4. The tallest horse on record weighed 1358.5 kilograms. This is 9 times as much as a small Shetland pony. Give the weight of the Shetland pony (to the nearest kilogram).



Type of horse	Weight (kg)
Pony	225
Saddle horse	480
Light harness horse	516
Heavy harness horse	620
Draft horse	984

This table gives 5 different types of horses and the weight of one horse of each type.

5. Find the average of the weights given in the table.

6. It has been said that a horse should be fed 0.5 kg of grain per day for each 50 kg it weighs. If this rule is followed, about how many kilograms of grain should be given each day to the draft horse whose weight is given in the table above ?

7. The average life span of man, 69 years, is 3 times the average life span of a horse. What is the average life span of a horse ?

8. A near-record high jump for a horse is 2.5 metres. A near-record high jump for a man is 224 centimetres. How much higher is the horse's jump ?

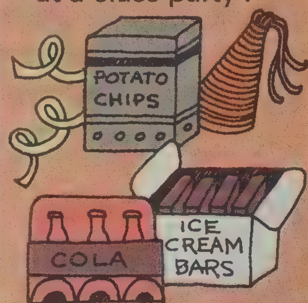
9. A light horseshoe weighs 225 g. Give the total weight in kilograms of the horseshoes needed to shoe a team of 6 horses.

★10. Fast horses can run about 62 km/h for a short time. If a horse runs at a rate of 48 km/h how far does it run in one minute ?

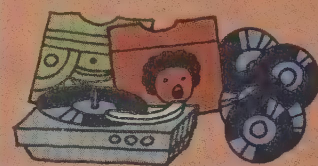
★11. A strong horse can lift 15 000 kg 30 centimetres high in 1 minute. A certain engine can do as much work as 34 strong horses. How many kilograms can this engine lift 30 centimetres in 1 minute ?

Investigating the Ideas

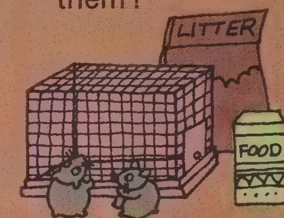
What would it cost to have refreshments at a class party?



What would it cost to buy a record player, six 45 rpm records, and six long-playing records?



What would it cost to buy a pair of hamsters and the supplies needed to keep them?



? Can you find the information needed and make an itemized cost list (including the total cost) for a situation like one of those above?

Discussing the Ideas

1. What "number problems" did you solve while doing the Investigation?

2. Often we need to multiply and divide numbers that give information about amounts of money. Can you explain the methods used to solve these problems?

   A Ellen bought 5 books, each costing \$2.98. What was the total cost? We use multiplication to solve this money problem.

   $$\begin{array}{r} \$2.98 \\ \times 5 \\ \hline \$14.90 \end{array} \qquad \begin{array}{r} 298 \\ \times 5 \\ \hline 1490 \end{array}$$

   B Ann, Jan, Nan, and Fran received \$20.92 from the sale of candy and cookies they had made. They divided the money so that each received the same amount. How much did each girl receive?

   $$\begin{array}{r} \$\ 5.23 \\ 4\overline{)\$20.92} \end{array} \qquad \begin{array}{r} 523 \\ 4\overline{)2092} \end{array}$$

1. Find the answers to these money problems.

A $\$6.75 \times 3$ B $\$0.88 \times 6$ C $\$8.99 \times 8$ D $\$0.35 \times 23$ E $\$1.39 \times 365$

F $5\overline{)\$2.95}$ G $7\overline{)\$44.38}$ H $30\overline{)\$87.00}$ I $65\overline{)\$22.10}$

J $\$0.05\overline{)\$1.00}$ K $\$0.50\overline{)\$7.50}$ L $\$0.75\overline{)\$17.25}$ M $\$0.25\overline{)\$20.00}$

2. Solve the problems.

A A jet plane carried 84 passengers from San Francisco to New York. The fare with tax was $160.90. How much money did the airline collect?

B Jane took 20 pictures. 4 of the pictures were overexposed and were not developed. It cost $3.84 to develop the other pictures. What was the cost of each developed picture?

C Linda works at the school popcorn stand. She needs some coins in order to make change. How many quarters can she get for a $10 bill?

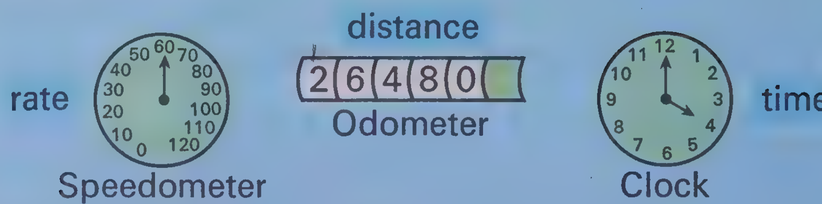
D A new bike cost $56.00. Jim received $12.50 for his old bike and paid the remainder in 6 payments. How much was each payment?

★ **3.** A hamburger and a bag of potato chips cost $0.50. Two hamburgers and a bag of potato chips cost $0.85. What is the cost of a bag of potato chips?

think

A dozen cookies and a loaf of bread cost $0.56. Half a dozen cookies and 2 loaves of bread cost $0.58. What is the cost of a loaf of bread?

Solving Short Story Problems



Give the number for ***r*** (rate), ***d*** (distance), or ***t*** (time).

1. WALK

A Slow: 3km. 3 hours. ***r*** km/h.

B Average: 5 km/h. 5 km. Walk for ***t*** hours.

C Fast: 1 hour. 8 km/h. Walk ***d*** kilometres.

2. BICYCLE

A Slow: 3 hours. 15 kilometres. ***r*** km/h.

B Average: 16 km/h. 32 km. Ride for ***t*** hours.

C Fast: 36 km/h. 2 hours. Go ***d*** kilometres.

3. CAR

A Slow: 36 km/h. 180 km. Takes ***t*** hours.

B Average: 6 hours. 528 km. ***r*** km/h.

C Fast: 222 km/h. 3 hours. Must go ***d*** more kilometres to finish the 800 kilometre race.

4. SHIP

A Slow: 168 km. 7 hours. ***r*** km/h.

B Average: 52 km/h. 468 km. Takes ***t*** hours.

C Fast: 65 km/h. 76 hours. Travels ***d*** kilometres.

5. PLANE

A Slow: 912 km. 6 hours. Travels ***r*** km/h.

B Average: 620 km/h. 4340 km in ***t*** hours.

C Fast: 2440 km/h. 4 hours. Travels ***d*** kilometres.

6. SATELLITE

A Average (in Earth's orbit): 28 232 km/h. 4 hours. Goes ***d*** kilometres.

B Fast (escaping Earth's orbit): 315 364 km. 8 hours. ***r*** km/h.

★ **7.** EARTH

Travels 952 million kilometres in about 365 days. Travels ***r*** km/h (rounded to nearest thousand).

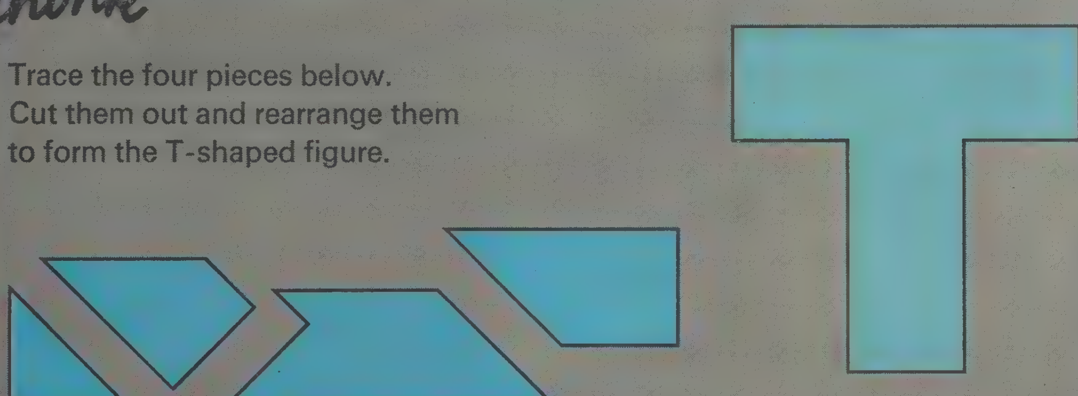
Solving Problems

Find the number for *n* in each exercise.

1. *n* is 232 more than 649.
2. *n* is 358 less than 914.
3. *n* is 913 more than the sum of 432 and 956.
4. *n* is 387 less than the sum of 390 and 423.
5. *n* is 786 more than the difference of 3142 and 787.
6. The sum of 862 and *n* is 1465.
7. The product of 47 and 12 is 519 less than *n*.
8. The product of 86 and 35 is 1946 more than *n*.
9. The product of 37 and 29 is half as large as *n*.
10. The product of 46 and 54 is twice as large as *n*.
11. The product of 72 and 24 is 9 times as large as *n*.
12. The product of 15 and 15 is 25 times as large as *n*.
13. The product of 86 and *n* is 602.
14. The product of 72 and 4 is equal to the product of 32 and *n*.
15. The product of 40 and 7 is equal to the product of 56 and *n*.
16. ★ The product of 28 and 54 is 6 times as much as the product of 36 and *n*.

think

Trace the four pieces below.
Cut them out and rearrange them to form the T-shaped figure.



1. Find the sums, products, differences, and quotients.

A $\begin{array}{r} 5764 \\ +3847 \\ \hline \end{array}$ B $\begin{array}{r} 861 \\ -463 \\ \hline \end{array}$ C $\begin{array}{r} 3947 \\ -1698 \\ \hline \end{array}$ D $\begin{array}{r} 653 \\ \times 25 \\ \hline \end{array}$

E $876 + 39 + 5483 + 9$ F $683 + 4765 + 98 + 17$

G $7\overline{)6478}$ H $7004 - 39$ I 37×56 J $50\overline{)3076}$

K $\begin{array}{r} 566 \\ \times 207 \\ \hline \end{array}$ L $\begin{array}{r} 8070 \\ -4293 \\ \hline \end{array}$ M $\begin{array}{r} 308 \\ \times 72 \\ \hline \end{array}$ N $\begin{array}{r} 6003 \\ -5786 \\ \hline \end{array}$

O $62\overline{)434}$ P $79\overline{)635}$ Q $35\overline{)2205}$ R $74\overline{)18\,944}$

S $57\overline{)874}$ T $16\overline{)12\,675}$ U $92\overline{)556\,947}$ V $486\overline{)3786}$

2. Give the missing numbers.

A $4^2 = \square$ B $5^3 = \square$ C $6^4 = \square$ D $3^5 = \square$

3. Find the arithmetic mean of each set of numbers.

A $\{562, 385, 419, 646\}$

B $\{88, 107, 365, 419, 136\}$

4. Find the total amounts.

A $\begin{array}{r} \$18.75 \\ 26.99 \\ \hline \end{array}$ B $\begin{array}{r} \$23.64 \\ 8.98 \\ \hline \end{array}$

5. Find the differences in the amounts.

A $\begin{array}{r} \$65.50 \\ 65.37 \\ \hline \end{array}$ B $\begin{array}{r} \$20.00 \\ 9.34 \\ \hline \end{array}$

6. Solve these money problems.

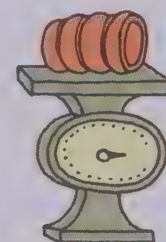
A $\begin{array}{r} \$3.49 \\ \times 4 \\ \hline \end{array}$ B $\begin{array}{r} \$0.87 \\ \times 25 \\ \hline \end{array}$ C $9\overline{)\$21.06}$

think

1
1 1 ← Level 1
1 2 1 ← Level 2
1 3 3 1 ← Level 3
1 4 6 4 1 ← Level 4
1 ? ? ? ? 1 ← Level 5
1 ← Level 6

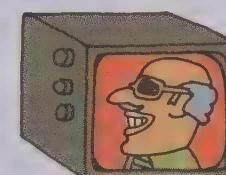
The array above is called Pascal's Triangle. The red arrows will help you see how to give the numbers for levels 5 and 6. Make a table of the sums of the numbers, level by level, 1 through 6. Predict the sums of the numbers in levels 7 and 8, and check your prediction.

7. A piece of meat weighs 2.3 kg.
   A It weighs how many grams less than 3 kilograms ?
   B What is the total cost of 3 kg of meat at \$2.79 per kilogram ?
   C 2 kg of another type of meat cost \$6.16.
   How much more or less does this meat cost per kilogram than the meat in part B ?



8. A plane flew at an average speed of 342 kilometres per hour for 7 hours. How far did it fly ?

9. Mr. Smith bought a television set. The cost plus interest charges was \$340.08. If he pays for it in 12 equal payments, how much will each payment be ?



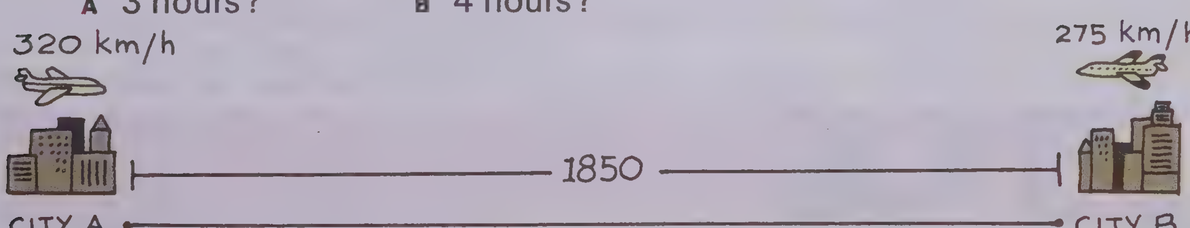
10. Here is part of Jim's record of the time it takes him to deliver papers each day. He spent a total of 370 minutes delivering papers during this week. How long did it take him to deliver papers on Saturday ?

Days	M	T	W	Th	F	S
Minutes	55	65	65	60	50	?

★ 11. The chart shows the number of minutes Sally practiced the piano during one week. How many minutes must she practice on Saturday in order to have practiced an average of 35 minutes a day ?

Days	M	T	W	Th	F	S
Minutes	30	25	35	40	30	?

★ 12. If the planes fly toward each other at the speed given, how far apart will they be after:
   A 3 hours ? B 4 hours ?



Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Place value
Functions

1. Solve the equations.

   A $n \times 6 = 24$ B $q \times 9 = 54$ C $t \times 7 = 63$ D $8 \times t = 0$ E $n \times 11 = 66$ F $15 \times b = 75$ G $8 \times q = 8$ H $7 \times n = 49$

2. Find the products.

   A $10^1 \times 10^2$ B $10^2 \times 10^2$ C $10^1 \times 10^3$ D 50×80 E $5 \times 10 \times 4 \times 10$ F $6 \times 10^2 \times 3 \times 10$ G 90×600 H 800×400

3. Give the missing words and numbers.

   A 438 614: The 8 in the __?__ place means 8 × ▒▒▒.
   B 73 602 549: The 7 in the __?__ place means 7 × ▒▒▒.

4. Write the number that is

   A 10 000 more than one million. B 1 less than one million.

5. Write the ordinary base-ten numeral for each exercise.

   A 8×10^2 B 6×10^3 C 70×10^1 D 80×10^2 E $(8 \times 10^3) + (3 \times 10^2) + (4 \times 10) + 6$ F $(7 \times 10^5) + (4 \times 10^2) + 3$

6. Give the missing numbers for the table.

Function Rule

$n^2 + 5$

	n	$f(n)$
A	6	▒▒▒
B	10	▒▒▒
C	0	▒▒▒
D	▒▒▒	54

★ 7. Give the missing numbers in each row.

Sums +

Quotients ÷

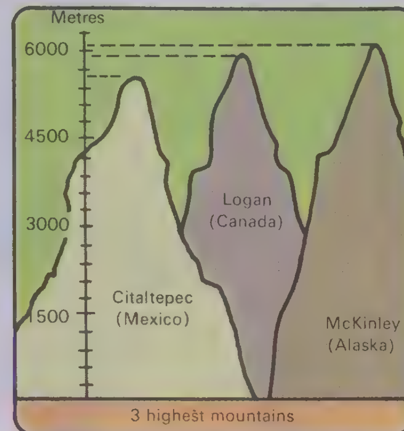
	Sum				Quotient
A	12	▒▒▒	÷	▒▒▒	3
B	12	▒▒▒	÷	▒▒▒	5
C	18	▒▒▒	÷	▒▒▒	1
D	5	▒▒▒	÷	▒▒▒	0

You are invited to explore

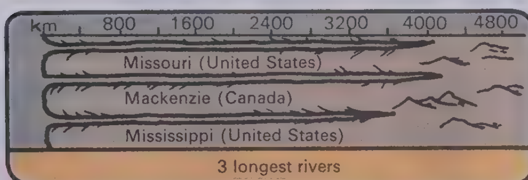
ACTIVITY CARD 5
Page 357

NORTH AMERICA

*Population of 3 largest countries



3 highest mountains



3 longest rivers

Canada	9 974 505
U. S.	9 362 804
Mexico	1 963 815

3 largest countries (area in square kilometres)

New York	7 896 000
Chicago	3 369 000
Mexico City	7 006 000

3 largest cities (population*)

Superior (Canada and U. S.)	82 100
Huron (Canada and U. S.)	59 697
Michigan (U. S.)	57 755

3 largest lakes (area in square kilometres)

1. Use the graphs above to give
- **A** the populations of the 3 largest countries in North America (to the nearest 5 million people).
- **B** the lengths of the 3 longest rivers (to the nearest hundred kilometres).
- **C** the heights of the 3 highest mountains (to the nearest 50 metres).
- **D** an estimate of the difference in height between Mt. McKinley and Mt. Logan.

2. Use the tables above to give
- **A** the areas of the 3 largest countries (rounded to the nearest 500 000).
- **B** the populations of the 3 largest cities (rounded to the nearest million).
- **C** the average area of the 3 largest lakes (to the nearest whole number).

3. The average number of persons per square kilometre in North America is about 12. The area of North America is 24 400 900 square kilometres. About what is the population of North America ?

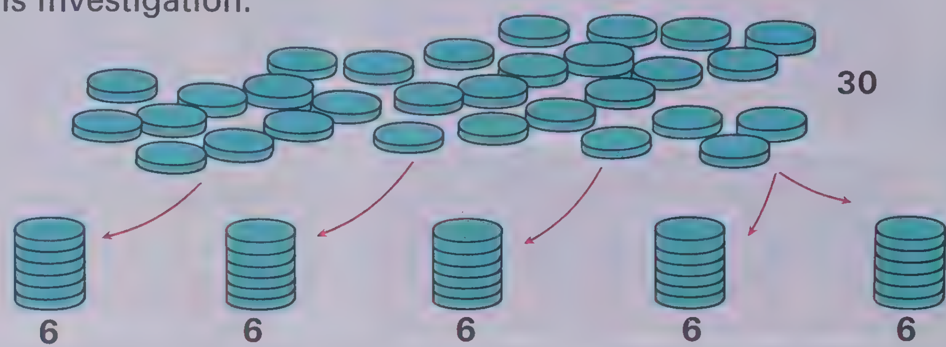
*Population figures based on 1970 census.

6 Number Theory

When is one number a factor of another?

Investigating the Ideas

Use a set of 30 objects (counters, toothpicks, slips of paper) for this Investigation.



How many ways can you find to divide the set into equivalent sets?

Discussing the Ideas

1. The numbers of the equivalent sets you found in the Investigations are **factors** of 30. How many of the factors of 30 can you give?

2. Solve the equations in the table to find the factors of 36.

	Factor		Factor		Product
A	n	×	1	=	36
B	n	×	2	=	36
C	n	×	3	=	36
D	n	×	4	=	36
E	n	×	6	=	36

3. Find the quotient and remainder.

A $2\overline{)36}$ B $3\overline{)36}$ C $4\overline{)36}$

D $5\overline{)36}$ E $6\overline{)36}$ F $7\overline{)36}$

4. If a number is divided by one of its factors, what can you say about the quotient and remainder?

1. Solve the equations that have whole-number solutions. Then list all the factors of 96.

A $n \times 1 = 96$	F $n \times 6 = 96$
B $n \times 2 = 96$	G $n \times 7 = 96$
C $n \times 3 = 96$	H $n \times 8 = 96$
D $n \times 4 = 96$	I $n \times 9 = 96$
E $n \times 5 = 96$	J $n \times 10 = 96$

think

Give the smallest number that has 2, 3, and 5 as factors.

Give the smallest 3-digit number that has 2, 3, and 5 as factors.

Give the smallest 4-digit number that has 2, 3, and 5 as factors.

2. Solve the equations that have whole-number solutions. Then list all the factors of 64.

A $64 \div 1 = n$	C $64 \div 3 = n$	E $64 \div 5 = n$	G $64 \div 7 = n$
B $64 \div 2 = n$	D $64 \div 4 = n$	F $64 \div 6 = n$	H $64 \div 8 = n$

3. Is the first number a factor of the second?
A 3, 51 B 5, 63 C 8, 96 D 7, 98 E 3, 160 F 9, 9

4. List all the factors of each number.
A 16 B 30 C 13 D 24 E 40 F 23 G 72

5. For each set, give the number (other than 1) that is a factor of each number in the set.
A {0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, . . .}
B {0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, . . .}

6. Study the examples below.

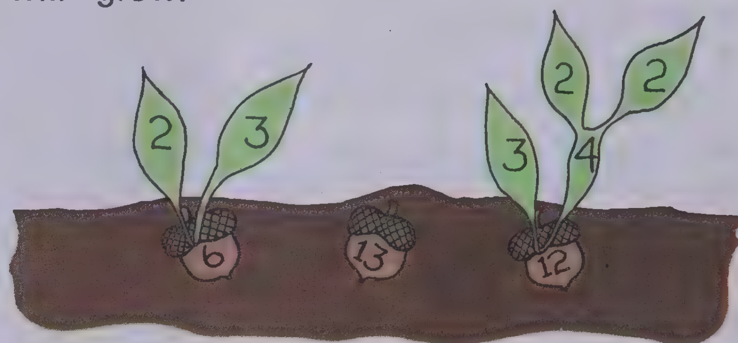
$6+5+4=15$ ← If 3 is a factor of this sum, then
654 ← 3 is a factor of this number.

$2+1+5+6=14$ ← If 3 is **not** a factor of this sum, then
2156 ← 3 is **not** a factor of this number.

Which numbers in the set {282, 540, 4394, 555, 46 338, 5700, 369 050, 144, 7655, 549} have a factor of
A 2? B 5? C 3?

Investigating the Ideas

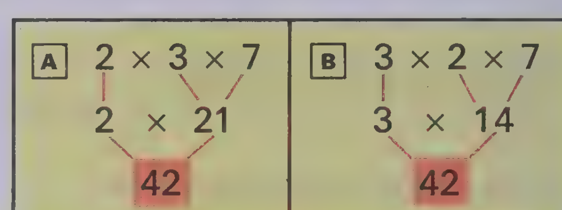
Some numbers will "grow."
Some will not.



? Which of these acorns will grow?
Can you draw pictures for some of them?

Discussing the Ideas

1. Sometimes a number is repeated in a **factor tree** so that the product of the factors in each row is the beginning number. Each tree "grows" as tall as possible. (1 is not used as a factor.) A number such as 42 can have more than one tree. Can you find a third tree for 42?



2. What do you know about the top row of each tree for 42?

3. Can you draw two different trees for 30?

1. Copy each factor tree and give the missing factors.

A ___ × ___
15

B ___ × ___
49

C ___ × ___
58

D ___ × ___
111

E 2 × ___ × ___
___ × ___
28

F ___ × ___ × ___
5 × ___
70

G 5 × ___ × ___
___ × ___
105

H ___ × ___ × 13
___ × ___
78

I ___ × ___ × ___
3 × ___
582

J ___ × ___ × ___
___ × ___
1335

2. Find at least two trees for each number.

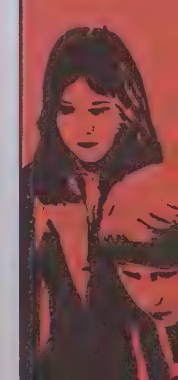
A 84 B 126
C 60 D 64

3. Below is a tree for 180. Can you find four more?

2 × 2 × 3 × 3 × 5
4 × 3 × 3 × 5
12 × 15
180

think

Factors of 6: 1, 2, 3, 6
Notice that
$2 \times 6 = 1 + 2 + 3 + 6.$
When the sum of the factors of a number is twice the number, the number is called a ______. Show that 28 and 496 are perfect numbers. The only perfect numbers less than 500 are 6, 28, and 496.



More practice, page A-8, Set 15

Investigating the Ideas

Terry is thinking about a set of numbers.

? Can you use the clues below to answer the question?	List some other numbers in Terry's set to show you know.

Clues

All of these are in Terry's set.

17 83 31 5 29 19

None of these are in Terry's set.

24 8 49 35 27 51

Question

Which of these are in Terry's set?

93 13 29 15 16 53 91

Discussing the Ideas

1. The numbers in Terry's set are called **prime numbers**. How would you describe prime numbers?

2. Prime numbers cannot be shown as a rectangular array of dots. Use dots to show that 7 is a prime and that 9 is not a prime.

6 is not a prime.

5 is a prime.

3. Prime numbers have exactly two factors. Is 1 a prime number? How many factors does 21 have? Is it prime?

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4

4. Try to build a factor tree for each of the numbers 38, 57, and 73. Which of these numbers is prime?

5. A Which is the smallest prime number?
   B Do you think there is a largest prime number?

1. List all the prime numbers
   A through 50.   B between 60 and 70.   C through 100.

2. A Express 30 as the sum of two prime numbers.
   B Express 30 as the product of prime numbers.

3. Find two prime numbers whose product is an even number.

4. When you express a number as a product of **prime** factors, you have found the **prime factorization** of the number. Give the missing numbers. The prime factorization of 90 is $3 \times 3 \times \square \times \square$.

$3 \times 3 \times \square \times \square$

9×10

90

5. Copy and complete each factor tree. Then give the prime factorization of the number at the bottom.

A $\square \times \square \times \square$
$\square \times \square$
45

B $\square \times \square \times \square \times \square$
$\square \times \square$
150

C $2 \times 2 \times \square \times \square$
$\square \times \square$
84

6. The red lines should remind you of "branches" of an "upside-down" factor tree. Copy each exercise without the red lines and give the missing numbers.

A $66 = 11 \times \square$
$66 = 11 \times \square \times \square$

B $130 = \square \times 5$
$130 = \square \times \square \times 5$

C $210 = 21 \times \square$
$210 = \square \times \square \times \square \times \square$

7. Give the prime factorization of each number.

A 20	E 126	I 45
B 30	F 51	J 250
C 72	G 98	K 231
D 81	H 100	L 330

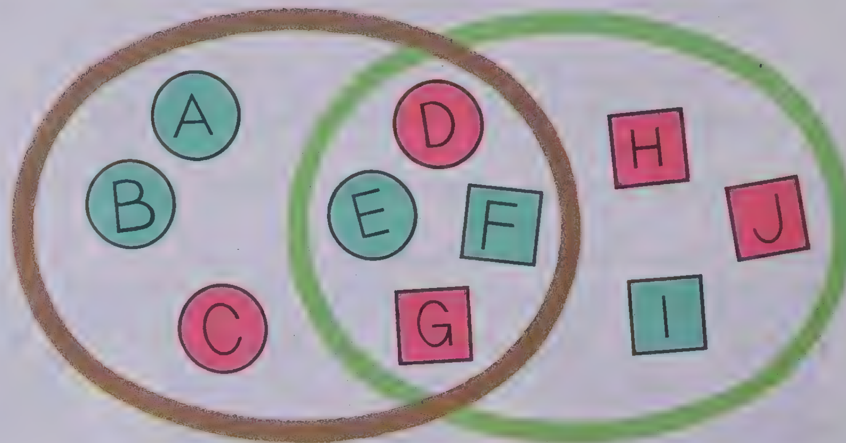
think

Notice that 17 and 71 are both prime. Name 3 other pairs of 2-digit primes that have the same digits.

Let's explore union and intersection of sets.

Investigating the Ideas

Let's play "GUESS MY SET."



Bill is thinking of a special set of these figures.
Jill is trying to guess his set with five or fewer questions.

1.	Jill: Are they all in the brown ring?	Bill: Yes.
2.	Jill: Are any of them outside the green ring?	Bill: No.
3.	Jill: Are any of them circles?	Bill: No.
4.	Jill: Are there more than one figure in your set?	Bill: Yes.

What are the figures in Bill's set?

? Can you choose a special set of the objects shown above and play "Guess My Set" with a classmate?

Discussing the Ideas

1. The **union** of the figures in the brown set with the figures in the green set is the set of all the figures in the two sets. What are these figures?

2. The **intersection** of the two sets (brown and green) is the set of figures that are inside the green set and also inside the brown set. What figures are in the intersection of the two sets?

1. The table below will help you better understand union and intersection of sets. Give the missing numbers in the sets.

Set **P** = {2, 3, 5, 7}	The set of **prime** numbers less than 10
Set **O** = {1, 3, 5, 7, 9}	The set of **odd** numbers less than 10
Set **E** = {0, 2, 4, 6, 8}	The set of **even** numbers less than 10

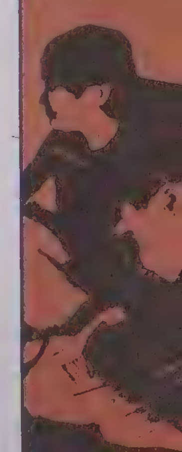
The union of sets O and E	contains	the numbers that are in one or the other or both sets (all the whole numbers less than 10).
$O \cup E$	=	{0, 1, 2, 3, 4, ▮, ▮, ▮, ▮, ▮}
The intersection of sets P and O	contains	only the numbers that are in both set P and set O (the odd prime numbers less than 10).
$P \cap O$	=	{3, ▮, ▮}

2. What numbers are in the intersection of sets O and E? We call this set the **empty** set. We write: $O \cap E = \{\ \}$

3. For each exercise, give the set of numbers.
   A $P \cup E$ B $E \cap P$ C $O \cup P$ D $E \cap O$

4. A Give the set of numbers less than 25 that have 2 as a factor. Call this set T.
   B Give the set of numbers less than 25 that have 3 as a factor. Call this set R.
   C Give the set of numbers for $T \cap R$.
   D Give the set of numbers less than 25 that have 6 as a factor.

★ 5. How many numbers less than 100 have
   A 2 as a factor?
   B 5 as a factor?
   C 2 and 5 as factors?
   D 10 as a factor?



Investigating the Ideas

2 is a factor of both of these numbers. → 72 450

Can you use division to show this fact?

? There are five other numbers that are factors of both 72 and 450. How many of them can you find?

Discussing the Ideas

1. Study the flow chart below for finding the **greatest common factor** (GCF) of 70 and 105. Explain each step and then give the output number.

List the factors of 70. → List the factors of 105. → List the common factors. → Output: Greatest common factor ____

A = {1,2,5,7, 10,14,35,70}

B = {1,3,5,7, 15,21,35,105}

A ∩ B = {1,5,7,35}

____

2. The flow chart below shows a second method for finding the GCF of 70 and 105. Use the method in this flow chart to find the GCF of 45 and 30.

Give the prime factorization of 70. → Give the prime factorization of 105. → Find the greatest factor common to both products. → Output: GCF ____

$70 = 2 \times 5 \times 7$

$105 = 3 \times 5 \times 7$

$2 \times \boxed{5 \times 7}$
$3 \times \boxed{5 \times 7}$

____

1. A List the factors of 12.
   B List the factors of 30.
   C List the common factors of 12 and 30.
   D What is the GCF of 12 and 30?

2. A List the factors of 50.
   B List the factors of 30.
   C List the common factors of 50 and 30.
   D What is the GCF of 50 and 30?

3. Use the method shown in the first flow chart on page 132 to give the GCF of each pair of numbers.
   A 18, 24 B 20, 27 C 12, 60

4. The prime factorization of each number is given. Give the GCF of the two numbers.
   A 70: $2 \times 5 \times 7$
   20: $2 \times 2 \times 5$
   Answer: 10
   B 126: $3 \times 3 \times 2 \times 7$
   24: $3 \times 2 \times 2 \times 2$
   C 66: $2 \times 11 \times 3$
   165: $11 \times 3 \times 5$
   D 525: $3 \times 5 \times 5 \times 7$
   330: $2 \times 3 \times 5 \times 11$
   E 135: $3 \times 3 \times 3 \times 5$
   36: $2 \times 2 \times 3 \times 3$
   F 210: $3 \times 2 \times 7 \times 5$
   60: $2 \times 5 \times 2 \times 3$

5. Give the GCF for each pair of numbers. Use the flow chart in exercise 2, page 132.
   A 36, 42
   B 20, 36
   C 90, 315
   D 72, 450
   E 175, 105
   F 525, 441

★ 6. Give the GCF of 36, 42, and 54.

★ 7. Give the GCF of 70, 28, 42, and 56.

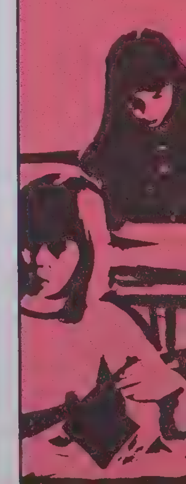
think Let's agree that ★ between two numbers means:

Multiply the first number by itself and add the second to this product.

So, $2 \star 1 = 5$, $3 \star 2 = 11$, and $4 \star 2 = 18$

1. Solve the equations.
   A $2 \star 4 = n$ C $6 \star 6 = n$
   B $5 \star 1 = n$ D $1 \star 1 = n$

2. Answer T (true) or F (false).
   A $2 \star 5 = 5 \star 2$ D $4 \star 0 = 0$
   B $4 \star 3 = 3 \star 4$ E $0 \star 1 = 1$
   C $0 \star 4 = 4$ F $0 \star 2 = 2$



More practice, page A-8, Set 16

Discussing the Ideas

1. The **least common multiple** (LCM) of two numbers is the smallest number (other than zero) that is a multiple of both numbers. Study method 1. Then use it to find the LCM of 3 and 4.

Method 1 Find the LCM of 6 and 8.

$\boxed{1}$ List some multiples of 6. → {0,6,12,18,24,30,36,42,48, . . .} = S
$\boxed{2}$ List some multiples of 8. → {0,8,16,24,32,40,48, . . .} = E
$\boxed{3}$ List **common multiples** of 6 and 8. → {0,24,48, . . .} = S ∩ E
$\boxed{4}$ Choose the **least common multiple.**

2. Study method 2. Then use it to find the LCM of 8 and 12.

Method 2 Find the LCM of 9 and 12.

$\boxed{1}$ Choose the greater of the two numbers: 12
$\boxed{2}$ Find the least multiple of 12 (other than 0) that has 9 as a factor.
12; 9 **is not** a factor of 12.
24; 9 **is not** a factor of 24.
36; 9 **is** a factor of 36.
$\boxed{3}$ 36 is the **least common multiple** of 9 and 12.

3. Study method 3. See if you can explain why it gives the LCM of two numbers.

Method 3 Find the LCM of 30 and 42.

$2 \times 3 \times 5$ ← The prime factorization of 30.
$2 \times 3 \quad\quad \times 7$ ← The prime factorization of 42.
$2 \times 3 \times 5 \times 7$ ← This product is a multiple of both 30 and 42.

1. A List the multiples (to 60) of 6.
   B List the multiples (to 60) of 4.
   C List the common multiples (to 60) of 6 and 4.
   D Give the least common multiple of 6 and 4.

2. A List the multiples (to 30) of 5.
   B List the multiples (to 30) of 2.
   C List the common multiples (to 30) of 5 and 2.
   D Give the LCM of 5 and 2.

3. A List the multiples (to 72) of 12.
   B Give the least multiple of 12 (other than 0) that has 8 as a factor.
   C What is the LCM of 8 and 12?

4. Use method 2 to find the LCM for each pair of numbers.

A 3, 4	C 10, 8	E 9, 15	G 6, 9	I 10, 15	K 24, 18
B 12, 16	D 7, 14	F 8, 20	H 5, 9	J 10, 100	L 24, 40

5. The prime factorization of each number is given. Give the LCM of the two numbers.

A $10 = 2 \times 5$
$15 = 5 \times 3$

C $6 = 2 \times 3$
$15 = 3 \times 5$

E $75 = 3 \times 5 \times 5$
$50 = 5 \times 5 \times 2$

B $30 = 2 \times 3 \times 5$
$70 = 2 \times 5 \times 7$

D $12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$

F $6 = 2 \times 3$
$35 = 5 \times 7$

6. What is the LCM for each pair of numbers?

A 2, 5	F 4, 12
B 6, 5	G 21, 6
C 10, 12	H 17, 2
D 10, 6	I 20, 24
E 20, 10	J 90, 63

7. What is the LCM of
   A 2, 6, and 10?
   B 15, 6, and 4?
   C 18, 15, and 8?
   D 8, 6, and 4?
   E 5, 10, and 3?
   F 10, 12, and 15?

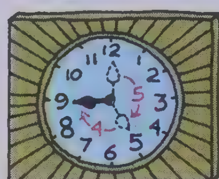
More practice, page A-9, Set 17

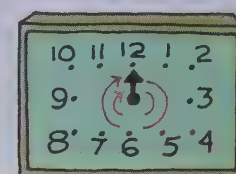
Investigating the Ideas

The figure shows how you can use a one-handed clock to think of addition in "clock arithmetic." You used the number line to show operations in ordinary arithmetic. A clock may be used in much the same way to show operations in clock arithmetic.

Equation: $3 + 4 = 7$

? Can you write a "clock arithmetic" equation for each clock below?



Discussing the Ideas

1. Use a clock to explain each of the following sums.
   A $5 + 6 = 11$ B $5 + 7 = 12$ C $5 + 8 = 1$ D $5 + 9 = 2$
2. Use a clock to explain each of these differences.
   A $10 - 8 = 2$ B $10 - 9 = 1$ C $10 - 10 = 12$ D $10 - 11 = 11$
3. Suppose you have a special one-handed clock with just 5 numbers on it. Also, suppose you use 0 instead of 5 on your clock. You can do some interesting arithmetic with this clock, much as you did with an ordinary clock. Choose some pairs of "five-clock numbers" and explain how you can find their sums.

4. Use the five-clock to explain each equation.
   A $4 + 2 = 1$ B $1 - 2 = 4$ C $2 \times 4 = 3$ D $3 \times 2 = 1$
5. Is it true that for any pair of five-clock numbers a and b
   $$a + b = b + a?$$
   Choose a pair of five-clock numbers and illustrate this principle.

1. Answer the question. Then solve the equation.

   A What time will it be 5 hours after 8 o'clock?
   $8 + 5 = n$

   B What time was it 6 hours before 2 o'clock?
   $2 - 6 = n$

2. Use twelve-clock numbers to find the sums.

   A $7 + 7$ B $7 + 7 + 7$ C $8 + 9$ D $11 + 11$ E $12 + 12$

3. Use twelve-clock numbers to find the differences.

   A $4 - 6$ B $4 - 4$ C $8 - 7$ D $7 - 8$ E $12 - 12$

4. Find the sum. Then find the product. Use twelve-clock numbers.

   A $2+2+2+2=n$
   $4 \times 2 = n$

   B $5+5+5=n$
   $3 \times 5 = n$

   C $7+7+7+7=n$
   $4 \times 7 = n$

Use the **five-clock** for all of these exercises.

5. Answer the question. Then solve the equation.

   A Start at 0. Move forward 3. Move forward 4 more. Where are you?
   $3 + 4 = n$

   B Start at 0. Move forward 2. Move backward 4. Where are you?
   $2 - 4 = n$

   C Start at 0. Move forward 4. Move forward 4 more. Where are you?
   $2 \times 4 = n$

6. Solve the equations.

   A $3 + 0 = n$ D $3 + 3 = s$ G $2 - 3 = n$ J $3 \times 3 = p$
   B $3 + 1 = a$ E $2 - 1 = t$ H $2 - 4 = d$ K $3 \times 4 = q$
   C $3 + 2 = b$ F $2 - 2 = c$ I $2 \times 3 = y$ L $4 \times 0 = r$

7. Copy and complete the addition and multiplication tables.

+	0	1	2	3	4
0	0		2		
1		2			
2				0	
3					2
4					

×	0	1	2	3	4
0		0			
1		1			
2			4	1	
3			1		
4				2	

1. Solve the equations that have whole-number solutions.

	Factor		Factor		Product
A	f	×	1	=	32
B	f	×	2	=	32
C	f	×	3	=	32
D	f	×	4	=	32
E	f	×	5	=	32
F	f	×	6	=	32

	Factor		Factor		Product
G	f	×	1	=	28
H	f	×	2	=	28
I	f	×	3	=	28
J	f	×	4	=	28
K	f	×	5	=	28
L	f	×	6	=	28

2. A List the factors of 32. B List the factors of 28.

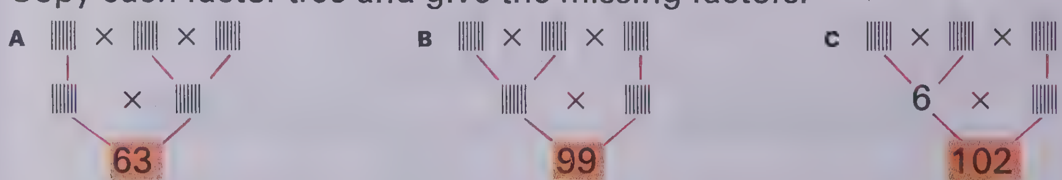
3. Is the first number a factor of the second?
A 7, 42 B 10, 55 C 6, 54 D 7, 91 E 1, 85

4. List the factors of: A 12 B 42 C 56 D 100

5. Find each product. Then give **all** the factors of that number.
A 3×5 C 5×11 E $3 \times 5 \times 7$ G $2 \times 2 \times 2$
B 2×7 D 7×7 F $2 \times 3 \times 3$ H $2 \times 5 \times 7$

6. Give all the numbers in the set {82, 75, 102, 300, 537, 870, 377, 468, 1266, 4575} that have
A 2 as a factor. B 5 as a factor. C 3 as a factor.

7. Copy each factor tree and give the missing factors.



8. Which of these numbers are prime?
{21, 27, 33, 37, 47, 49, 51, 53, 61, 63}

9. Give two prime numbers whose difference is 1.

10. Give the prime factorization of the numbers at the bottom of the factor trees in exercise 7.

11. The red lines should remind you of the branches of an "upside-down" factor tree. Copy each exercise without the red lines and give the missing numbers.

A $78 = 13 \times \square$
$78 = 13 \times \square \times \square$

B $154 = 11 \times \square$
$154 = \square \times \square \times \square$

C $170 = \square \times \square$
$170 = \square \times \square \times \square$

12. Give the prime factorization of each number.

A 45 B 28 C 75 D 120 E 72 F 180

13. Set T = {0, 3, 6, 9, 12, 15} Set E = {0, 2, 4, 6, 8, 10, 12, 14}
Give the set of numbers for: A $T \cap E$ B $T \cup E$

14. Give the **greatest common factor** for each pair of numbers.

A 6, 9 B 12, 18 C 32, 36 D 60, 48 E 36, 42 F 42, 55

15. Give the **least common multiple** for each pair of numbers.

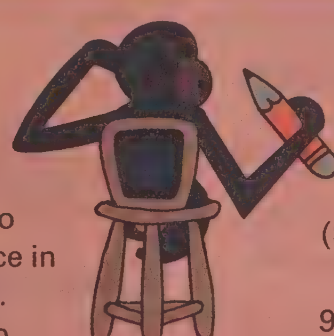
A 4, 6 B 12, 16 C 14, 10 D 8, 12 E 30, 66 F 30, 45

★ 16. Give two numbers whose least common multiple

A is their product. B is one of the numbers.

think

Write symbols for the numbers 1 to 40, but use only 1, 3, 9, 27 and + or −. Use no number more than once in a given set of symbols. The examples will help you get started.



Symbols	Numbers
1	1
3 − 1	2
3	3
3 + 1	4
(9 − 3) − 1	5
9 − 3	6
9 − (3 − 1)	7
⋮	⋮

1. Find the products.

   A 10×10 C 100×10 E 10×1000 G 100×1000

   B 10×100 D 100×100 F 1000×10 H 1000×1000

2. Give the smallest number that has 4 thousands, 6 tens, 5 hundreds, 3 ten thousands, 2 ones, 6 hundred thousands, 9 millions, 6 hundred millions, and 0 ten millions.

3. Write each number as a power of ten or as a number less than 10 times a power of ten.

   A 500 Answer: 5×10^2 B 600 C 1000 D 5000 E 10 000

4. Give the ordinary base-ten numeral for each ▮.

   A $4^2 = $ ▮ C $3^3 = $ ▮ E $6^3 = $ ▮ G $5 \times 10^3 = $ ▮

   B $10^2 = $ ▮ D $2^5 = $ ▮ F $13^3 = $ ▮ H $8 \times 10^4 = $ ▮

5. Find the products.

   A 300×9 B 237×6 C 39×20 D 54×47 E 387×29 F 465×174

6. Find the quotients and remainders.

   A $8\overline{)3642}$ B $40\overline{)2963}$

   C $78\overline{)457}$ D $39\overline{)2164}$

   E $65\overline{)4762}$ F $84\overline{)5967}$

7. Find the arithmetic mean for each set of numbers.

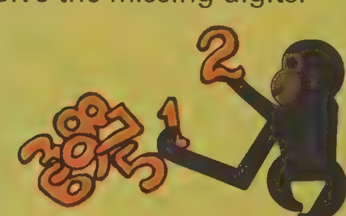
   A {613, 786, 494}

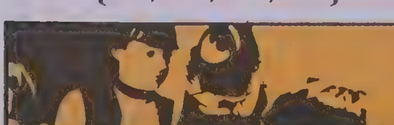
   B {86, 97, 81, 76}

think

Copy the problem.
Give the missing digits.

```
       ▮▮
73) ▮▮▮▮
    ▮▮▮
     ▮▮
     ▮3
      0
```

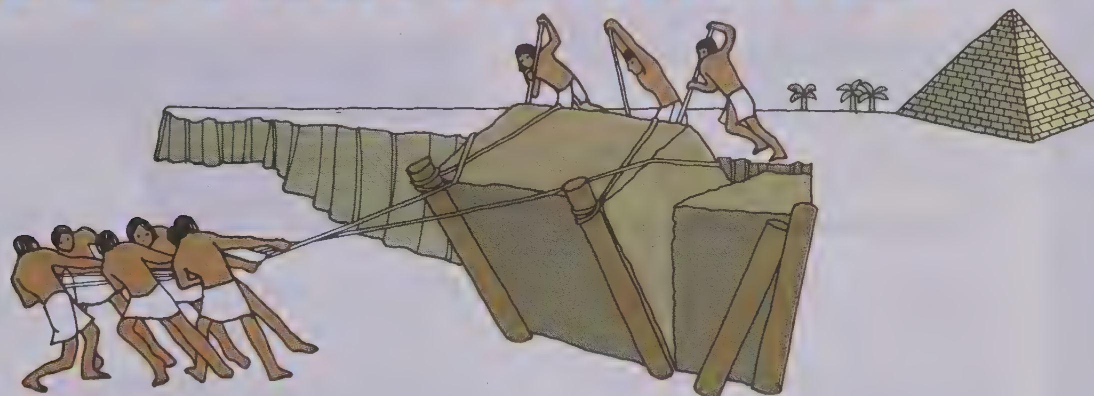




You are invited to explore

ACTIVITY CARD 6
Page 358

EGYPTIAN PYRAMIDS



The Great Pyramid at El Gizeh is the largest of the approximately 80 pyramids that still stand in Egypt. The Pharaoh Cheops had it built as a tomb for himself around 2600 B.C. One hundred thousand slaves worked for 20 years to complete the project, which required 2 300 000 stone blocks, weighing about 2275 kg each. The Great Pyramid is considered one of the seven wonders of the ancient world.

1. The Great Pyramid was originally 144 m tall. The Great Pyramid was as tall as a building of how many stories, if you use 4 metres per story ?

2. When the capstone was removed from the top, the Great Pyramid was 137 m tall and the top was a square surface about 10 m on each side. If a car takes up a space of 4.5 by 2 m, about how many cars could be placed on top of the pyramid ?

3. The bottom of the pyramid is a square about 228 metres on a side. A hectare is 10 000 square metres. Does the pyramid cover an area closer to 5 or to 6 hectares ?

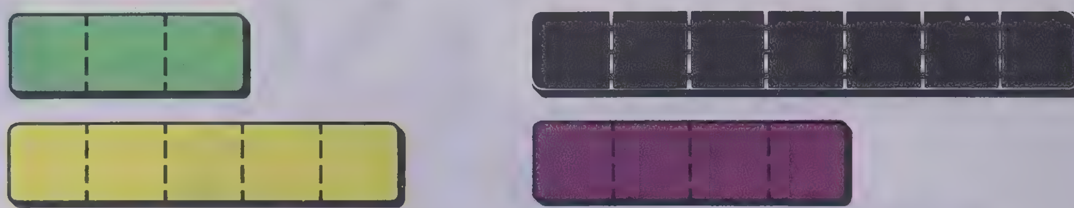
4. Estimate the number of hockey rinks (61 m long and 26 m wide) it would take to provide room for the pyramid.

5. How many years ago (to the nearest 100 years) was the Great Pyramid built ?

7 Fractional Numbers

Let's explore fractions.

Investigating the Ideas



The light green strip has 3 parts, compared with 5 parts for the yellow strip. The **fraction** $\frac{3}{5}$ compares the light green strip with the yellow strip.

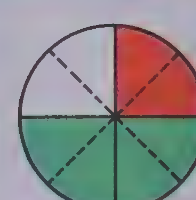
The black strip has 7 parts, compared with 4 parts for the purple strip. The **fraction** $\frac{7}{4}$ compares the black strip with the purple strip.

?	
Choose four of your strips. Can you write fractions that compare each strip with each of the other strips?	Record each pair of strips and the corresponding fractions.

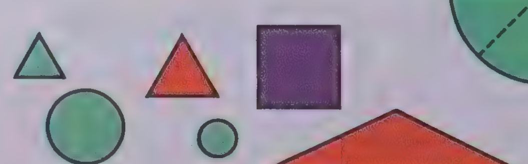
Discussing the Ideas

1. You can say that the light green strip above is $\frac{3}{5}$ the size of the yellow strip. How can you use a fraction to describe this pair?

2. How can you use fractions to describe what you see in this picture of a circular region?

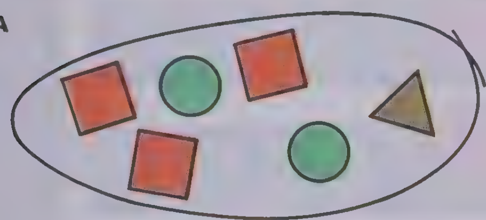


3. How can you use fractions to describe what you see in this picture?



1. Find the numbers for ***a*** and ***b***.
Then write the fraction for the ▥.

A



a of the regions are squares.
b regions in all.
▥ of the regions are squares.

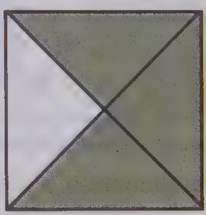
B

The purple strip has ***a*** parts.
The yellow strip has ***b*** parts.
The purple strip is ▥ of the yellow strip.

C

a of the ***b*** parts of the dark green strip are covered.
▥ of the dark green strip is covered.

D ***a*** of ***b*** parts are colored.



▥ of the square is colored.

2. A Give two fractions to tell what part of the set of children are girls.

B Give two fractions to tell what part of the set of children are boys.

C Give two fractions to tell what part of the set of children are wearing glasses.

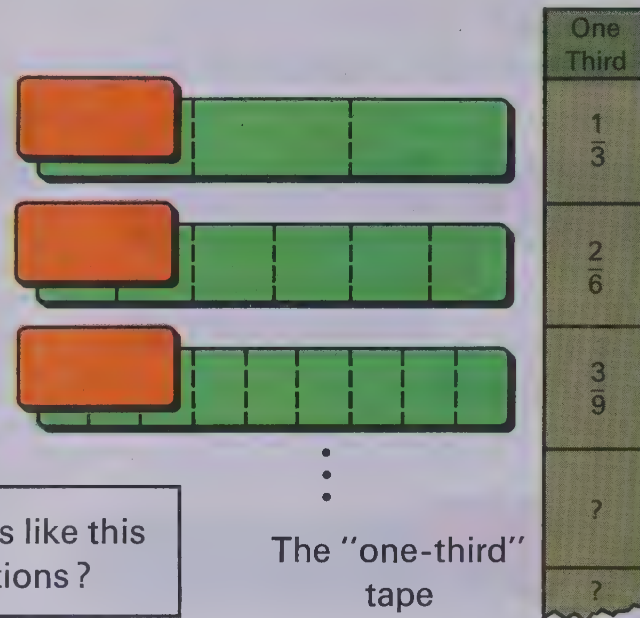
D Give two fractions to tell what part of the set of girls are wearing glasses.

E Give two fractions to tell what part of the set of children are girls wearing glasses.

F Give a fraction to tell what part of the set of children are boys not wearing glasses.

Investigating the Ideas

Imagine the dark green strip divided equally into different numbers of parts. Give the next four fractions on the "one-third" tape.



The "one-third" tape

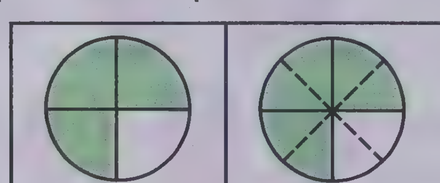
? Can you make tapes like this for some other fractions?

Discussing the Ideas

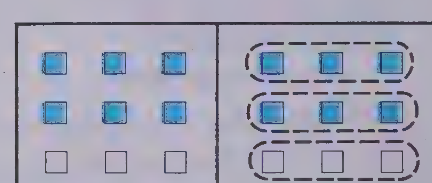
A pair of fractions that suggest the same number of objects in a set or the same part of an object are called **equivalent fractions.**

1. Use the strips pictured above to explain why $\frac{2}{6}$ is equivalent to $\frac{3}{9}$.
2. What pair of equivalent fractions is suggested by each pair of pictures? Explain.

A



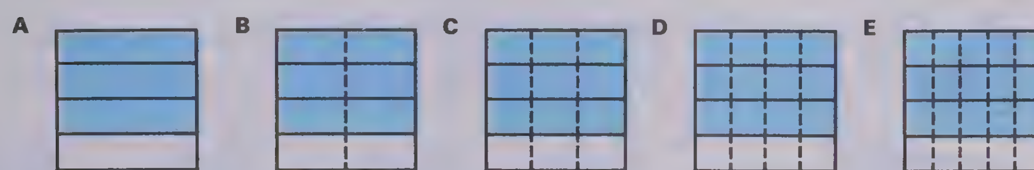
B



3. Can you explain an easy way to make the "equivalent-fraction" tapes?
4. Explain why any two of the fractions from the one-third tape are equivalent.

1. Each exercise below suggests a pair of equivalent fractions. For example, figure A suggests the fractions $\frac{1}{2}$ and $\frac{2}{4}$. Give a pair of equivalent fractions for each figure.

2. Give the fraction suggested by each figure. Are any two of the fractions equivalent?



3. Give the next 3 fractions for each set of equivalent fractions.

A $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \ldots\}$ B $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \ldots\}$

4. Give the missing fractions for each of these "tapes."

	$\frac{1 \times 2}{1 \times 3}$	$\frac{2 \times 2}{2 \times 3}$	$\frac{3 \times 2}{3 \times 3}$	$\frac{4 \times 2}{4 \times 3}$	$\frac{5 \times 2}{5 \times 3}$	$\frac{6 \times 2}{6 \times 3}$
Two Thirds	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	A	B	C

	$\frac{1 \times 5}{1 \times 4}$	$\frac{2 \times 5}{2 \times 4}$	$\frac{3 \times 5}{3 \times 4}$	$\frac{4 \times 5}{4 \times 4}$	$\frac{5 \times 5}{5 \times 4}$	$\frac{6 \times 5}{6 \times 4}$
Five Fourths	$\frac{5}{4}$	$\frac{10}{8}$	D	E	F	G

5. Copy each set of fractions. Give 3 more fractions for each set.

A $\{\frac{1}{5}, \frac{2}{10}, \frac{3}{15}, \ldots\}$ C $\{\frac{1}{2}, \frac{2}{4}, \ldots\}$ E $\{\frac{1}{3}, \ldots\}$ G $\{\frac{9}{10}, \ldots\}$

B $\{\frac{6}{5}, \frac{12}{10}, \frac{18}{15}, \ldots\}$ D $\{\frac{3}{10}, \frac{6}{20}, \ldots\}$ F $\{\frac{8}{5}, \ldots\}$ H $\{\frac{1}{20}, \ldots\}$

Investigating the Ideas

When you build a set of equivalent fractions, you start with a **lowest-terms** fraction.

	Lowest-Terms Fraction	$\frac{2 \times 2}{2 \times 5}$	$\frac{3 \times 2}{3 \times 5}$	$\frac{4 \times 2}{4 \times 5}$	$\frac{5 \times 2}{5 \times 5}$	$\frac{6 \times 2}{6 \times 5}$	$\frac{7 \times 2}{7 \times 5}$
Two Fifths	$\frac{2}{5}$	$\frac{4}{10}$	$\frac{6}{15}$	$\frac{8}{20}$	$\frac{10}{25}$	$\frac{12}{30}$	$\frac{14}{35}$

? Can you find the lowest-terms fraction that was used to "build" each of these fractions?

A $\frac{9}{15}$ $\frac{12}{20}$ **B** $\frac{3}{6}$ **C** $\frac{9}{24}$ **D** $\frac{10}{14}$ **E** $\frac{12}{16}$

Discussing the Ideas

1. When the numerator and the denominator of a fraction have no common factor other than 1, the fraction is a lowest-terms fraction. In the examples below, the factors of the numerator and the denominator are given.

   The fraction $\frac{12}{15}$ is not a lowest-terms fraction. Why? → $\frac{12}{15}$ $\{1, 2, 3, 4, 6, 12\}$ $\{1, 3, 5, 15\}$

   The fraction $\frac{8}{9}$ is a lowest-terms fraction. Why? → $\frac{8}{9}$ $\{1, 2, 4, 8\}$ $\{1, 3, 9\}$

2. To find the lowest-terms fraction you can "divide out" common factors. Explain examples A and B and then use this method to find the lowest-terms fraction for $\frac{24}{30}$.

   A $\frac{60}{90} \rightarrow \frac{60 \div 10}{90 \div 10} \rightarrow \frac{6}{9} \rightarrow \frac{6 \div 3}{9 \div 3} \rightarrow \frac{2}{3}$

   B $\frac{24}{18} \rightarrow \frac{24 \div 2}{18 \div 2} \rightarrow \frac{12}{9} \rightarrow \frac{12 \div 3}{9 \div 3} \rightarrow \frac{4}{3}$

1. Build a set of six equivalent fractions from each of these lowest-terms fractions.
Example: $\frac{3}{4} \longrightarrow \{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}\}$

A $\frac{1}{2}$ **B** $\frac{1}{4}$ **C** $\frac{2}{3}$ **D** $\frac{5}{8}$ **E** $\frac{1}{10}$ **F** $\frac{10}{7}$ **G** $\frac{5}{6}$ **H** $\frac{3}{5}$

2. In each of these sets of fractions, the lowest-terms fraction is covered. Give the lowest-terms fraction for each set.

A $\{■, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$
B $\{■, \frac{6}{20}, \frac{9}{30}, \frac{12}{40}, \ldots\}$
C $\{■, \frac{10}{14}, \frac{15}{21}, \frac{20}{28}, \ldots\}$
D $\{■, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \ldots\}$
E $\{■, \frac{14}{16}, \frac{21}{24}, \frac{28}{32}, \ldots\}$
F $\{■, \frac{18}{20}, \frac{27}{30}, \frac{36}{40}, \ldots\}$

3. Tell whether or not the fraction is a lowest-terms fraction. The factors of the numerator and denominator are given in red.

A $\frac{14}{15}$ {1, 2, 7, 14} {1, 3, 5, 15}
B $\frac{16}{18}$ {1, 2, 4, 8, 16} {1, 2, 3, 6, 9, 18}
C $\frac{8}{12}$ {1, 2, 4, 8} {1, 2, 3, 4, 6, 12}
D $\frac{9}{16}$ {1, 3, 9} {1, 2, 4, 8, 16}
E $\frac{12}{20}$ {1, 2, 3, 4, 6, 12} {1, 2, 4, 5, 10, 20}
F $\frac{15}{28}$ {1, 3, 5, 15} {1, 2, 4, 7, 14, 28}

4. Tell whether or not these fractions are in lowest terms.

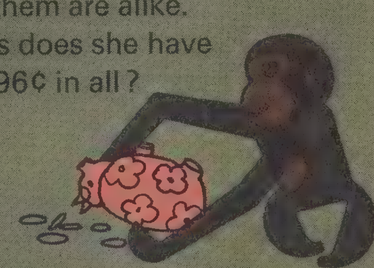
A $\frac{6}{10}$ **B** $\frac{9}{15}$ **C** $\frac{8}{15}$ **D** $\frac{10}{18}$ **E** $\frac{24}{9}$ **F** $\frac{15}{20}$ **G** $\frac{21}{5}$ **H** $\frac{30}{25}$

5. Give the lowest-terms fraction for each fraction.

A $\frac{8}{12}$
B $\frac{16}{20}$
C $\frac{8}{20}$
D $\frac{20}{60}$
E $\frac{25}{100}$
F $\frac{25}{10}$
G $\frac{4}{24}$
H $\frac{15}{50}$
I $\frac{6}{50}$
J $\frac{75}{100}$
K $\frac{9}{21}$
L $\frac{8}{10}$
M $\frac{28}{14}$
N $\frac{50}{75}$
O $\frac{16}{30}$
P $\frac{50}{100}$

think

Susan has some coins. Only 2 of them are alike. What coins does she have if she has 96¢ in all?

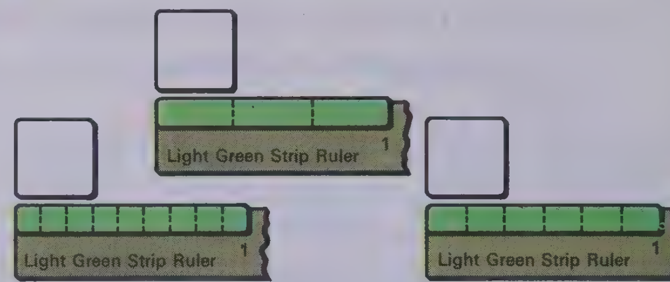


More practice, page A-9, Set 18

Let's explore fractional numbers.

Discussing the Ideas

1. What is the **length** of the white strip when the light green strip is the unit? After choosing the unit, the length of the strip is a **number** which does not change, but different fractions can be used to represent it. What are some of these fractions?

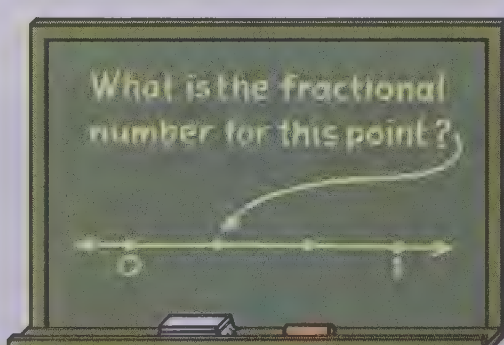


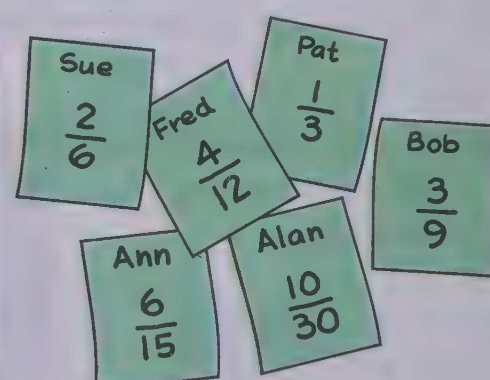
2.

For each set of equivalent fractions	we think of one fractional number	and one point on the number line.

Don is thinking of a fractional number. How do you think he found the point for this number on the number line?

3.





Any fraction from a set of equivalent fractions can be used to name the fractional number for that set. Can you find the one paper that is incorrect? Give a name for this fractional number that is different from any of the names above.

1. Each set of equivalent fractions indicates one fractional number. Give the correct point (X, Y, or Z) for that fractional number.

A $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \dots\}$

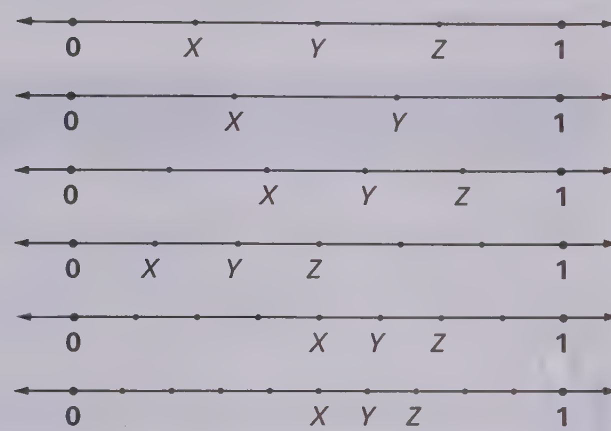
B $\{\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \dots\}$

C $\{\frac{4}{5}, \frac{8}{10}, \frac{12}{15}, \frac{16}{20}, \dots\}$

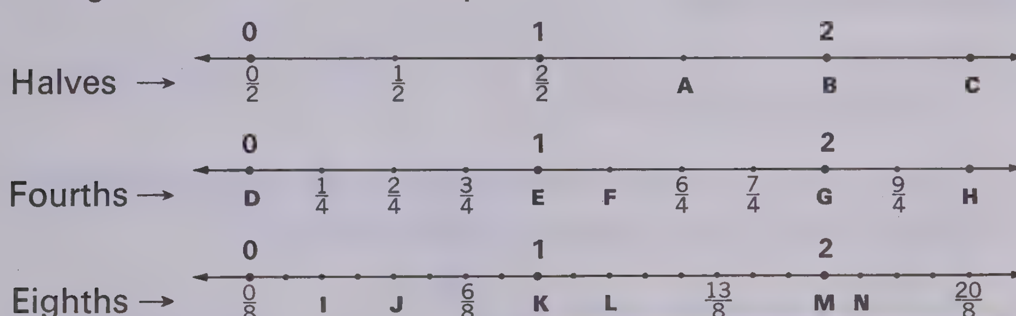
D $\{\frac{1}{6}, \frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \dots\}$

E $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \dots\}$

F $\{\frac{7}{10}, \frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \dots\}$

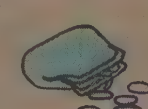


2. Using the denominator suggested at the left of each number line, give the fraction for the point above each letter.



3. Give the lowest-terms fraction that names the fractional number for the point over the red arrow.

think



Janet had 69¢.
Sally asked her for change for a 50-cent piece. Janet tried to make change but found she could not. What coins did she have if they were all less than 50¢?

Investigating the Ideas

Use the equivalent-fraction tape for seven thirds.
Choose any two fractions.

Seven Thirds	$\frac{7}{3}$	$\frac{14}{6}$	$\frac{21}{9}$	$\frac{28}{12}$	$\frac{35}{15}$	$\frac{42}{18}$	$\frac{49}{21}$	$\frac{56}{24}$

Find the product of the numbers in each ring.
Are these "cross products" the same?

$\frac{14}{6} \quad \frac{35}{15}$

? Can you pick two fractions so that the products of the numbers in the rings are different?

Discussing the Ideas

1. What do you think is true about the "cross products" for a pair of equivalent fractions?

2. Study the chart below. Then use the ideas to help you write about the numbers named by these fractions.

   A $\frac{7}{9}$ and $\frac{3}{4}$ B $\frac{6}{8}$ and $\frac{27}{36}$

FRACTIONS			NUMBERS	
Since	we know		we write	and think
$\frac{3}{4} \quad \frac{6}{8}$ → 24 → 24	$\frac{3}{4}$ and $\frac{6}{8}$ are equivalent.	→	$\frac{3}{4} = \frac{6}{8}$	$\frac{3}{4}$ and $\frac{6}{8}$ name the same number.
$\frac{10}{12} \quad \frac{8}{10}$ → 96 → 100	$\frac{10}{12}$ and $\frac{8}{10}$ are not equivalent.	→	$\frac{10}{12} \neq \frac{8}{10}$	$\frac{10}{12}$ and $\frac{8}{10}$ do not name the same number.

1. Copy each exercise and give the correct sign ($=$ or $\neq$). The red and black rings are shown to remind you of the cross-product check for equivalent fractions.

A $\frac{4}{8}\ \bigcirc\ \frac{6}{9}$ **B** $\frac{10}{18}\ \bigcirc\ \frac{5}{9}$ **C** $\frac{16}{12}\ \bigcirc\ \frac{8}{6}$ **D** $\frac{8}{20}\ \bigcirc\ \frac{4}{10}$

E $\frac{3}{6}\ \bigcirc\ \frac{4}{10}$ **F** $\frac{8}{7}\ \bigcirc\ \frac{22}{20}$ **G** $\frac{3}{5}\ \bigcirc\ \frac{60}{100}$ **H** $\frac{20}{30}\ \bigcirc\ \frac{6}{9}$

I $\frac{4}{12}\ \bigcirc\ \frac{8}{20}$ **J** $\frac{9}{21}\ \bigcirc\ \frac{6}{14}$ **K** $\frac{12}{8}\ \bigcirc\ \frac{6}{4}$ **L** $\frac{15}{10}\ \bigcirc\ \frac{30}{20}$

2. Use the set of equivalent fractions to help you give the missing numerator or denominator. $\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}, \frac{35}{42}, \ldots\}$

A $\frac{5}{6} = \frac{\square}{18}$ **B** $\frac{20}{24} = \frac{\square}{12}$ **C** $\frac{10}{12} = \frac{15}{\square}$ **D** $\frac{\square}{30} = \frac{5}{6}$ **E** $\frac{35}{42} = \frac{\square}{12}$ **F** $\frac{25}{\square} = \frac{30}{36}$

3. Give the number for a. Then give the number for b.

A $1 \times 6 = 2 \times a \rightarrow \frac{1}{2} = \frac{b}{6}$ **C** $a \times 8 = 4 \times 6 \rightarrow \frac{b}{4} = \frac{6}{8}$

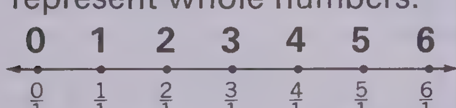
B $2 \times a = 3 \times 6 \rightarrow \frac{2}{3} = \frac{6}{b}$ **D** $1 \times a = 6 \times 4 \rightarrow \frac{1}{6} = \frac{4}{b}$

4. Give the missing numerator or denominator.

A $\frac{3}{2} = \frac{6}{\square}$ **C** $\frac{7}{8} = \frac{14}{\square}$ **E** $\frac{4}{10} = \frac{\square}{20}$ **G** $\frac{10}{16} = \frac{15}{\square}$ **I** $\frac{\square}{10} = \frac{80}{100}$

B $\frac{3}{4} = \frac{6}{\square}$ **D** $\frac{6}{8} = \frac{3}{\square}$ **F** $\frac{12}{6} = \frac{\square}{4}$ **H** $\frac{18}{\square} = \frac{24}{4}$ **J** $\frac{75}{100} = \frac{\square}{4}$

5. This number-line picture shows two ways to represent whole numbers.



Which whole numbers are named by these fractions?

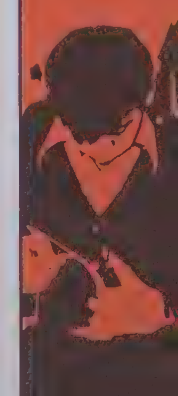
A $\frac{8}{2}$ **D** $\frac{12}{3}$ **G** $\frac{8}{4}$

B $\frac{2}{1}$ **E** $\frac{7}{7}$ **H** $\frac{20}{5}$

C $\frac{12}{4}$ **F** $\frac{7}{1}$ **I** $\frac{20}{4}$

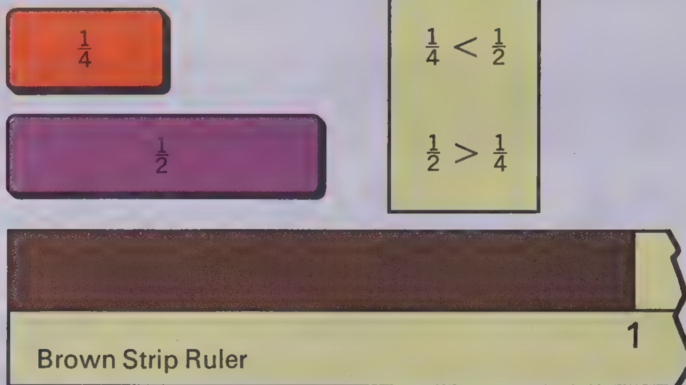
think

Trace and cut out four of these "birds." Can you put four of these birds together to form a square?



More practice, page A-10, Set [illegible]

Investigating the Ideas



$\frac{1}{4} < \frac{1}{2}$

$\frac{1}{2} > \frac{1}{4}$

1 Use the brown strip as the unit.

2 Compare two other strips.

3 Use the lengths of the two strips to write inequality statements.

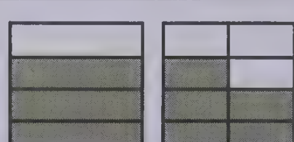
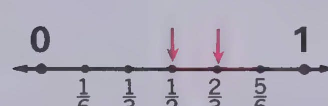
? With the brown strip as the unit, how many different inequality statements can you write by using the red, light green, and purple strips?

Discussing the Ideas

1. How would you use strips to help you compare $\frac{2}{3}$ and $\frac{5}{6}$?

2. Three other ways to compare fractional numbers are shown below.

   A Give the correct symbol ($<$, $>$) for each ▒.

1 Regions	2 Number line	3 Common denominator
 $\frac{3}{4}$ ▒ $\frac{5}{8}$	 $\frac{1}{2}$ ▒ $\frac{2}{3}$	$\frac{8}{12} < \frac{9}{12}$ $\uparrow \quad \uparrow$ $\frac{2}{3}$ ▒ $\frac{3}{4}$

   B Which of these methods would you use to compare $\frac{2}{3}$ with $\frac{3}{5}$? Write the inequality statement.

1. Give the numerators for a and b. Then give the correct sign ($>$ or $<$) for the ●.

A $\left.\begin{array}{l}\frac{1}{2} = \frac{a}{6} \\ \frac{1}{3} = \frac{b}{6}\end{array}\right\} \rightarrow \frac{1}{2}$ ● $\frac{1}{3}$

C $\left.\begin{array}{l}\frac{1}{4} = \frac{a}{20} \\ \frac{1}{5} = \frac{b}{20}\end{array}\right\} \rightarrow \frac{1}{4}$ ● $\frac{1}{5}$

E $\left.\begin{array}{l}\frac{4}{5} = \frac{a}{20} \\ \frac{3}{4} = \frac{b}{20}\end{array}\right\} \rightarrow \frac{4}{5}$ ● $\frac{3}{4}$

B $\left.\begin{array}{l}\frac{3}{4} = \frac{a}{12} \\ \frac{5}{6} = \frac{b}{12}\end{array}\right\} \rightarrow \frac{3}{4}$ ● $\frac{5}{6}$

D $\left.\begin{array}{l}\frac{7}{8} = \frac{a}{24} \\ \frac{2}{3} = \frac{b}{24}\end{array}\right\} \rightarrow \frac{7}{8}$ ● $\frac{2}{3}$

F $\left.\begin{array}{l}\frac{5}{6} = \frac{a}{12} \\ \frac{2}{3} = \frac{b}{12}\end{array}\right\} \rightarrow \frac{5}{6}$ ● $\frac{2}{3}$

2. Give the correct sign for each ●.

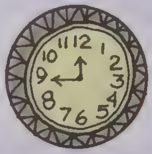
A $\frac{1}{2}$ ● $\frac{3}{8}$ **C** $\frac{1}{2}$ ● $\frac{1}{3}$ **E** $\frac{6}{8}$ ● $\frac{11}{16}$ **G** $\frac{7}{2}$ ● 4 **I** $\frac{14}{3}$ ● 5

B 6 ● $\frac{7}{1}$ **D** 9 ● $\frac{9}{1}$ **F** $\frac{2}{3}$ ● $\frac{7}{9}$ **H** $\frac{5}{8}$ ● $\frac{3}{4}$ **J** $\frac{4}{5}$ ● $\frac{2}{3}$

Short Stories

1. Miss Brown's class: $\frac{1}{2}$ are girls. Miss White's class: 3 out of each 5 are girls. Same number of children in each class. Which class has more girls?

2. 20 minutes. $\frac{1}{4}$ hour. Which is longer?



3. 3500 ml. 4 litres. Which is more?

4. 130 cm. 1.5 m. Which is longer?

5. Orchard: $\frac{2}{10}$ apple trees, $\frac{1}{2}$ pear trees, $\frac{5}{50}$ peach trees, $\frac{5}{25}$ apricot trees.

A The orchard has most of which kind of tree?

B The orchard has fewest of which kind of tree?

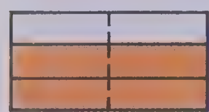
C The orchard has the same number of which two kinds of trees?

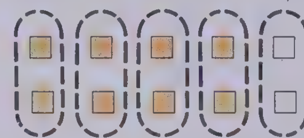
think

Can you unscramble these math words?

More practice, page A-10, Set 20

1. Give the pair of equivalent fractions suggested by each figure.

A 

B 

C 

2. Copy each set of fractions and give three more fractions for the set.

A $\{\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \ldots\}$ B $\{\frac{7}{5}, \frac{14}{10}, \frac{21}{15}, \ldots\}$ C $\{\frac{9}{10}, \frac{18}{20}, \frac{27}{30}, \ldots\}$

3. Give a set of six equivalent fractions for each lowest-terms fraction.

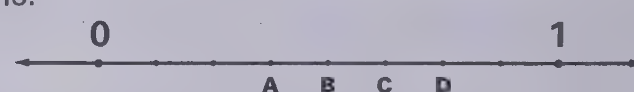
A $\frac{1}{8}$ B $\frac{5}{6}$ C $\frac{3}{10}$ D $\frac{2}{5}$ E $\frac{5}{2}$ F $\frac{7}{3}$

4. Give the lowest-terms fraction for each fraction.

A $\frac{7}{14}$ B $\frac{6}{24}$ C $\frac{40}{60}$ D $\frac{15}{24}$ E $\frac{7}{70}$ F $\frac{70}{100}$ G $\frac{6}{10}$ H $\frac{35}{50}$

5. Give the point for the fractional number indicated by the set of equivalent fractions.

A $\{\frac{5}{8}, \frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \ldots\}$



B $\{\frac{9}{4}, \frac{18}{8}, \frac{27}{12}, \frac{36}{16}, \ldots\}$

6. Give the lowest-terms fraction for each letter.

7. Give the number for *n*.

A $\frac{1}{2} = \frac{n}{10}$ C $\frac{5}{8} = \frac{n}{24}$ E $\frac{n}{10} = \frac{4}{5}$

B $\frac{2}{3} = \frac{8}{n}$ D $\frac{6}{n} = \frac{9}{12}$ F $\frac{2}{8} = \frac{n}{36}$

8. Give the correct sign (<, =, >).

A $\frac{11}{2}$ ◯ 5 C $\frac{6}{12}$ ◯ $\frac{5}{10}$ E $\frac{3}{4}$ ◯ $\frac{4}{6}$

B $\frac{3}{8}$ ◯ $\frac{1}{4}$ D $\frac{7}{8}$ ◯ $\frac{3}{4}$ F $\frac{8}{6}$ ◯ $\frac{10}{8}$

think

Study the example.
Then give the lowest-terms fraction for A, B, and C.

Example: $\frac{3 \times 5}{4 \times 5^2}$ $\frac{3}{4 \times 5}$ $\frac{3}{20}$

A $\frac{2 \times 5^2}{7 \times 5}$ B $\frac{3 \times 5^2}{7 \times 5^3}$ C $\frac{3 \times 5^3}{6 \times 5^5}$

Keeping in Touch with

Addition, Subtraction, Multiplication, Division, Place value, Functions

1. Give the ordinary base-ten numeral for each number.

A 10^3 C 37×10^3 E 8×10^7 G 427×10^2
B 5×10^3 D 6×10^4 F 35×10^6 H $10^6 \times 10^3$

2. In a recent year, the combined population of North and South America was 475 387 000.

A Give this population figure to the nearest ten thousand.
B Give this population figure to the nearest million.

3. Solve the equations.

A $327 + 8465 + 29 + 7 = n$
B $84\,206 - 7349 = n$
C $82\,654 + n = 97\,612$
D $74\,261 + 93\,483 + 68 = n$
E $n - 820\,064 = 132\,576$
F $843 + 7 + 6929 + 78 = n$

4. Give the missing numbers.

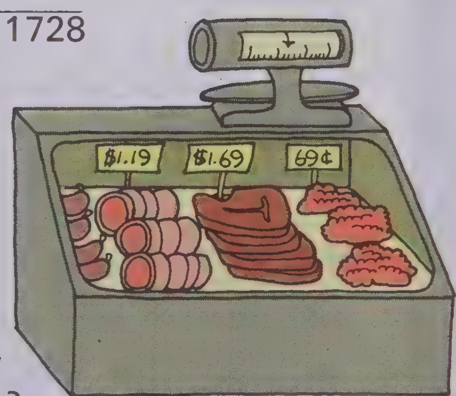
Function Rule

$n \times 37$

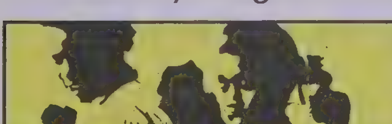
	n	$f(n)$
A	6	
B		1776
C	465	

5. Find the products and quotients.

A $\begin{array}{r} 286 \\ \times 53 \\ \hline \end{array}$ B $\begin{array}{r} 4681 \\ \times 24 \\ \hline \end{array}$ C $31\overline{)1829}$ D $48\overline{)1728}$



6. A If a roast is \$1.19 for 500 g, how much will a 3.5 kg roast cost?
B If steak is \$1.69 for 500 g, what will a 1.5 kg steak cost?
C If hamburger costs 69¢ for 500 g, how many kilograms can you buy for \$2.76?



You are invited to explore

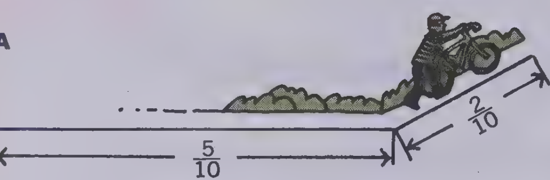
ACTIVITY CARD 7
Page 358

8 Addition and Subtraction of Fractional Numbers

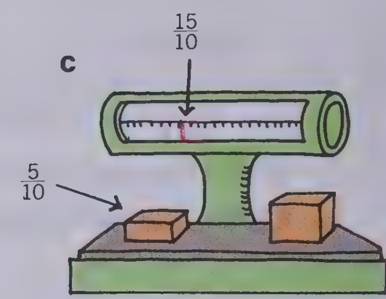
Let's add and subtract fractional numbers.

Investigating the Ideas

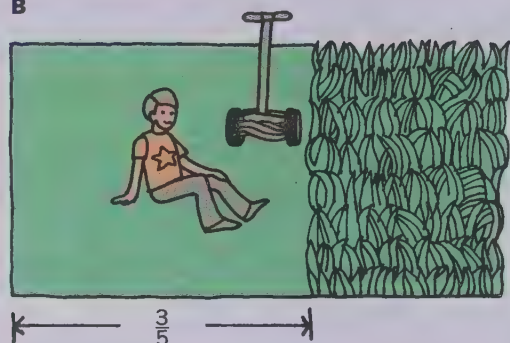
A



C



B



D

? Can you write and solve a story problem for one of these pictures?

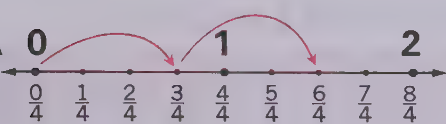
Discussing the Ideas

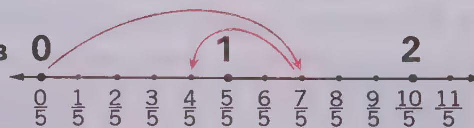
1. A Which pictures suggest addition?
B Which pictures suggest subtraction?

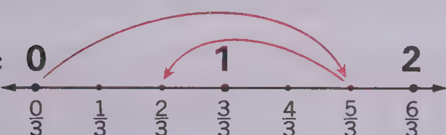
2. Can you give a simple rule for finding sums and differences such as the ones below?

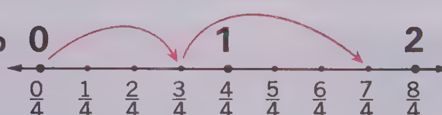
A $\frac{4}{5} + \frac{2}{5} = n$ B $\frac{7}{10} - \frac{3}{10} = m$

1. The number line will help you understand addition and subtraction of fractional numbers. Write the addition or subtraction equation suggested by each number line.

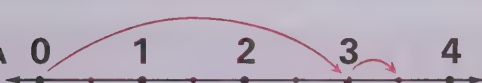
A


B


C


D


2. The number line will help you understand **mixed numerals,** such as $3\frac{1}{2}$, $2\frac{3}{4}$, and $1\frac{5}{6}$. Solve the equations.

A


$3 + n = 3\frac{1}{2}$

B


$n + \frac{3}{4} = 2\frac{3}{4}$

C


$1 + n = 1\frac{5}{6}$

D


$2 + \frac{3}{5} = n$

3. Find the sums and differences.

- A $\frac{2}{8} + \frac{3}{8}$
- B $\frac{5}{6} - \frac{1}{6}$
- C $\frac{4}{7} + \frac{2}{7}$
- D $\frac{6}{10} - \frac{2}{10}$
- E $\frac{7}{5} + \frac{4}{5}$
- F $\frac{9}{4} - \frac{2}{4}$
- G $\frac{6}{10} + \frac{6}{10}$
- H $\frac{7}{8} + \frac{2}{8}$
- I $\frac{10}{12} - \frac{3}{12}$
- J $\frac{3}{9} + \frac{6}{9}$
- K $\frac{9}{12} - \frac{9}{12}$
- L $\frac{0}{7} + \frac{5}{7}$
- M $\frac{5}{50} - \frac{4}{50}$
- N $\frac{7}{100} + \frac{10}{100}$
- O $\frac{75}{100} - \frac{38}{100}$
- P $\frac{62}{200} + \frac{59}{200}$

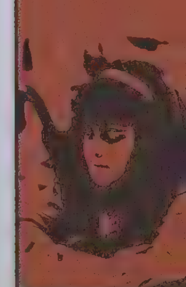
4. Give the correct mixed numeral for each sum.

- A $6 + \frac{1}{2}$
- B $9 + \frac{3}{4}$
- C $15 + \frac{7}{8}$
- D $27 + \frac{15}{16}$
- E $86 + \frac{9}{40}$
- F $97 + \frac{17}{100}$
- G $126 + \frac{37}{100}$
- H $528 + \frac{15}{73}$

think

$1 → 1 coin (silver dollar)
$1 → 2 coins (2 half dollars)
$1 → 3 coins (2 quarters, 1 half)
$1 → 4 coins (4 quarters)
$1 → 5 coins (1 half, 1 quarter, 2 dimes, 1 nickel)

Can you continue the pattern up to 20 coins?



● *How can we add or subtract when the fractions have different denominators?*

Investigating the Ideas

Make and cut out equivalent-fraction strips like these. Choose at least one other fractional number and make a strip for it.

One Half	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$
Three Fourths	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	$\frac{15}{20}$	$\frac{18}{24}$	$\frac{21}{28}$	$\frac{24}{32}$	$\frac{27}{36}$	$\frac{30}{40}$
Four Fifths	$\frac{4}{5}$	$\frac{8}{10}$	$\frac{12}{15}$	$\frac{16}{20}$	$\frac{20}{25}$	$\frac{24}{30}$	$\frac{28}{35}$	$\frac{32}{40}$	$\frac{36}{45}$	$\frac{40}{50}$
Two Thirds	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{16}{24}$	$\frac{18}{27}$	$\frac{20}{30}$

? Can you "line up" pairs of strips so that fractions with the same denominator match and then find the sum?

$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$
$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$
	$\frac{17}{12}$	

Discussing the Ideas

1. Use the example above to give this sum: $\frac{3}{4} + \frac{2}{3}$

2. Explain how you could use your equivalent-fraction strips to find $\frac{2}{3} - \frac{1}{2}$.

3. Explain this example. Then write enough fractions to find $\frac{1}{3} + \frac{1}{4}$.

$$\frac{5}{6} \longrightarrow \{\frac{5}{6}, \quad , \frac{15}{18}, \frac{20}{24}, \ldots\} \longrightarrow \frac{10}{12}$$

$$+\frac{1}{4} \longrightarrow \{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\} \longrightarrow \frac{3}{12}$$

$$n \longleftarrow \frac{13}{12}$$

1. For each exercise, find the numerators for ***a***, ***b***, and ***c***.

A To find $\frac{1}{2} + \frac{1}{3}$, we think $\frac{a}{6} + \frac{b}{6} = \frac{c}{6}$.

$\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots\}$ $\{\frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \ldots\}$

B To find $\frac{5}{6} - \frac{1}{4}$, we think $\frac{a}{12} - \frac{b}{12} = \frac{c}{12}$.

$\{\frac{5}{6}, \frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \ldots\}$ $\{\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \ldots\}$

2. Give the numbers for ***a*** and ***b***. Then give the number for ***c***.

A $\frac{3}{8} = \frac{a}{16}$, $\frac{1}{4} = \frac{b}{16}$ $\longrightarrow$ $\frac{3}{8} + \frac{1}{4} = \frac{c}{16}$

B $\frac{5}{6} = \frac{a}{12}$, $\frac{1}{4} = \frac{b}{12}$ $\longrightarrow$ $\frac{5}{6} - \frac{1}{4} = \frac{c}{12}$

3. For each exercise, make lists of equivalent fractions to find two fractions with the same denominator. Then find the sum or difference.

A $\frac{7}{8} + \frac{1}{4}$ **B** $\frac{3}{4} - \frac{1}{6}$ **C** $\frac{1}{10} + \frac{3}{4}$ **D** $\frac{3}{5} - \frac{1}{10}$ **E** $\frac{3}{4} + \frac{1}{10}$ **F** $\frac{1}{5} - \frac{1}{6}$

G $\frac{1}{2} + \frac{3}{4}$ **H** $\frac{1}{4} - \frac{1}{10}$ **I** $\frac{1}{3} + \frac{1}{5}$ **J** $\frac{1}{5} + \frac{3}{50}$ **K** $\frac{3}{4} - \frac{1}{10}$ **L** $\frac{1}{2} - \frac{1}{8}$ **M** $\frac{1}{6} + \frac{1}{9}$

N $\frac{51}{100} + \frac{1}{4}$ **O** $\frac{1}{2} - \frac{1}{8}$ **P** $\frac{1}{6} + \frac{3}{5}$ **Q** $\frac{5}{8} - \frac{1}{2}$ **R** $\frac{2}{5} - \frac{1}{4}$ **S** $\frac{1}{3} + \frac{1}{6}$ **T** $\frac{3}{20} - \frac{1}{10}$

think

There are 6 more children in room A than there are in room B. 5 children move from room A to room B. Now there are twice as many children in room B as in room A. How many children were in room A before the move?

More practice, page A-11, Set 21

What is the least common denominator for two fractions?

Investigating the Ideas

Use your equivalent-fraction strips for one half and two thirds in this Investigation.

One Half	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{4}{8}$	$\frac{5}{10}$	$\frac{6}{12}$	$\frac{7}{14}$	$\frac{8}{16}$	$\frac{9}{18}$	$\frac{10}{20}$

Two Thirds	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	$\frac{10}{15}$	$\frac{12}{18}$	$\frac{14}{21}$	$\frac{16}{24}$	$\frac{18}{27}$	$\frac{20}{30}$

? How many different ways can you "line up" your strips to help you find $\frac{1}{2} + \frac{2}{3}$?

Record your findings.

Discussing the Ideas

1. Explain why each sum you found with your strips is correct for $\frac{1}{2} + \frac{2}{3}$.

2. Which sum had the smallest denominator?

3. The smallest denominator that is the same for two fractions is called the least common denominator. What is the least common denominator for $\frac{1}{2}$ and $\frac{2}{3}$?

4. The least common denominator for two fractions is the least common multiple of the two denominators.
   - A List the multiples (to 54) of 6.
   - B List the multiples (to 54) of 9.
   - C List the common multiples (to 54) of 6 and 9.
   - D What is the least common multiple of 6 and 9?
   - E What is the least common denominator of $\frac{1}{6}$ and $\frac{1}{9}$?

1. A Is 6 a multiple of 9?
   B Is 2 × 6 a multiple of 9?
   C Is 3 × 6 a multiple of 9?
   D What is the least common multiple of 6 and 9?
   E What is the least common denominator for $\frac{1}{6}$ and $\frac{1}{9}$?

2. A Is 8 a multiple of 6?
   B Is 2 × 8 a multiple of 6?
   C Is 3 × 8 a multiple of 6?
   D What is the least common multiple of 6 and 8?
   E What is the least common denominator for $\frac{1}{6}$ and $\frac{1}{8}$?
   F What is the least common denominator for $\frac{5}{6}$ and $\frac{3}{8}$?

3. Find the least common multiple of the two numbers given in red. Then give the least common denominator for the two fractions.

   A $\frac{1}{6}, \frac{7}{9}$ B $\frac{7}{8}, \frac{1}{6}$ C $\frac{1}{3}, \frac{1}{6}$ D $\frac{1}{2}, \frac{5}{8}$ E $\frac{3}{4}, \frac{1}{6}$ F $\frac{1}{4}, \frac{3}{10}$

4. A Give the common multiples (less than 60) of 4, 6, and 8.
   B Give the least common multiple of 4, 6, and 8.
   C Give the least common denominator for $\frac{1}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$.

5. Find the least common denominator for each pair of fractions.

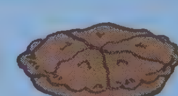
   A $\frac{1}{5}, \frac{1}{10}$ E $\frac{3}{4}, \frac{1}{3}$

   B $\frac{4}{5}, \frac{7}{10}$ F $\frac{1}{4}, \frac{2}{3}$

   C $\frac{5}{8}, \frac{1}{6}$ G $\frac{5}{6}, \frac{7}{10}$

   D $\frac{1}{8}, \frac{5}{6}$ H $\frac{1}{6}, \frac{9}{10}$

think



What do you know about numerator *a* if:

1. $\frac{a}{5} = 1$
2. $\frac{a}{5} = 4$
3. $\frac{a}{5} = 2\frac{3}{5}$
4. $\frac{a}{5} = 0$
5. $\frac{a}{5} < 1$
6. $\frac{a}{5} < 2$
7. $\frac{a}{5} > 1$
8. $\frac{a}{5} = a$

Is there a shortcut for adding and subtracting?

Investigating the Ideas

Study the flow chart for finding $\frac{1}{4} + \frac{5}{6}$.

Addition problem → Find the least common denominator. → Find the numerators. → Add to get the sum.

$$\begin{array}{r} \frac{1}{4} \\ +\frac{5}{6} \\ \hline \end{array} \qquad \begin{array}{rl} \frac{1}{4} = & \frac{}{12} \\ +\frac{5}{6} = & \frac{}{12} \\ \hline \end{array} \qquad \begin{array}{rl} \frac{1}{4} = & \frac{3}{12} \\ +\frac{5}{6} = & \frac{10}{12} \\ \hline \end{array} \qquad \begin{array}{rl} \frac{1}{4} = & \frac{3}{12} \\ +\frac{5}{6} = & \frac{10}{12} \\ \hline & \frac{13}{12} \end{array}$$

? Can you use the flow chart to help you find the sum $\frac{1}{2} + \frac{2}{5}$?

Discussing the Ideas

Explain the steps in each example below. Then find the sum or difference.

A

$$\begin{array}{r} \frac{1}{6} \\ +\frac{3}{8} \\ \hline \end{array}$$

Think: The least common denominator is 24.

Write:

$$\begin{array}{rl} \frac{1}{6} = & \frac{}{24} \\ +\frac{3}{8} = & \frac{}{24} \\ \hline \end{array}$$

Think: $\frac{1}{6} = \frac{4}{24}$ $\frac{3}{8} = \frac{9}{24}$

Write:

$$\begin{array}{rl} \frac{1}{6} = & \frac{4}{24} \\ +\frac{3}{8} = & \frac{9}{24} \\ \hline & \square \end{array}$$

B

$\frac{4}{9} - \frac{1}{6}$

Think: The least common denominator is 18.

Write:

$\frac{4}{9} - \frac{1}{6}$

$\frac{}{18} - \frac{}{18}$

Think: $\frac{4}{9} = \frac{8}{18}$ $\frac{1}{6} = \frac{3}{18}$

Write:

$\frac{4}{9} - \frac{1}{6}$

$\frac{8}{18} - \frac{3}{18} = \square$

1. Copy each exercise and give the missing numerators. Then find the sum or difference.

A $\begin{array}{r} \frac{1}{2} = \frac{\square}{6} \\ +\frac{1}{3} = \frac{\square}{6} \\ \hline \end{array}$ B $\begin{array}{r} \frac{1}{4} = \frac{\square}{8} \\ +\frac{3}{8} = \frac{\square}{8} \\ \hline \end{array}$ C $\begin{array}{r} \frac{5}{8} = \frac{\square}{8} \\ -\frac{1}{2} = \frac{\square}{8} \\ \hline \end{array}$ D $\begin{array}{r} \frac{5}{6} = \frac{\square}{18} \\ -\frac{1}{9} = \frac{\square}{18} \\ \hline \end{array}$ E $\begin{array}{r} \frac{5}{12} = \frac{\square}{12} \\ +\frac{1}{4} = \frac{\square}{12} \\ \hline \end{array}$

F $\frac{7}{12} + \frac{1}{3}$ $\quad \frac{\square}{12} + \frac{\square}{12}$

G $\frac{5}{6} - \frac{1}{3}$ $\quad \frac{\square}{6} - \frac{\square}{6}$

H $\frac{3}{10} - \frac{1}{4}$ $\quad \frac{\square}{20} - \frac{\square}{20}$

I $\frac{7}{10} + \frac{1}{2}$ $\quad \frac{\square}{10} + \frac{\square}{10}$

J $\frac{1}{8} + \frac{1}{6}$ $\quad \frac{\square}{24} + \frac{\square}{24}$

2. Find the sums and differences. Give your answers in lowest terms.

A $\begin{array}{r} \frac{1}{3} \\ +\frac{1}{6} \\ \hline \end{array}$ B $\begin{array}{r} \frac{1}{3} \\ -\frac{1}{6} \\ \hline \end{array}$ C $\begin{array}{r} \frac{7}{10} \\ -\frac{5}{100} \\ \hline \end{array}$ D $\begin{array}{r} \frac{1}{4} \\ +\frac{5}{12} \\ \hline \end{array}$ E $\begin{array}{r} \frac{5}{6} \\ -\frac{1}{9} \\ \hline \end{array}$ F $\begin{array}{r} \frac{1}{8} \\ +\frac{1}{12} \\ \hline \end{array}$ G $\begin{array}{r} \frac{7}{8} \\ -\frac{1}{2} \\ \hline \end{array}$

H $\frac{1}{4} + \frac{5}{6}$ I $\frac{5}{8} - \frac{1}{4}$ J $\frac{1}{8} + \frac{1}{12}$ K $\frac{3}{10} - \frac{1}{5}$ L $\frac{3}{10} - \frac{1}{4}$

M $\frac{2}{3} + \frac{2}{5}$ N $\frac{9}{10} - \frac{3}{4}$ O $\frac{7}{9} + \frac{1}{3}$ P $\frac{5}{8} - \frac{1}{6}$ Q $\frac{2}{10} + \frac{7}{100}$

3. Give the missing numerators in exercises A and B. Then find the sums for exercises C through L.

A $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{a}{6} + \frac{b}{6} + \frac{c}{6}$

B $\frac{3}{4} + \frac{1}{2} + \frac{1}{6}$

$\downarrow \quad \downarrow \quad \downarrow$

$\frac{a}{12} + \frac{b}{12} + \frac{c}{12}$

C $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

D $\frac{3}{4} + \frac{1}{2} + \frac{1}{6}$

E $\frac{2}{3} + \frac{1}{2} + \frac{5}{6}$

F $\frac{1}{4} + \frac{5}{6} + \frac{1}{2}$

G $\frac{1}{4} + \frac{2}{3} + \frac{1}{6}$

H $\frac{3}{8} + \frac{1}{3} + \frac{3}{4}$

I $\frac{1}{4} + \frac{3}{8} + \frac{1}{2}$

J $\frac{1}{4} + \frac{1}{8} + \frac{1}{2}$

K $\frac{1}{2} + \frac{7}{8} + \frac{1}{4}$

L $\frac{1}{2} + \frac{1}{4} + \frac{3}{5}$

think

68 legs on some chickens and horses.
22 animals in all.
How many chickens?

More practice, page A-11, Set 22

Short Stories

1. Studied $\frac{1}{2}$ hour.
Watched TV $\frac{1}{3}$ hour.
How much time in all?

2. Allan lives $\frac{6}{10}$ km from school.
Cheryl lives $\frac{1}{4}$ km from school.
How much closer does Cheryl live?

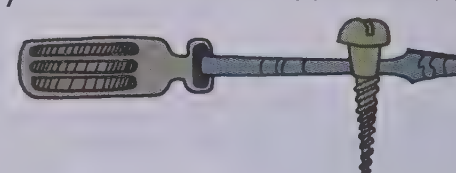
3. Nancy: $\frac{1}{6}$ of a pie.
Bill and Bob: $\frac{3}{4}$ of the pie.
How much of the pie in all?

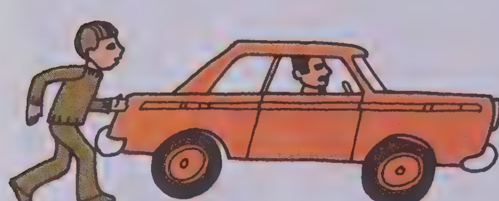
4. Ran $\frac{1}{2}$ km.
Walked $\frac{1}{5}$ km.
Skipped $\frac{1}{10}$ km.
How far in all?

5. Spent $\frac{1}{2}$ dollar. Spent $\frac{1}{4}$ dollar.
Spent $\frac{1}{10}$ dollar. Spent what part of a dollar in all?

6. Screw: 4 centimetres long.
Threaded section: $\frac{3}{4}$ of the length.
How many centimetres without threads?

7. Mowed $\frac{2}{3}$ of the lawn.
Then mowed $\frac{1}{4}$ of the lawn.
How much more to mow?





8. $\frac{5}{8}$ of class: brown hair.
$\frac{1}{3}$ of class: blonde hair.
Rest of class: red hair.
What part has red hair?

9. Walked $\frac{1}{5}$ of a kilometre.
Rode $\frac{3}{4}$ of a kilometre.
How far in all?

10. Beth: $\frac{7}{10}$ of a dollar.
Larry: $\frac{4}{5}$ of a dollar.
How much more does Larry have?

Keeping in Touch with

Addition
Subtraction
Multiplication
Division
Measurement
Principles

1. Find the sum, difference, product, and quotient.

A $\begin{array}{r} 27\,643 \\ +54\,896 \\ \hline \end{array}$

B $\begin{array}{r} 57\,384 \\ -32\,641 \\ \hline \end{array}$

C $\begin{array}{r} 64\,237 \\ \times 24 \\ \hline \end{array}$

D $42\overline{)13\,692}$

2. Each of these exercises illustrates one of the basic principles. Give the number of the principle.

A $3278 \times 26 = 26 \times 3278$
Answer: 2

B $264 + (278 + 315) = (264 + 278) + 315$

C $4 \times (26 + 57) = (4 \times 26) + (4 \times 57)$

D $(6428 + 32) + 0 = 6428 + 32$

E $276 \times 3426 = 3426 \times 276$

F $(76 \times 28) \times 463 = 76 \times (28 \times 463)$

G $1 \times (327 + 683) = 327 + 683$

H $7 + (326 + 483) = 7 + (483 + 326)$

I $(29 + 83) + 65 = 65 + (29 + 83)$

J $(3 \times 28) + (3 \times 43) = 3 \times (28 + 43)$

$\boxed{1}$ Commutative principle (addition)
$\boxed{2}$ Commutative principle (multiplication)
$\boxed{3}$ Associative principle (addition)
$\boxed{4}$ Associative principle (multiplication)
$\boxed{5}$ Distributive principle
$\boxed{6}$ Zero principle
$\boxed{7}$ One principle

3. Solve the equations.

A $648+274=n+648$

B $(23\times 65)\times 34=23\times (n\times 34)$

C $572\times 927=927\times n$

D $6\times (254+9)=(6\times n)+54$

E $(82+n)+21=82+(48+21)$

4. Give the measure of each angle.

A $\angle CAB$

B $\angle DAB$

C $\angle EAB$

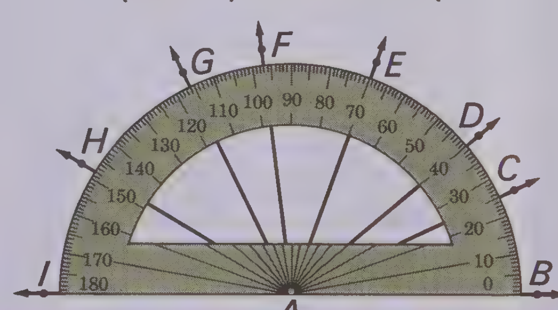
D $\angle CAE$

E $\angle FAB$

F $\angle GAB$

G $\angle HAB$

H $\angle IAB$



You are invited to explore

ACTIVITY CARD 8
Page 359

Discussing the Ideas

1. Give the correct numerator for ***a***. Then give the correct improper fraction for ***b***.

A $2\frac{1}{3} = \frac{a}{3} + \frac{1}{3} = b$ C $12\frac{1}{4} = \frac{a}{4} + \frac{1}{4} = b$ E $7\frac{3}{4} = \frac{a}{4} + \frac{3}{4} = b$

B $3\frac{1}{4} = \frac{a}{4} + \frac{1}{4} = b$ D $10\frac{2}{3} = \frac{a}{3} + \frac{2}{3} = b$ F $3\frac{2}{5} = \frac{a}{5} + \frac{2}{5} = b$

2. Study the flow chart for writing an improper fraction for a mixed numeral.

Mixed numeral → Multiply the denominator by the whole number. → Add the numerator to the product. → Write the sum over the denominator.

$2\frac{3}{5}$ $2 \times 5 = 10$ $10 + 3 = 13$ $\frac{13}{5}$ ← Improper fraction

Can you explain the flow chart and compare it with exercise 1 ?

3. Give the numerator for ***a*** and the whole number for ***b***. Then give the mixed numeral for ***c***.

A $\frac{11}{3} = \frac{a}{3} + \frac{2}{3} \rightarrow \frac{11}{3} = b + \frac{2}{3} \rightarrow \frac{11}{3} = c$

B $\frac{23}{5} = \frac{a}{5} + \frac{3}{5} \rightarrow \frac{23}{5} = b + \frac{3}{5} \rightarrow \frac{23}{5} = c$

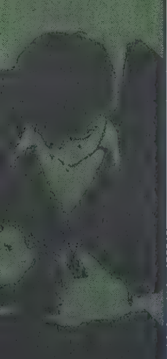
C $\frac{9}{2} = \frac{a}{2} + \frac{1}{2} \rightarrow \frac{9}{2} = b + \frac{1}{2} \rightarrow \frac{9}{2} = c$

4. Study the flow chart for writing a mixed numeral for an improper fraction.

Improper fraction → Divide the numerator by the denominator. → Use the quotient as the whole-number part. → Write the remainder over the denominator.

$\frac{23}{6}$ $6\overline{)23}$ 3 R5 $\frac{23}{6} = 3\frac{\square}{\square}$ $3\frac{5}{6}$ ← Mixed numeral

Can you explain the flow chart and compare it with exercise 3 ?



1. Write an improper fraction for each mixed numeral.

A $1\frac{1}{2}$ C $1\frac{1}{3}$ E $2\frac{3}{5}$ G $1\frac{3}{4}$

B $4\frac{3}{8}$ D $1\frac{1}{4}$ F $1\frac{2}{3}$ H $6\frac{7}{10}$

2. Write a mixed numeral for each improper fraction.

A $\frac{5}{2}$ C $\frac{17}{6}$ E $\frac{8}{3}$ G $\frac{11}{4}$

B $\frac{7}{3}$ D $\frac{9}{4}$ F $\frac{20}{7}$ H $\frac{21}{10}$

3. Give the missing numerators.

A $\frac{8}{5} = 1 + \frac{\square}{5}$ E $\frac{13}{10} = 1 + \frac{\square}{10}$ I $\frac{9}{6} = 1 + \frac{\square}{2}$ M $\frac{32}{20} = 1 + \frac{\square}{5}$

B $\frac{7}{4} = 1 + \frac{\square}{4}$ F $\frac{15}{10} = 1 + \frac{\square}{10}$ J $\frac{8}{6} = 1 + \frac{\square}{3}$ N $\frac{16}{10} = 1 + \frac{\square}{5}$

C $\frac{6}{5} = 1 + \frac{\square}{5}$ G $\frac{9}{5} = 1 + \frac{\square}{5}$ K $\frac{10}{6} = 1 + \frac{\square}{3}$ O $\frac{14}{8} = 1 + \frac{\square}{4}$

D $\frac{5}{3} = 1 + \frac{\square}{3}$ H $\frac{10}{8} = 1 + \frac{\square}{4}$ L $\frac{24}{20} = 1 + \frac{\square}{10}$ P $\frac{60}{50} = 1 + \frac{\square}{25}$

4. Solve the equations. Use lowest-terms fractions for all answers.

A $5\frac{9}{8} = 6 + n$ F $15\frac{7}{4} = n + \frac{3}{4}$ K $83\frac{9}{6} = n + \frac{1}{2}$

B $6\frac{15}{10} = 7 + n$ G $36\frac{8}{5} = 37 + n$ L $46\frac{16}{10} = 47 + n$

C $3\frac{17}{10} = 4 + n$ H $43\frac{9}{7} = 44 + n$ M $12\frac{7}{2} = 13 + n$

D $2\frac{12}{7} = 3 + n$ I $95\frac{10}{6} = 96 + n$ N $14\frac{12}{9} = n + \frac{1}{3}$

E $12\frac{6}{5} = 13 + n$ J $36\frac{12}{10} = 37 + n$ O $13\frac{6}{4} = 14 + n$

5. Copy each exercise. Give the missing numerators.

A $4\frac{1}{2} = 4\frac{a}{4} = 3\frac{b}{4}$

B $5\frac{1}{3} = 5\frac{a}{12} = 4\frac{b}{12}$

C $6\frac{3}{4} = 6\frac{a}{12} = 5\frac{b}{12}$

D $7\frac{2}{3} = 7\frac{a}{9} = 6\frac{b}{9}$

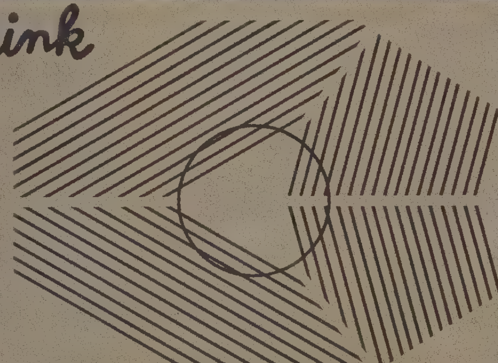
E $12\frac{2}{5} = 12\frac{a}{10} = 11\frac{b}{10}$

F $30\frac{3}{10} = 30\frac{a}{50} = 29\frac{b}{50}$

G $1 = 1\frac{a}{3} = \frac{b}{3}$

H $9 = 9\frac{a}{4} = 8\frac{b}{4}$

think



Is this "circle" perfectly round? After you have decided, use a penny to check.

More practice, page A-12, Set 23

Let's use the basic principles for fractional numbers.

Investigating the Ideas

Check this equation. → $5 + 2 + \frac{1}{2} + \frac{1}{4} = 7\frac{3}{4}$

This equation was formed from the four addends in the first equation. → $7 + \frac{3}{4} = 7\frac{3}{4}$

? How many other equations can you write by combining the addends 5, 2, $\frac{1}{2}$, and $\frac{1}{4}$ in different ways?	Record the equations you find.

Discussing the Ideas

1. What principle is illustrated by each equation?

A $\frac{3}{4} + \frac{1}{2} = \frac{1}{2} + \frac{3}{4}$

B $\frac{5}{6} + (\frac{1}{10} + \frac{3}{5}) = (\frac{5}{6} + \frac{1}{10}) + \frac{3}{5}$

2. Give numbers for ***a***, ***b***, and ***c***. Explain your answers.

A $2\frac{3}{8} + 4\frac{1}{4} = (2 + \boldsymbol{a}) + (4 + \frac{1}{4}) = 6 + \frac{5}{8} = \boldsymbol{b}$

B $5\frac{1}{2} + 2\frac{1}{4} = (5 + 2) + (\frac{1}{2} + \boldsymbol{a}) = 7 + \frac{3}{4} = \boldsymbol{b}$

C $7\frac{1}{4} + 2\frac{1}{6} = (7 + 2) + (\frac{1}{4} + \boldsymbol{a}) = 9 + \boldsymbol{b} = \boldsymbol{c}$

3. Solve the equations. Explain your answers.

A $5\frac{1}{2} + 1\frac{1}{4} = \boldsymbol{n} + \frac{3}{4}$

B $2\frac{1}{8} + 3\frac{1}{4} = \boldsymbol{c} + \frac{3}{8}$

C $4\frac{1}{8} + 3\frac{1}{2} = 7 + \boldsymbol{b}$

D $7\frac{1}{5} + 2\frac{3}{5} = 9 + \boldsymbol{t}$

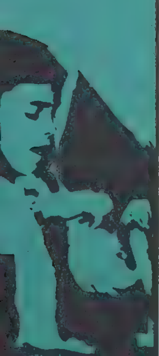
E $8\frac{3}{8} + 1\frac{1}{2} = \boldsymbol{x} + \frac{7}{8}$

F $6\frac{2}{7} + 2\frac{4}{7} = 8 + \boldsymbol{d}$

4. Explain how you can use the statement in the box to complete the example and find the sum.

> Since you can change the order and the grouping of addends, you can arrange the addends in any way that is convenient.

$$2\frac{1}{4} + 3\frac{1}{2} = (2 + \frac{1}{4}) + (3 + \frac{1}{2}) = \ ?$$



Find the sums and differences. Use mixed numerals and lowest-terms fractions for your answers.

1. Example:

$$\frac{5}{6} = \frac{5}{6}$$
$$+\frac{2}{3} = \frac{4}{6}$$
$$\frac{9}{6} = 1\frac{1}{2}$$

A $\frac{1}{2} + \frac{3}{4}$ B $\frac{5}{8} + \frac{2}{3}$ C $\frac{5}{6} + \frac{1}{2}$ D $\frac{2}{3} + \frac{2}{3}$ E $\frac{7}{10} + \frac{4}{5}$

F $\frac{61}{100} + \frac{7}{10}$ G $\frac{1}{4} + \frac{5}{6}$ H $\frac{11}{20} + \frac{4}{5}$ I $\frac{11}{15} + \frac{7}{10}$ J $\frac{3}{5} + \frac{9}{10}$

2. Example:

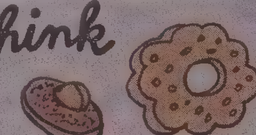
$$3\frac{5}{8} = 3\frac{15}{24}$$
$$+8\frac{2}{3} = 8\frac{16}{24}$$
$$11\frac{31}{24} = 12\frac{7}{24}$$

A $5\frac{1}{2} + 2\frac{3}{4}$ B $6\frac{5}{8} + 7\frac{2}{3}$ C $3\frac{5}{6} + 8\frac{1}{2}$ D $9\frac{2}{3} + 1\frac{2}{3}$ E $6\frac{7}{10} + 7\frac{3}{5}$

F $8\frac{2}{3} + 3\frac{3}{5}$ G $7\frac{1}{4} + 2\frac{5}{6}$

H $61\frac{1}{2} + 91\frac{1}{8}$ I $82\frac{3}{8} + 72\frac{1}{4}$ J $15\frac{7}{8} + 27\frac{5}{6}$ K $83\frac{2}{3} + 36\frac{3}{4}$

think



Fran gave Jan half her cookies and half a cookie. Fran had 7 cookies left. How many did Jan get?

BE CAREFUL ! ! !

3. Example:

$$7\frac{1}{2} = 7\frac{2}{4} = 6\frac{6}{4}$$
$$-2\frac{3}{4} = 2\frac{3}{4} = 2\frac{3}{4}$$
$$4\frac{3}{4}$$

A $9\frac{1}{2} - 2\frac{3}{4}$ B $8\frac{5}{8} - 1\frac{2}{3}$

C $6\frac{3}{10} - 1\frac{1}{5}$ D $7\frac{1}{4} - 6\frac{5}{6}$ E $8\frac{7}{10} - 5\frac{4}{5}$ F $4\frac{7}{10} - 2\frac{11}{15}$ G $7\frac{3}{5} - 3\frac{9}{10}$

H $26\frac{1}{2} - 15\frac{1}{4}$ I $47\frac{3}{4} - 39\frac{2}{3}$ J $83\frac{2}{3} - 15\frac{3}{4}$ K $91\frac{5}{6} - 56\frac{1}{8}$ L $65\frac{1}{8} - 38\frac{5}{6}$ M $27\frac{7}{10} - 9\frac{4}{5}$ N $96\frac{5}{6} - 56\frac{9}{10}$

More practice, page A-12, Set 24

Investigating the Ideas

The red train and the light green train are matching trains. Each train is 6 units long. Six is the least common multiple for 2 and 3.

Can you find out how long the purple, dark green, and blue trains will be when they all match?

Discussing the Ideas

1. A How long were the three trains when they matched?
   B What is the least common multiple for 4, 6, and 9?
   C What is the least common denominator for $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{9}$?

2. You can find the least common denominator of three or more fractional numbers by finding the least common denominator of two fractions at a time. Explain each step below.

$\frac{1}{6}$, $\frac{1}{9}$: The least common multiple of 6 and 9 is 18.

$+\frac{1}{4}$: The least common multiple of 18 and 4 is 36.

The least common multiple of 6, 9, and 4 is 36.

3. Once you find the least common multiple for the denominators, you have found the least common denominator for the fractions. Explain the steps in the example and give the sum.

$$\begin{aligned} \frac{1}{6} &= \frac{6}{36} \\ \frac{1}{9} &= \frac{4}{36} \\ +\frac{1}{4} &= \frac{9}{36} \end{aligned}$$

1. Find the sums. Give the sums in lowest terms.

A. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$
B. $\frac{3}{8} + \frac{1}{4} + \frac{1}{2}$
C. $\frac{1}{6} + \frac{1}{3} + \frac{1}{2}$
D. $\frac{1}{6} + \frac{2}{3} + \frac{1}{2}$
E. $\frac{3}{4} + \frac{1}{2} + \frac{5}{8}$
F. $\frac{1}{3} + \frac{3}{4} + \frac{1}{2}$
G. $\frac{1}{5} + \frac{3}{20} + \frac{1}{10}$
H. $\frac{1}{6} + \frac{1}{4} + \frac{1}{2}$

I. $3\frac{1}{2} + 7\frac{1}{4} + 6\frac{1}{8}$
J. $4\frac{1}{6} + 2\frac{1}{3} + 8\frac{2}{3}$
K. $19\frac{4}{5} + 26\frac{1}{10} + 35\frac{1}{4}$
L. $47\frac{5}{6} + 58\frac{1}{8} + 26\frac{1}{3}$
M. $158\frac{1}{4} + 327\frac{3}{8} + 962\frac{1}{2}$
N. $785\frac{5}{6} + 654\frac{1}{2} + 327\frac{2}{3}$

Give the missing numbers and function rules.

2. Function Rule: $n + \frac{1}{4} + \frac{1}{2}$

	n	$f(n)$
A	$\frac{1}{6}$	
B	$\frac{1}{3}$	
C	$\frac{3}{8}$	
D	$\frac{1}{2}$	
E	$\frac{3}{10}$	

3. Function Rule: $8\frac{7}{8} + n$

	n	$f(n)$
A	$3\frac{1}{4}$	
B	$6\frac{2}{3}$	
C	$5\frac{1}{2}$	
D	$7\frac{3}{10}$	
E	$9\frac{3}{4}$	

4. Function Rule: $n - 4\frac{1}{3}$

	n	$f(n)$
A	$8\frac{5}{6}$	
B	$7\frac{1}{2}$	
C	$6\frac{1}{6}$	
D	$5\frac{1}{4}$	
E	$9\frac{3}{5}$	

★ 5. Function Rule: $(5\frac{2}{3} + n) - 2\frac{2}{3}$

	n	$f(n)$
A	$3\frac{1}{4}$	
B	$4\frac{3}{4}$	
C	$7\frac{1}{6}$	
D	$2\frac{4}{5}$	
E	$1\frac{7}{8}$	

★ 6. Function Rule: A.

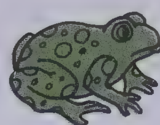
	n	$f(n)$
	$4\frac{1}{4}$	$3\frac{1}{2}$
	$2\frac{1}{8}$	$1\frac{3}{8}$
	$7\frac{7}{8}$	$7\frac{1}{8}$
B	$6\frac{3}{8}$	
C		9

★ 7. Function Rule: A.

	n	$f(n)$
	$\frac{1}{2}$	$\frac{1}{2}$
	$\frac{1}{3}$	$\frac{2}{3}$
	$\frac{3}{5}$	$\frac{2}{5}$
B	$\frac{5}{8}$	
C	1	

Short Stories

1. Fish: First weighed $1\frac{3}{4}$ kg. Second weighed $2\frac{1}{10}$ kg.
   A How much did they weigh together?
   B How much more did the second weigh than the first?

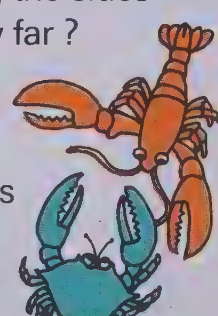


2. Frog: Jumped $3\frac{7}{10}$ m. Then jumped $2\frac{3}{4}$ m.
   A How far in all?
   B How much farther was the first jump than the second?

3. Driving: $57\frac{1}{2}$ km the first hour. $49\frac{7}{10}$ km the second hour. How far in the two hours?

4. Stars: Alpha Centauri is $4\frac{1}{3}$ light years* away. Sirius is $8\frac{3}{5}$ light years away. How much farther is Sirius than Alpha Centauri?

5. Rectangle: $4\frac{1}{2}$ cm by $6\frac{7}{10}$ cm. Ant crawls from one corner to the opposite corner, moving along the sides of the rectangle. How far?

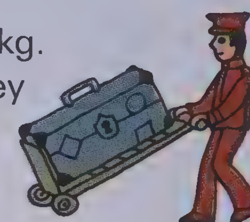


6. Lobster: $3\frac{3}{10}$ kg. Blue crab: $1\frac{1}{4}$ kg. How much more does the lobster weigh?

*A light year is the distance light travels in one year.

7. Grasshopper:
   A Jumped to $3\frac{5}{8}$. Then jumped to $6\frac{1}{4}$. How far was the second jump?
   B Jumped to $4\frac{1}{2}$. Then jumped back to $\frac{7}{8}$. How far was the second jump?

8. Steak: T-bone, $\frac{7}{10}$ kg.
   Rib, $\frac{3}{5}$ kg.
   Sirloin, $1\frac{1}{4}$ kg.
   How much do they weigh together?



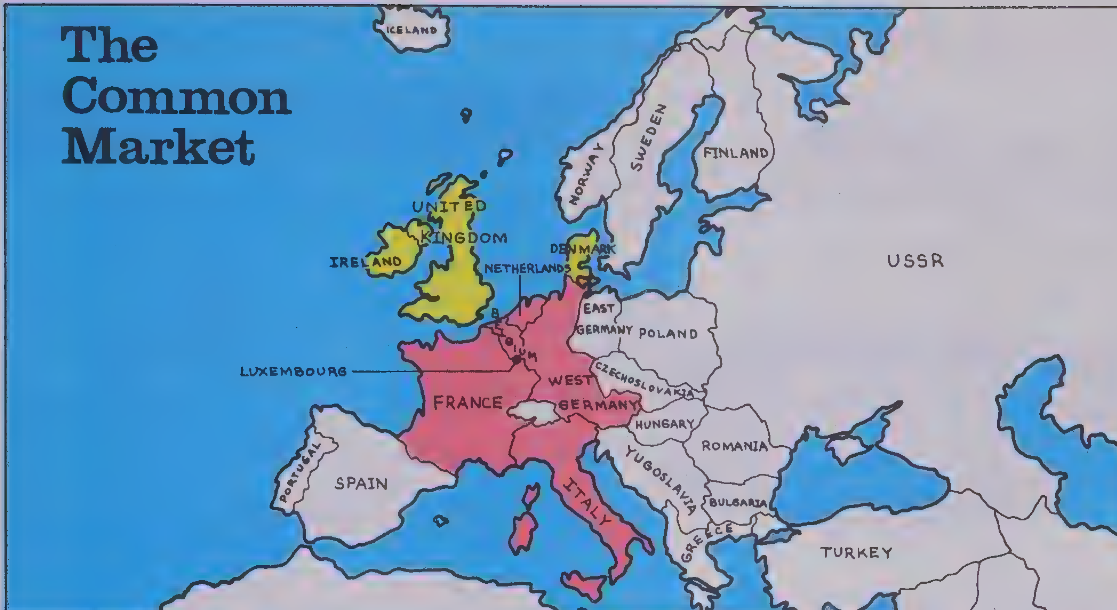
9. Plane trip: Baggage allowance, 20 kg. Mr. Lo's suitcase weighed $29\frac{6}{10}$ kg. How much overweight was his bag?

10. Tom: $113\frac{1}{2}$ cm tall. Grew $1\frac{1}{4}$ cm. Then grew $2\frac{1}{10}$ cm. How tall now?

11. Reading: $\frac{3}{4}$ hour. Arithmetic: $\frac{2}{3}$ hour. Writing: $\frac{1}{2}$ hour. How long for all three?

12. Triangle's sides: $3\frac{1}{5}$ cm, $5\frac{7}{10}$ cm, $4\frac{3}{4}$ cm. What is the perimeter?

13. Rectangle: $6\frac{7}{10}$ by $2\frac{4}{5}$. What is the perimeter?



On January 1, 1958, Belgium, France, Italy, Luxembourg, West Germany, and the Netherlands signed a treaty establishing the European Economic Community. This is generally known as the Common Market. The United Kingdom, Denmark, and Ireland joined the Common Market on January 1, 1973, bringing the total membership to nine.

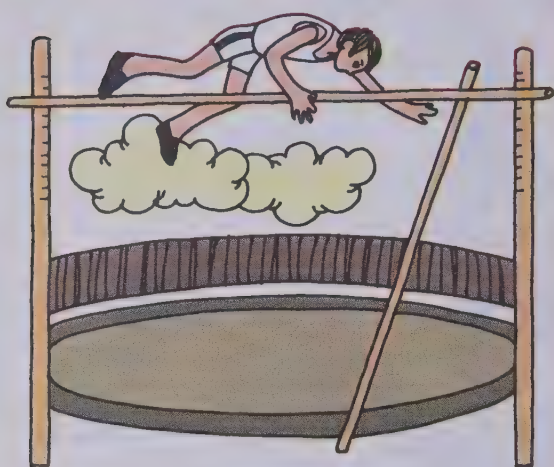
1. What fraction of the total membership
   A were original members?
   B joined in 1973?

2. Denmark, the United Kingdom, the Netherlands, Belgium, and Luxembourg are monarchies (they have a king or queen). What fraction of the Common Market countries are not monarchies?

3. Twenty-four European countries are members of the United Nations. All the Common Market countries except West Germany are members of the United Nations. What fraction of the European UN members are not members of the Common Market?

4. The population of Europe is $1\frac{8}{9}$ times as great as the population of the USSR. The population of Europe is $1\frac{5}{6}$ times as great as the population of the Common Market countries. Are there more people in the USSR or in the Common Market countries? By how much?

Track Records



Olympic Track Records*	
Running high jump	$224\frac{15}{100}$ cm
Long jump	$8\frac{9}{10}$ m
Pole vault	$5\frac{1}{2}$ m
Javelin	$90\frac{48}{100}$ m
Discus throw	$64\frac{78}{100}$ m
100-metre run	$9\frac{9}{10}$ s
200-metre run	$19\frac{8}{10}$ s
800-metre run	1 min $44\frac{3}{10}$ s
1500-metre run	3 min $34\frac{9}{10}$ s
5000-metre run	13 min $26\frac{4}{10}$ s

1. How much higher is the pole-vault record than the high-jump record ?

2. How much farther is the javelin record than the discus record ?

3. The Olympic women's record for the long jump is $208\frac{28}{100}$ cm shorter than the men's record. What is this record ?

4. The Olympic swimming record for the 100-metre freestyle is $51\frac{2}{10}$ seconds. The same man holds the record for the 200-metre freestyle, 1 minute $52\frac{8}{10}$ seconds. If he could swim the 200 metres at the same speed as the 100, how much time would be cut from the present record ?

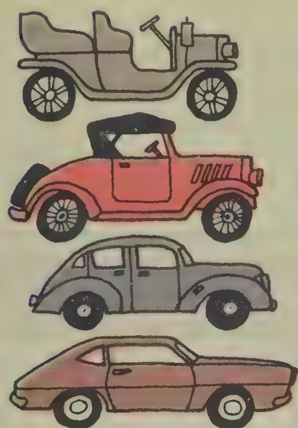
5. What is the difference between the time for the 5000-metre run and the time for the 1500-metre run ?

★6. A If the 800-metre runner could run at his record pace for 1500 metres, how fast could he run the 1500 metres ?
   B What is the difference between this time and the record for the 1500-metre run ?

*The records given on this page are Olympic track and field records for men including the 1972 games.

Automobiles

Number of passenger cars built in selected years



Each symbol stands for 1 000 000 cars.

1900 — 4192
1910 — 181 000
1920 — 1 905 560
1930 — 2 787 456
1940 — 3 717 385
1950 — 6 665 863
1960 — 6 674 796
1970 — 6 550 203

In the early 1900's, automobiles were referred to as "horseless carriages." Today the automobile plays a vital role in all our lives.

1. How many more cars were built **A** in 1940 than in 1930? **B** in 1950 than in 1930? **C** in 1970 than in 1900?

2. There are more than 280 000 000 people in North America. About what is the average number of people per car?

3. In 1925 a new car could be expected to last about $6\frac{1}{2}$ years. Today's cars can be expected to last about $12\frac{1}{4}$ years. How much longer can cars be expected to last today than in 1925?

4. A car travelling 80 km/h travels $16\frac{3}{4}$ metres between the time a driver decides to stop and the time he applies the brakes. It travels another 57 metres after he applies the brakes. What is the total stopping distance from the time the driver decides to stop?

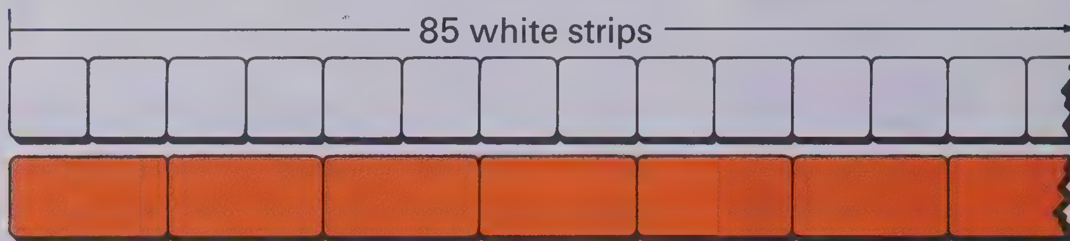
5. In 1911 the winner of the Indianapolis "500" auto race averaged $120\frac{2}{100}$ km/h. In 1972 the winner averaged $262\frac{20}{100}$ km/h. How much faster was the average speed in 1972 than in 1911?

More practice, page A-13, Set 25

Let's explore mixed numerals and improper fractions again.

Investigating the Ideas

Suppose you have a train made of 85 white strips.



It takes $42\frac{1}{2}$ red strips to match the white strips.

? Can you give a mixed numeral for the number of light green strips it takes to match the 85 white strips?

Discussing the Ideas

1. If the red strip is the unit, can you find two ways to express the length of the white train above?

2. A If the light green strip is the unit, what improper fraction gives the length of the white train?
   B What mixed numeral gives the length of the train?

3. Think of the purple strip as the unit.
   A Give an improper fraction for the length of the white train.
   B Give a mixed numeral for n. $\frac{85}{4} = n$

4. Solve: $\frac{85}{5} = n$

5. Explain these examples. They show how you can use division to write a mixed numeral for an improper fraction.

A $\;6\overline{)85}$ = 14 R 1 $\longrightarrow$ $\frac{85}{6} = \frac{14 \times 6}{6} + \frac{1}{6}$ $\longrightarrow$ $\boxed{\frac{85}{6} = 14\frac{1}{6}}$

B $\;8\overline{)85}$ = 10 R 5 $\longrightarrow$ $\frac{85}{8} = \frac{10 \times 8}{8} + \frac{5}{8}$ $\longrightarrow$ $\boxed{\frac{85}{8} = 10\frac{5}{8}}$

1. Give the numerator for *a* and the whole number for *b*.

A $\frac{35}{2} = \frac{a}{2} + \frac{1}{2} = b + \frac{1}{2}$

B $\frac{52}{3} = \frac{a}{3} + \frac{1}{3} = b + \frac{1}{3}$

C $\frac{95}{4} = \frac{a}{4} + \frac{3}{4} = b + \frac{3}{4}$

D $\frac{83}{6} = \frac{a}{6} + \frac{5}{6} = b + \frac{5}{6}$

E $\frac{87}{5} = \frac{a}{5} + \frac{2}{5} = b + \frac{2}{5}$

F $\frac{85}{2} = \frac{a}{2} + \frac{1}{2} = b + \frac{1}{2}$

G $\frac{191}{8} = \frac{a}{8} + \frac{7}{8} = b + \frac{7}{8}$

H $\frac{267}{7} = \frac{a}{7} + \frac{1}{7} = b + \frac{1}{7}$

2. Give the number for *q* (quotient) and the number for *r* (remainder). Then give a mixed numeral for *m*.

A $\frac{15}{2} = \frac{q \times 2}{2} + \frac{r}{2} = m$

B $\frac{25}{4} = \frac{q \times 4}{4} + \frac{r}{4} = m$

C $\frac{36}{7} = \frac{q \times 7}{7} + \frac{r}{7} = m$

D $\frac{42}{8} = \frac{q \times 8}{8} + \frac{r}{8} = m$

E $\frac{67}{3} = \frac{q \times 3}{3} + \frac{r}{3} = m$

F $\frac{84}{5} = \frac{q \times 5}{5} + \frac{r}{5} = m$

G $\frac{93}{6} = \frac{q \times 6}{6} + \frac{r}{6} = m$

H $\frac{27}{2} = \frac{q \times 2}{2} + \frac{r}{2} = m$

3. Give the mixed numeral for each fraction.

A $\frac{18}{4}$ G $\frac{97}{6}$ M $\frac{125}{100}$

B $\frac{42}{5}$ H $\frac{53}{4}$ N $\frac{164}{8}$

C $\frac{61}{7}$ I $\frac{76}{5}$ O $\frac{327}{5}$

D $\frac{92}{10}$ J $\frac{134}{8}$ P $\frac{841}{10}$

E $\frac{81}{50}$ K $\frac{113}{5}$ Q $\frac{253}{5}$

F $\frac{65}{8}$ L $\frac{261}{20}$ R $\frac{423}{2}$

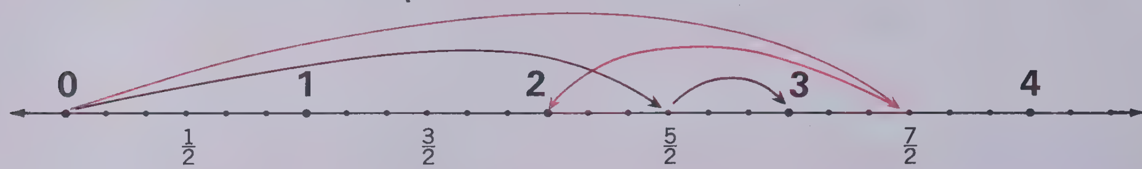
think

Suppose that first-class mailing costs 8¢ for each 25 grams, plus 8¢ for any additional weight under 25 grams. Using these rates, give the cost for first-class mailing of items weighing:

1. 126 grams
2. 257 grams
3. 87 grams
4. 210 grams

More practice, page A-13, Set 26

1. Write an addition equation for the black arrows.
Write a subtraction equation for the red arrows.



2. Find the sums and differences.

A $\frac{1}{4}+\frac{1}{2}$ B $\frac{1}{4}-\frac{1}{6}$ C $\frac{1}{3}-\frac{1}{6}$ D $\frac{7}{10}+\frac{1}{5}$ E $\frac{3}{4}-\frac{3}{8}$ F $\frac{2}{3}+\frac{1}{6}$

G $\frac{1}{8}+\frac{5}{6}$ H $\frac{3}{5}-\frac{1}{10}$ I $\frac{3}{10}+\frac{1}{4}$ J $\frac{9}{10}+\frac{4}{5}$ K $\frac{1}{4}-\frac{1}{5}$ L $\frac{9}{10}-\frac{3}{4}$ M $\frac{1}{3}+\frac{1}{10}$ N $\frac{5}{6}-\frac{4}{5}$

3. Solve the equations.

A $\frac{1}{4}+n=\frac{3}{4}$ B $\frac{5}{6}-n=\frac{1}{6}$ C $n+\frac{1}{2}=\frac{3}{2}$ D $\frac{9}{10}+n=1$

4. Find the sums and differences.

A $17\frac{1}{8}+16\frac{1}{2}$ B $42\frac{7}{10}+31\frac{1}{2}$ C $65\frac{3}{4}-23\frac{3}{8}$ D $84\frac{5}{6}+12\frac{1}{4}$ E $61\frac{1}{4}-17\frac{1}{5}$ F $83\frac{3}{4}+98\frac{1}{2}$

G $56\frac{2}{3}-19\frac{1}{6}$ H $81\frac{1}{4}-78\frac{1}{2}$ I $32\frac{9}{10}-19\frac{4}{5}$ J $64\frac{1}{5}-28\frac{1}{4}$ K $53\frac{3}{4}+67\frac{9}{10}$ L $52\frac{1}{5}+47\frac{1}{10}$

5. Give the missing numbers.

Function Rule: $n+2\frac{1}{2}$

n	$f(n)$
$3\frac{5}{8}$	A
$6\frac{1}{4}$	B
$9\frac{1}{6}$	C
$5\frac{7}{10}$	D

Function Rule: $n-4\frac{1}{3}$

n	$f(n)$
$8\frac{1}{2}$	E
$7\frac{5}{6}$	F
G	8
$5\frac{1}{10}$	H

think

Copy each equation on your paper and give correct signs (+ or −) for each $\bigcirc$.

1. $(\frac{1}{2} \bigcirc \frac{1}{4}) \bigcirc \frac{1}{8}=\frac{5}{8}$
2. $\frac{1}{2} \bigcirc (\frac{1}{2} \bigcirc \frac{1}{2})=\frac{1}{2}$
3. $\frac{1}{2} \bigcirc (\frac{1}{3} \bigcirc \frac{1}{6})=\frac{1}{3}$

6. Give a whole number for each improper fraction.

A $\frac{10}{5}$ B $\frac{10}{2}$ C $\frac{12}{3}$ D $\frac{12}{4}$ E $\frac{24}{3}$ F $\frac{36}{3}$ G $\frac{52}{2}$ H $\frac{75}{5}$

7. Give an improper fraction for each number.

A 3 B $3\frac{1}{2}$ C $3\frac{2}{3}$ D $3\frac{3}{4}$ E $5\frac{1}{2}$ F $6\frac{3}{5}$ G $7\frac{4}{5}$ H $15\frac{3}{10}$

8. Give a mixed numeral for each improper fraction.

A $\frac{15}{2}$ B $\frac{20}{3}$ C $\frac{41}{2}$ D $\frac{83}{10}$ E $\frac{126}{4}$ F $\frac{257}{8}$ G $\frac{356}{5}$ H $\frac{483}{10}$

9. Find the sums. Use lowest-terms fractions for all answers.

A $\begin{array}{r} 3\frac{1}{2} \\ 8\frac{1}{4} \\ +6\frac{1}{2} \\ \hline \end{array}$ B $\begin{array}{r} 2\frac{1}{4} \\ 9\frac{1}{2} \\ +7\frac{1}{4} \\ \hline \end{array}$ C $\begin{array}{r} 9\frac{1}{2} \\ 6\frac{1}{3} \\ +5\frac{1}{4} \\ \hline \end{array}$ D $\begin{array}{r} 14\frac{1}{6} \\ 16\frac{1}{2} \\ +18\frac{1}{3} \\ \hline \end{array}$ E $\begin{array}{r} 32\frac{1}{20} \\ 61\frac{1}{10} \\ +15\frac{1}{5} \\ \hline \end{array}$ F $\begin{array}{r} 49\frac{1}{2} \\ 67\frac{1}{6} \\ +94\frac{1}{4} \\ \hline \end{array}$

G $7\frac{1}{2} + 8\frac{2}{3} + 6\frac{1}{4} + 7\frac{5}{12}$ H $7\frac{1}{2} + 2\frac{5}{6} + 3\frac{2}{3} + 7\frac{1}{4}$

Short Stories

1. First jump: $12\frac{1}{2}$ m.
Second jump: $11\frac{3}{4}$ m.
How much farther was the first jump than the second?

2. Jim: $161\frac{1}{4}$ cm tall.
Joe: $159\frac{1}{2}$ cm tall.
How much taller is Jim than Joe?

3. Runner: $8\frac{85}{100}$ m/s.
Snake: $\frac{73}{100}$ m/s.
How much faster does the runner go?

4. First suitcase: $16\frac{7}{10}$ kg.
Second suitcase: $25\frac{1}{4}$ kg.
How much do they weigh together?

5. $6\frac{1}{2}$ litres of gasoline in the tank. Put in $8\frac{7}{10}$ litres more.

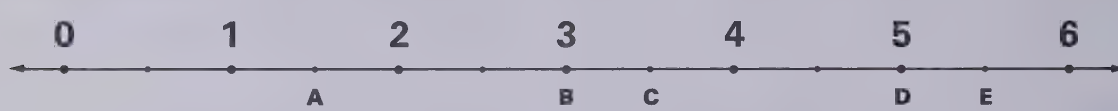
A How much gasoline is in the tank now?

B If the tank holds $16\frac{1}{4}$ litres, how much more could be put in?

1. Find the sums, differences, products, and quotients.

A $28 + 375 + 89 + 6 + 39$	D $52\,006 - 41\,839$	G $387\,522 \div 6$
B $3278 + 9654 + 3612$	E $7 \times 326\,894$	H $2072 \div 74$
C $72\,641 - 13\,782$	F 28×5264	I $28\,784 \div 7$

2. Give improper fractions to name the fractional numbers for the points over letters A through E.



3. Change each fraction to a lowest-terms fraction.

A $\frac{15}{10}$ B $\frac{75}{100}$ C $\frac{40}{60}$ D $\frac{15}{20}$ E $\frac{18}{30}$ F $\frac{8}{6}$ G $\frac{125}{100}$

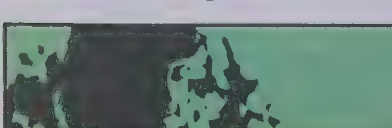
4. Solve the equations.

A $2 \times n = 16$	E $(2 \times n) + n = 12$	I $18 = (3 \times n) - 3$
B $(2 \times n) + 6 = 16$	F $(2 \times n) + (2 \times n) = 12$	J $24 = n + n + n$
C $(2 \times n) - 6 = 16$	G $18 = 3 \times n$	K $24 = (2 \times n) + n$
D $2 \times n = 12$	H $18 = (3 \times n) + 3$	L $24 = (2 \times n) - 8$

5. Give the correct sign (< or >) for each ■.

A $\frac{1}{2}$ ■ $\frac{3}{5}$ B $\frac{4}{5}$ ■ $\frac{7}{8}$ C $\frac{5}{6}$ ■ $\frac{7}{10}$ D $\frac{7}{10}$ ■ $\frac{3}{4}$ E $\frac{2}{3}$ ■ $\frac{8}{10}$ F $\frac{4}{3}$ ■ $\frac{6}{5}$

6. A Give the degree measure of this angle.
   B Suppose we invent a new unit that is only $\frac{1}{2}$ of a degree. What is the measure of the angle when we use this new unit?
   C If we invent and use a unit that is 3 degrees, what is the measure of the angle?

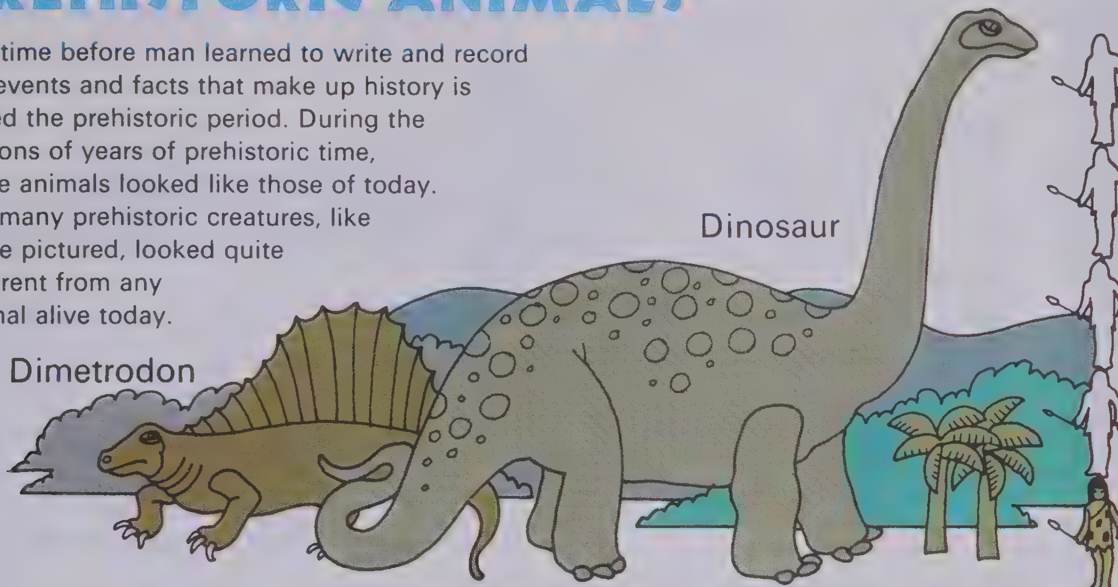


You are invited to explore

ACTIVITY CARD 9
Page 359

PREHISTORIC ANIMALS

The time before man learned to write and record the events and facts that make up history is called the prehistoric period. During the millions of years of prehistoric time, some animals looked like those of today. But many prehistoric creatures, like those pictured, looked quite different from any animal alive today.



1. Dinosaurs, such as the one pictured above, grew as tall as five 2-metre men, standing as in the picture. How many centimetres tall were these dinosaurs ?

2. Some dinosaurs attained lengths nearly three times the height described above. How many metres long were they ?

3. The dimetrodon grew to a length of only about 280 centimetres. About how many times shorter than the dinosaur in number two?

4. The dinosaur era (called Mesozoic) began about 200 000 000 years ago and lasted about 140 000 000 years. About how long ago did the Mesozoic era end ?

5. Some dinosaurs weighed as much as eighty 570-kilogram cows.
   A How many kilograms is this ? B How many tonnes ?

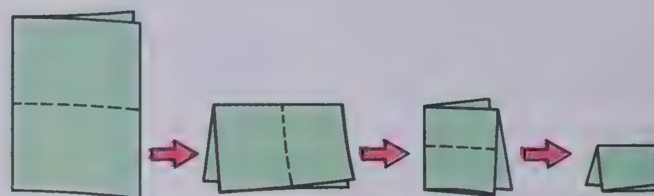
6. Whales, the largest living animals of today, attain weights of as much as 120 tonnes. How many more kilograms is this weight than the weight of the dinosaur ?

9 Multiplication and Division of Fractional Numbers

Let's explore multiplication using unit fractions.

Investigating the Ideas

Fold a piece of notebook paper in half four times.



? Can you unfold the paper and color it so that the following statements are true?

A $\frac{1}{2}$ of $\frac{1}{2}$ of the sheet is red.

B $\frac{1}{4}$ of $\frac{1}{4}$ of the sheet is blue.

C $\frac{1}{2}$ of $\frac{1}{4}$ of the sheet is green.

D $\frac{1}{8}$ of $\frac{1}{2}$ of the sheet is brown.

Discussing the Ideas

1. What fraction of the sheet in the Investigation did you color
   A red? B blue? C green? D brown?

2. In the Investigation, you colored $\frac{1}{4}$ of the sheet red.
   We say, "$\frac{1}{2}$ of $\frac{1}{2}$ of the sheet is $\frac{1}{4}$ of the sheet."
   Make statements like this one for parts B, C, and D above.

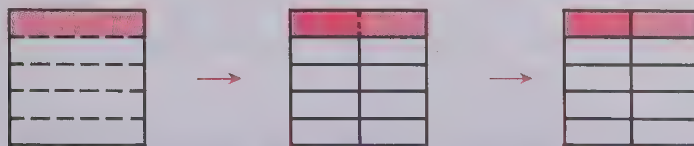
3. Give the missing fractions.

▯ of the region is shaded pink. → $\frac{1}{3}$ of $\frac{1}{4}$ is shaded red. → ▯ of the region is shaded red.

We write this multiplication equation: $\frac{1}{3} \times \frac{1}{4} =$ ▯

4. Can you explain how to find these products?
   A $\frac{1}{2} \times \frac{1}{2}$ B $\frac{1}{4} \times \frac{1}{2}$ C $\frac{1}{3} \times \frac{1}{2}$ D $\frac{1}{2} \times \frac{1}{5}$ E $\frac{1}{8} \times \frac{1}{2}$

1. Give the missing numbers. Then copy the equation with the correct product.

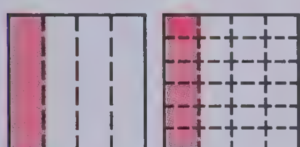


$\frac{1}{2} \times \frac{1}{5} = n$

▥ of the region is shaded pink. ▥ of ▥ is shaded red. ▥ of the region is shaded red.

2. Study the figures. Then give the product.

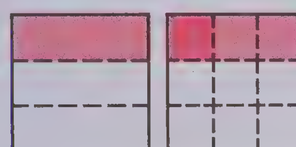
A



$\frac{1}{4}$ $\frac{1}{6}$ of $\frac{1}{4}$

$\frac{1}{6} \times \frac{1}{4} = n$

B



$\frac{1}{3}$ $\frac{1}{3}$ of $\frac{1}{3}$

$\frac{1}{3} \times \frac{1}{3} = n$

C

$\frac{1}{7}$ $\frac{1}{2}$ of $\frac{1}{7}$

$\frac{1}{2} \times \frac{1}{7} = n$

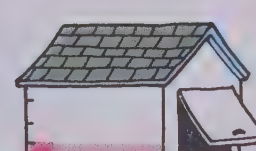
3. Give the products.

A $\frac{1}{3} \times \frac{1}{5}$
B $\frac{1}{8} \times \frac{1}{2}$
C $\frac{1}{3} \times \frac{1}{3}$
D $\frac{1}{5} \times \frac{1}{4}$
E $\frac{1}{6} \times \frac{1}{5}$
F $\frac{1}{10} \times \frac{1}{10}$
G $(\frac{1}{2} \times \frac{1}{3}) \times \frac{1}{4}$
H $\frac{1}{2} \times (\frac{1}{5} \times \frac{1}{10})$

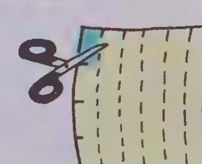
4. Write and solve a multiplication equation for each problem.

A Joan found $\frac{1}{2}$ of a pie in the refrigerator. She ate $\frac{1}{3}$ of it. What fraction of the whole pie did she eat?

B Gary painted $\frac{1}{4}$ of the side of the garage. When he had repainted $\frac{1}{5}$ of this part, what part of the side of the garage had he painted twice?



C Ann cut a sheet of paper into 6 strips of the same size. Then she colored $\frac{1}{4}$ of one of the strips and cut off the colored piece. What part of the whole sheet was this colored piece?

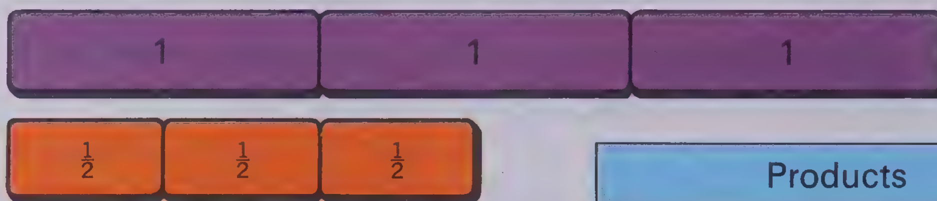


Can you find the product of a whole number and a unit fraction?

Investigating the Ideas

If the purple strip is the unit, this picture shows that:

$$3 \times \frac{1}{2} = \frac{3}{2}$$



? Can you find the products in the table by placing the correct strips along the unit strips?

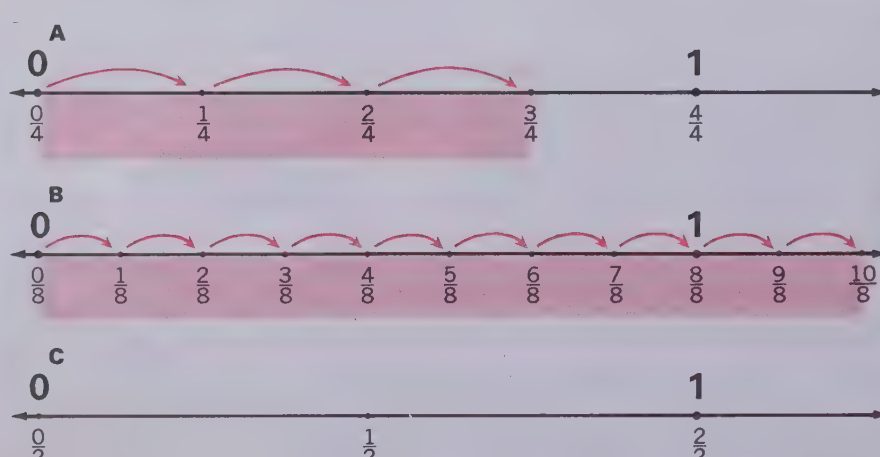
Products	
$5 \times \frac{1}{2} = $ ▯	$2 \times \frac{1}{2} = $ ▯
$7 \times \frac{1}{4} = $ ▯	$9 \times \frac{1}{4} = $ ▯
$3 \times \frac{1}{4} = $ ▯	$6 \times \frac{1}{2} = $ ▯

Discussing the Ideas

1. How could you use the blue strips and the light green strips to show $4 \times \frac{1}{3}$?

2. Write each product in the Investigation as an improper fraction.

3. Study the number-line picture. Then solve the equation.



A: $3 \times \frac{1}{4} = n$

B: $10 \times \frac{1}{8} = n$

C: $0 \times \frac{1}{2} = n$

1. Study the examples. Then solve the equations.

$7 \times \frac{1}{4} = \frac{7}{4}$ $\quad 6 \times \frac{1}{6} = \frac{6}{6}$ $\quad 3 \times \frac{1}{5} = \frac{3}{5}$ $\quad \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$ $\quad \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$

A $9 \times \frac{1}{5} = n$	**F** $9 \times \frac{1}{9} = n$	**K** $\frac{1}{3} \times \frac{1}{10} = n$	**P** $9 \times \frac{1}{10} = n$
B $4 \times \frac{1}{7} = n$	**G** $\frac{1}{2} \times \frac{1}{9} = n$	**L** $\frac{1}{10} \times \frac{1}{10} = n$	**Q** $13 \times \frac{1}{10} = n$
C $3 \times \frac{1}{3} = n$	**H** $\frac{1}{3} \times \frac{1}{7} = n$	**M** $\frac{1}{10} \times \frac{1}{100} = n$	**R** $10 \times \frac{1}{10} = n$
D $8 \times \frac{1}{5} = n$	**I** $\frac{1}{5} \times \frac{1}{5} = n$	**N** $\frac{1}{10} \times \frac{1}{1000} = n$	**S** $10 \times \frac{1}{100} = n$
E $7 \times \frac{1}{6} = n$	**J** $\frac{1}{4} \times \frac{1}{8} = n$	**O** $\frac{1}{100} \times \frac{1}{100} = n$	**T** $(\frac{1}{10})^3 = n$

Write and solve a multiplication equation for each problem.

2. Andy ran $\frac{1}{2}$ of the way to school. Sue ran $\frac{1}{4}$ as far as Andy. What part of the way to school did Sue run?

3. A bullfrog jumped $\frac{1}{3}$ of the way across a stream. A green frog jumped only $\frac{1}{2}$ as far. What part of the way across the stream did the green frog jump?

4. Dan's shoes were each $\frac{1}{4}$ metre long. He used a shoe as a segment to measure the width of a room. How wide was the room if Dan counted 20 of his shoe lengths in walking across?

5. The school track is $\frac{4}{10}$ km around. Jack and Neil raced $\frac{1}{2}$ of the way around. How far did they race?

6. A fast turtle can go $\frac{15}{100}$ km in an hour. At this rate, how far will he go in 24 hours?

7. A car needs $\frac{3}{4}$ litre of gasoline to travel 5 km. How many litres will it need to travel 95 kilometres?

More practice, page A-14, Set 27

Discussing the Ideas

Here are some basic principles that we accept for addition and multiplication of fractional numbers.

Commutative Principles	Associative Principles
You can change the **order** of addends or factors, and the sum or product will be the same.	You can change the **grouping** of the addends or factors, and the sum or product is the same.
Zero Principle	**1 Principle**
When you choose a fractional number and add 0, the sum is the number you have chosen.	When you choose a number and multiply by 1, the product is the number you have chosen.

1. Which principles can help you solve these equations?
   A $\frac{3}{8} \times 1 = n$ B $\frac{2}{3} + 0 = n$ C $7\frac{1}{3} \times \frac{10}{10} = n$ D $0 + \frac{5}{8} = n$

2. Which principles can help you solve these equations?
   A $4\frac{3}{5} + \frac{2}{3} = \frac{2}{3} + n$ B $\frac{3}{4} \times \frac{7}{2} = n \times \frac{3}{4}$

3. A If $\frac{1}{2} + (\frac{1}{3} + \frac{1}{4}) = \frac{13}{12}$, what is $(\frac{1}{2} + \frac{1}{3}) + \frac{1}{4}$?
   B If $\frac{1}{2} \times (\frac{3}{4} \times \frac{3}{5}) = \frac{9}{40}$, what is $(\frac{1}{2} \times \frac{3}{4}) \times \frac{3}{5}$?
   C Which principles did you use in exercises 3A and 3B?

4. Study the statement below. Then solve the equations.
   > Since you can change both the **order** and **grouping** of addends or factors, you can arrange the addends or factors in any way that is convenient.

   A If $2 + \frac{4}{5} + 3 + \frac{1}{5} = 6$, then $\frac{4}{5} + \frac{1}{5} + 2 + 3 = n$.
   B If $5 \times \frac{1}{4} \times 3 \times \frac{1}{7} = \frac{15}{28}$, then $5 \times 3 \times \frac{1}{4} \times \frac{1}{7} = n$.

1. Give the number for a. Then give the number for b.

A $5 \times \frac{1}{6} = a \rightarrow \frac{1}{6} \times 5 = b$

B $9 \times \frac{1}{2} = a \rightarrow \frac{1}{2} \times 9 = b$

C $3 \times \frac{1}{10} = a \rightarrow \frac{1}{10} \times 3 = b$

D $10 \times \frac{1}{10} = a \rightarrow \frac{1}{10} \times 10 = b$

2. Solve the equations.

A $\frac{1}{5} \times 1 = n$ B $1 \times \frac{1}{4} = n$ C $\frac{1}{7} \times \frac{2}{2} = n$ D $0 \times \frac{1}{8} = n$ E $\frac{1}{8} \times \frac{0}{6} = n$

3. Solve the equations.

A $(2 \times \frac{1}{3}) \times \frac{1}{4} = 2 \times (n \times \frac{1}{4})$

B $\frac{1}{5} \times (\frac{1}{2} \times 7) = (\frac{1}{5} \times n) \times 7$

C $(9 \times \frac{1}{7}) \times \frac{1}{3} = n \times (\frac{1}{7} \times \frac{1}{3})$

D $\frac{1}{7} \times (\frac{1}{6} \times 13) = (\frac{1}{7} \times \frac{1}{6}) \times n$

4. Find the products. First find the product of the factors given in red.

A $3 \times \frac{1}{5} \times \frac{1}{2}$ C $\frac{1}{6} \times 6 \times \frac{1}{4}$ E $\frac{1}{6} \times 10 \times \frac{1}{3}$ G $\frac{1}{10} \times \frac{1}{10} \times 100$

B $7 \times \frac{1}{3} \times \frac{1}{4}$ D $\frac{1}{5} \times \frac{1}{6} \times 9$ F $17 \times \frac{1}{10} \times \frac{1}{10}$ H $\frac{1}{5} \times 25 \times \frac{1}{5}$

5. Find the product for each exercise. First find the product of the factors given in heavy black. Then find the product of the factors given in red.

A $\mathbf{2} \times \frac{1}{8} \times \mathbf{4} \times \frac{1}{2} = p$

B $\mathbf{3} \times \frac{1}{7} \times \mathbf{5} \times \frac{1}{5} = p$

C $\mathbf{6} \times \frac{1}{6} \times \mathbf{4} \times \frac{1}{3} = p$

D $\mathbf{2} \times \frac{1}{5} \times \mathbf{9} \times \frac{1}{8} = p$

E $\mathbf{8} \times \frac{1}{10} \times \mathbf{7} \times \frac{1}{10} = p$

F $\mathbf{8} \times \frac{1}{10} \times \mathbf{3} \times \frac{1}{100} = p$

think

A doubloon is a Spanish coin worth 16 dollars.

A pirate buried $\frac{1}{2}$ of his doubloons and threw $\frac{1}{3}$ of them into the sea. When he counted those remaining, he found 4000 doubloons. How many did he have when he started? How many dollars were they worth?

● *Let's explore multiplication of fractional numbers.*

Investigating the Ideas

Factors

$2 \quad \frac{1}{3}$

$\frac{1}{8} \quad 5$

Products

$2 \times \frac{1}{3} = \blacksquare$

$2 \times 5 = \blacksquare$

⋮

? How many different products can you find by using the factors in the set?

Discussing the Ideas

1. Give the number for n in each equation.

   A $2 \times 5 = n$ B $\frac{1}{3} \times \frac{1}{8} = n$ C $(2 \times 5) \times (\frac{1}{3} \times \frac{1}{8}) = n$

   D $2 \times \frac{1}{3} = n$ E $5 \times \frac{1}{8} = n$ F $\frac{2}{3} \times \frac{5}{8} = n$

2. The diagram below will help you review a way to find products such as $\frac{5}{7} \times \frac{3}{8}$. Give the missing product.

   Find the product of these factors.

   $$\frac{5}{7} \times \frac{3}{8} = (5 \times \frac{1}{7}) \times (3 \times \frac{1}{8}) = 15 \times \frac{1}{56} = \blacksquare$$

   Find the product of these factors.

3. Can you describe a shortcut for multiplying two fractional numbers?

1. Solve the equations.

A $\frac{3}{4} = n \times \frac{1}{4}$ C $\frac{6}{10} = 6 \times n$ E $\frac{13}{15} = 13 \times n$ G $\frac{9}{10} = 9 \times n$

B $\frac{5}{6} = n \times \frac{1}{6}$ D $\frac{5}{100} = n \times \frac{1}{100}$ F $\frac{0}{7} = n \times \frac{1}{7}$ H $\frac{54}{100} = 54 \times n$

2. Copy the equations. Give the number instead of n.

A $\frac{3}{4} \times \frac{1}{3} = n \times \frac{1}{4} \times \frac{1}{3}$ C $\frac{1}{6} \times \frac{3}{8} = \frac{1}{6} \times 3 \times n$ E $\frac{5}{6} \times \frac{1}{5} = n \times \frac{1}{6} \times \frac{1}{5}$

B $\frac{2}{5} \times \frac{1}{4} = n \times \frac{1}{5} \times \frac{1}{4}$ D $9 \times \frac{2}{3} = 9 \times 2 \times n$ F $\frac{1}{2} \times \frac{4}{9} = \frac{1}{2} \times n \times \frac{1}{9}$

3. Solve the equation to find the product of the factors in the red screen.

A $\frac{3}{4} \times \frac{1}{3}$
$3 \times \frac{1}{4} \times \frac{1}{3} = n$

C $\frac{4}{7} \times \frac{1}{5}$
$4 \times \frac{1}{7} \times \frac{1}{5} = n$

E $\frac{1}{6} \times \frac{3}{8}$
$\frac{1}{6} \times 3 \times \frac{1}{8} = n$

G $\frac{1}{3} \times \frac{8}{5}$
$\frac{1}{3} \times 8 \times \frac{1}{5} = n$

B $\frac{2}{5} \times \frac{1}{4}$
$2 \times \frac{1}{5} \times \frac{1}{4} = n$

D $\frac{5}{8} \times \frac{1}{2}$
$5 \times \frac{1}{8} \times \frac{1}{2} = n$

F $9 \times \frac{2}{3}$
$9 \times 2 \times \frac{1}{3} = n$

H $6 \times \frac{9}{10}$
$6 \times 9 \times \frac{1}{10} = n$

4. Give the whole number for a.
Give the fraction for b. Then give the missing product.

A $\frac{3}{4} \times \frac{2}{3} = (3 \times \frac{1}{4}) \times (2 \times \frac{1}{3}) = a \times b =$

B $\frac{2}{5} \times \frac{5}{6} = (2 \times \frac{1}{5}) \times (5 \times \frac{1}{6}) = a \times b =$

C $\frac{3}{8} \times \frac{3}{4} = (3 \times \frac{1}{8}) \times (3 \times \frac{1}{4}) = a \times b =$

5. Copy each equation. Give the product instead of n.

A $\frac{3}{4} \times \frac{2}{3} = n$ E $\frac{5}{8} \times \frac{5}{8} = n$

B $\frac{2}{5} \times \frac{5}{6} = n$ F $\frac{4}{5} \times \frac{4}{3} = n$

C $\frac{3}{8} \times \frac{3}{4} = n$ G $\frac{7}{10} \times \frac{5}{2} = n$

D $\frac{2}{7} \times \frac{7}{2} = n$ H $\frac{3}{5} \times \frac{5}{7} = n$

think

The weight of an object on the moon is $\frac{1}{6}$ its weight on Earth. Give the total "moon weight" of these objects. (The Earth weights are given.)

Boy: 42 kg Hamburger: $\frac{1}{10}$ kg
Cat: 3 kg Mouse: $\frac{1}{20}$ kg

More practice, page A-14, Set 28

Multiplication Practice

1. Find the products.

A $\frac{3}{5} \times \frac{2}{7} = (3 \times 2) \times (\frac{1}{5} \times \frac{1}{7}) = n$

B $\frac{3}{4} \times \frac{5}{3} = (3 \times 5) \times (\frac{1}{4} \times \frac{1}{3}) = n$

C $\frac{7}{2} \times \frac{2}{5} = (7 \times 2) \times (\frac{1}{2} \times \frac{1}{5}) = n$

D $\frac{4}{3} \times \frac{8}{11} = (4 \times 8) \times (\frac{1}{3} \times \frac{1}{11}) = n$

E $\frac{3}{5} \times \frac{2}{6} = (3 \times 2) \times (\frac{1}{5} \times \frac{1}{6}) = n$

F $\frac{2}{10} \times \frac{9}{10} = (2 \times 9) \times (\frac{1}{10} \times \frac{1}{10}) = n$

G $\frac{4}{7} \times \frac{11}{4} = (4 \times 11) \times (\frac{1}{7} \times \frac{1}{4}) = n$

H $\frac{5}{9} \times \frac{3}{5} = (5 \times 3) \times (\frac{1}{9} \times \frac{1}{5}) = n$

I $\frac{6}{100} \times \frac{4}{10} = (6 \times 4) \times (\frac{1}{100} \times \frac{1}{10}) = n$

J $\frac{0}{7} \times \frac{3}{4} = (0 \times 3) \times (\frac{1}{7} \times \frac{1}{4}) = n$

2. Find the products.

A $\frac{2}{3} \times \frac{4}{5}$

B $\frac{7}{4} \times \frac{3}{2}$

C $\frac{5}{7} \times \frac{6}{3}$

D $\frac{4}{9} \times \frac{1}{4}$

E $\frac{7}{3} \times \frac{3}{7}$

F $\frac{4}{8} \times \frac{8}{4}$

G $\frac{30}{1} \times \frac{2}{6}$

H $\frac{3}{10} \times \frac{4}{7}$

I $\frac{7}{11} \times \frac{3}{5}$

J $\frac{4}{3} \times \frac{9}{8}$

K $\frac{0}{6} \times \frac{2}{5}$

L $\frac{1}{10} \times \frac{1}{10}$

M $\frac{3}{10} \times \frac{8}{100}$

N $\frac{3}{8} \times \frac{23}{1}$

O $\frac{5}{6} \times \frac{5}{6}$

P $\frac{6}{7} \times \frac{7}{6}$

Q $\frac{4}{10} \times \frac{5}{10}$

R $\frac{7}{20} \times \frac{8}{30}$

S $\frac{5}{5} \times \frac{11}{30}$

T $\frac{25}{100} \times \frac{4}{10}$

★ **3.** Give the symbol (<, >, =) for each $\bigcirc$.

A $\frac{4}{5} \times \frac{2}{3} \bigcirc \frac{4}{5} + \frac{2}{3}$

B $\frac{1}{3} \times \frac{1}{3} \bigcirc \frac{1}{3}$

C $\frac{3}{4} \times 4 \bigcirc \frac{3}{7} \times 7$

D $\frac{7}{5} \times \frac{4}{3} \bigcirc \frac{5}{7} \times \frac{3}{4}$

E $3 - \frac{3}{4} \bigcirc 3 \times \frac{3}{4}$

F $\frac{3}{2} \times 2 \bigcirc \frac{3}{2} + 2$

G $\frac{4}{3} \times 4 \bigcirc \frac{4}{3} + 4$

H $\frac{5}{4} \times 5 \bigcirc \frac{5}{4} + 5$

think

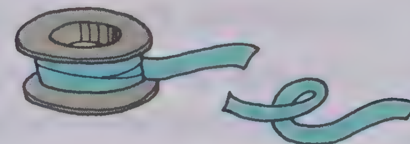
Give the pair of fractional numbers for *a* and *b* in each exercise.

	Sums		$\times$ / $+$		Products
1.	1	*a*		*b*	$\frac{1}{4}$
2.	$\frac{2}{3}$	*a*		*b*	$\frac{1}{9}$
3.	$\frac{1}{2}$	*a*		*b*	$\frac{1}{16}$
4.	1	*a*		*b*	$\frac{3}{16}$
5.	$\frac{5}{6}$	*a*		*b*	$\frac{1}{6}$

Short Stories

1 Bowling ball: weighs 6 kg. Baseball: weighs $\frac{1}{40}$ as much. What does the baseball weigh ?

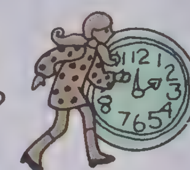
2 $\frac{3}{4}$ metre of ribbon. Cut off $\frac{1}{3}$ of it. What part of a metre was cut off ?



3 $\frac{3}{4}$ kg of butter. Used $\frac{2}{3}$ of it. Used what part of a kilogram ?

4 $\frac{1}{5}$ of the girls are blondes. 20 girls. How many blondes ?

5 1 m: $\frac{1}{1000}$ of a kilometre. 500 m: what part of a kilometre ?



6 Long kangaroo jump: 12 m. Long frog jump: $\frac{1}{3}$ as far. How far ?

7 Walk $\frac{5}{2}$ kilometres per hour. Walk for $\frac{3}{4}$ hour. How far ?

8 10 km. Walked $\frac{2}{3}$ of the way. Walked how far ?

9 An apron contains $\frac{7}{10}$ of a square metre of cloth. 40 aprons. How many square metres ?

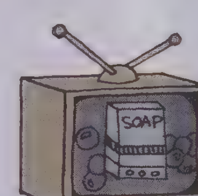
10 Running track: $\frac{4}{10}$ km. 16 laps: how many kilometres ?

11 Drive 45 minutes. $\frac{2}{3}$ km each minute. How many km ?

12 1 day: $\frac{1}{7}$ of a week. $\frac{3}{4}$ of a day: what part of a week ?

13 Iceberg: $\frac{1}{8}$ of it shows above the surface. If the iceberg is 210 cubic metres, how many cubic metres show on the surface ?

14 Television program: $\frac{1}{2}$ hour. Commercials: $\frac{1}{10}$ of the time. How many minutes of commercials ?



★ **15** \$24. Left $\frac{1}{3}$ of it at home. Spent $\frac{3}{4}$ of the rest. How much was spent ?

Discussing the Ideas

The examples that follow show how the **distributive principle for fractional numbers** can help you find products.

1. Explain each step of this example.

$3 \times 4\frac{1}{5} = 3 \times (4 + \frac{1}{5}) = (3 \times 4) + (3 \times \frac{1}{5})$		
1 $\begin{array}{r} 4\frac{1}{5} \\ \times\ 3 \\ \hline 12\phantom{\frac{1}{5}} \end{array}$	2 $\begin{array}{r} 4\frac{1}{5} \\ \times\ 3 \\ \hline 12\frac{3}{5} \end{array}$	3 $\begin{array}{r} 4\frac{1}{5} \\ \times\ 3 \\ \hline 12\frac{3}{5} \end{array}$
$3 \times 4 = 12$	$3 \times \frac{1}{5} = \frac{3}{5}$	$12 + \frac{3}{5} = 12\frac{3}{5}$

2. Explain each step of this example.

$\frac{1}{2} \times 4\frac{1}{5} = \frac{1}{2} \times (4 + \frac{1}{5}) = (\frac{1}{2} \times 4) + (\frac{1}{2} \times \frac{1}{5})$		
1 $\begin{array}{r} 4\frac{1}{5} \\ \times\ \frac{1}{2} \\ \hline 2\phantom{\frac{1}{10}} \end{array}$	2 $\begin{array}{r} 4\frac{1}{5} \\ \times\ \frac{1}{2} \\ \hline 2\frac{1}{10} \end{array}$	3 $\begin{array}{r} 4\frac{1}{5} \\ \times\ \frac{1}{2} \\ \hline 2\frac{1}{10} \end{array}$
$\frac{1}{2} \times 4 = 2$	$\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$	$2 + \frac{1}{10} = 2\frac{1}{10}$

3. The products found in exercises 1 and 2 will help you find the product below. Explain each step of this example.

$3\frac{1}{2} \times 4\frac{1}{5} = (3 + \frac{1}{2}) \times 4\frac{1}{5} = (3 \times 4\frac{1}{5}) + (\frac{1}{2} \times 4\frac{1}{5})$		
1 $\begin{array}{r} 4\frac{1}{5} \\ \times\ 3\frac{1}{2} \\ \hline 12\frac{3}{5} \end{array}$	2 $\begin{array}{r} 4\frac{1}{5} \\ \times\ 3\frac{1}{2} \\ \hline 12\frac{3}{5} \\ 2\frac{1}{10} \\ \hline \end{array}$	3 $\begin{array}{r} 4\frac{1}{5} \\ \times\ 3\frac{1}{2} \\ \hline 12\frac{3}{5} \\ 2\frac{1}{10} \\ \hline 14\frac{7}{10} \end{array}$
$3 \times 4\frac{1}{5} = 12\frac{3}{5}$	$\frac{1}{2} \times 4\frac{1}{5} = 2\frac{1}{10}$	$12\frac{3}{5} + 2\frac{1}{10} = 14\frac{7}{10}$

1. Solve the equations.

A $5 \times 3\frac{1}{4} = (5 \times 3) + (5 \times n)$

B $6 \times 2\frac{1}{5} = (6 \times n) + (6 \times \frac{1}{5})$

C $8 \times 5\frac{2}{3} = (n \times 5) + (8 \times \frac{2}{3})$

D $4\frac{1}{6} \times 3 = (4 \times n) + (\frac{1}{6} \times 3)$

E $\frac{1}{2} \times 3\frac{1}{7} = (\frac{1}{2} \times n) + (\frac{1}{2} \times \frac{1}{7})$

F $\frac{3}{5} \times 5\frac{2}{3} = (\frac{3}{5} \times 5) + (\frac{3}{5} \times n)$

2. Use the distributive principle to find the products.

A $5 \times 3\frac{1}{4}$ B $6 \times 2\frac{1}{5}$ C $8 \times 5\frac{2}{3}$ D $\frac{1}{2} \times 3\frac{1}{7}$ E $\frac{3}{5} \times 5\frac{2}{3}$

F $3\frac{1}{8} \times 5$ G $9\frac{1}{4} \times \frac{1}{3}$ H $53\frac{1}{2} \times 9$ I $3\frac{1}{4} \times 2\frac{1}{3}$ J $5\frac{2}{3} \times 3\frac{1}{4}$

3. Find the products below by first replacing each mixed numeral with an improper fraction as in the example.

$$1\frac{2}{3} \times 2\frac{1}{2} = 4\frac{1}{6}$$
$$\frac{5}{3} \times \frac{5}{2} = \frac{25}{6}$$

A $1\frac{1}{2} \times 1\frac{1}{3}$

B $2\frac{1}{2} \times 1\frac{1}{4}$

C $3\frac{1}{2} \times 1\frac{3}{4}$

D $3\frac{1}{5} \times \frac{2}{1}$

E $\frac{3}{4} \times 2\frac{1}{2}$

F $1\frac{1}{8} \times 2\frac{1}{3}$

G $\frac{3}{4} \times 3\frac{2}{3}$

H $\frac{1}{6} \times 1\frac{2}{5}$

I $3\frac{1}{2} \times 2\frac{1}{7}$

J $4\frac{1}{3} \times 5\frac{1}{2}$

K $8 \times 4\frac{1}{4}$

L $1\frac{1}{6} \times 3\frac{2}{3}$

M $2\frac{1}{3} \times 5\frac{1}{3}$

N $1\frac{1}{10} \times 2\frac{1}{5}$

O $\frac{5}{2} \times 3\frac{3}{4}$

P $6\frac{1}{2} \times 1\frac{1}{7}$

★ **4.** Ned drew this shortcut for finding products like those in exercise 3. Explain Ned's shortcut.

$$\begin{array}{rr} 4 & \frac{1}{5} \\ \times 3 & \frac{1}{2} \\ \hline \end{array}$$

think

First odd number

$1 = 1$

First 2 odd numbers

$\underbrace{1 + 3} = 4$

First 3 odd numbers

$\underbrace{1 + 3 + 5} = 9$

First 4 odd numbers

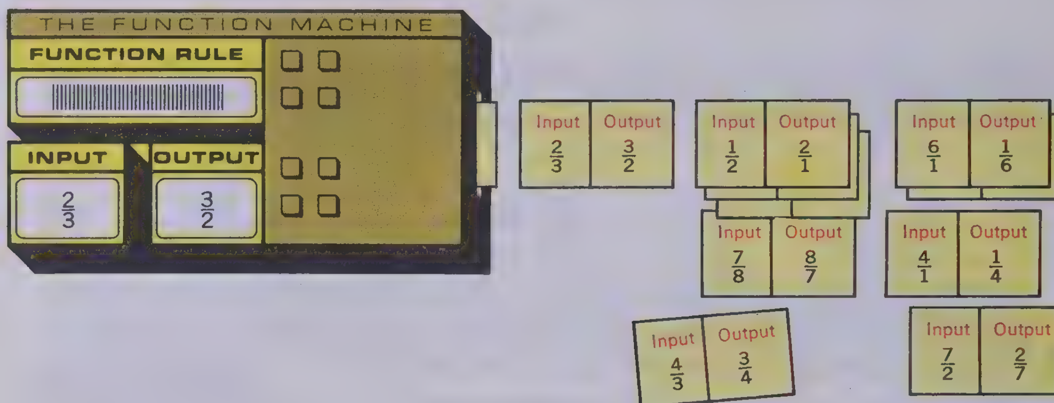
$\underbrace{1 + 3 + 5 + 7} = 16$

What is the sum of the first 100 odd numbers?

More practice, page A-15, Set 29

Investigating the Ideas

For each input number the function machine finds the output number and records the input and output on cards like these.



? Can you give some more input-output numbers that this function machine might produce?

Discussing the Ideas

1. Can you describe the rule the function machine is using?

2. What is the product of each of the input-output pairs on the cards?

3. When the product of two numbers is 1, we say that each of the numbers is a **reciprocal** of the other. Does the number **0** have a reciprocal? Explain.

F	F		P
$\frac{5}{1} \times$	$\frac{1}{5} =$	$\frac{5}{5} =$	1

F	F		P
$\frac{3}{4} \times$	$\frac{4}{3} =$	$\frac{12}{12} =$	1

4. What is the reciprocal of each of these numbers?

   A $\frac{4}{5}$ B $\frac{1}{2}$ C $\frac{3}{13}$ D 7 E 1 F $\frac{11}{5}$ G $6\frac{1}{3}$

1. Give the reciprocal of each number.

   A $\frac{3}{5}$ B $\frac{7}{4}$ C 6 D $\frac{1}{6}$ E 1 F $2\frac{1}{4}$ G $\frac{9}{10}$

2. Find the products.

   A $\frac{5}{8} \times \frac{8}{5}$ B $\frac{6}{1} \times \frac{1}{6}$ C $\frac{9}{2} \times \frac{2}{9}$ D $4 \times \frac{1}{4}$ E $\frac{1}{8} \times 8$

3. The missing factor is the reciprocal of the factor given. Find the missing factor.

   A $n \times \frac{2}{3} = 1$ B $n \times \frac{1}{3} = 1$ C $\frac{5}{6} \times n = 1$ D $\frac{1}{7} \times n = 1$ E $2\frac{1}{2} \times n = 1$

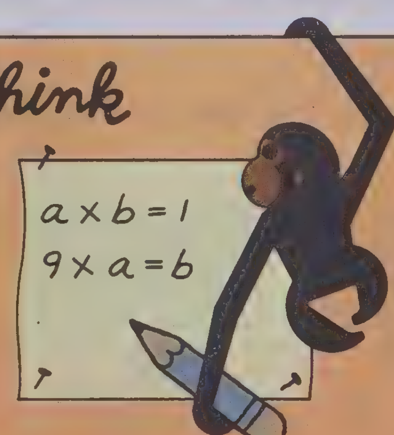
4. Give the number for the ▥. Then give the product n.

   A $12 \times \frac{1}{4} =$ ▥ $\times (4 \times \frac{1}{4}) = n$
   B $15 \times \frac{1}{3} = 5 \times ($▥ $\times \frac{1}{3}) = n$
   C $40 \times \frac{1}{10} = 4 \times ($▥ $\times \frac{1}{10}) = n$
   D $20 \times \frac{1}{5} =$ ▥ $\times (5 \times \frac{1}{5}) = n$
   E $18 \times \frac{1}{6} = 3 \times ($▥ $\times \frac{1}{6}) = n$
   F $27 \times \frac{1}{3} =$ ▥ $\times (3 \times \frac{1}{3}) = n$
   G $56 \times \frac{1}{7} = 8 \times ($▥ $\times \frac{1}{7}) = n$
   H $72 \times \frac{1}{9} =$ ▥ $\times (9 \times \frac{1}{9}) = n$

5. Give the number for a. Then give the number for b.

   A $5 \times \frac{1}{5} = a \rightarrow 10 \times \frac{1}{5} = b$
   B $7 \times \frac{1}{7} = a \rightarrow 14 \times \frac{1}{7} = b$
   C $3 \times \frac{1}{3} = a \rightarrow 9 \times \frac{1}{3} = b$
   D $6 \times \frac{1}{6} = a \rightarrow 24 \times \frac{1}{6} = b$
   E $9 \times \frac{1}{9} = a \rightarrow 9 \times \frac{2}{9} = b$
   F $10 \times \frac{1}{10} = a \rightarrow 10 \times \frac{2}{10} = b$
   G $4 \times \frac{1}{4} = a \rightarrow 4 \times \frac{3}{4} = b$
   H $5 \times \frac{3}{5} = a \rightarrow 10 \times \frac{3}{5} = b$
   I $7 \times \frac{4}{7} = a \rightarrow 14 \times \frac{4}{7} = b$
   J $5 \times \frac{3}{5} = a \rightarrow 20 \times \frac{3}{5} = b$
   K $10 \times \frac{7}{10} = a \rightarrow 30 \times \frac{7}{10} = b$

think



Two numbers are reciprocals of each other. One number is 9 times as large as the other. Find the two numbers.

Discussing the Ideas

1. Can you explain why the products in both examples A and B are the same? What are the products?

$\boxed{A}\ \frac{5}{15} \times \frac{4}{8}$

$\boxed{B}\ \frac{5}{8} \times \frac{4}{15}$

2. Jody said that example A above would be easier if the fractions were simplified before multiplying. Give the lowest-terms fractions for *a* and *b* and the product *c*.

$\boxed{A}\ \frac{5}{15} \times \frac{4}{8} = a \times b = c$

3. Brian said, "I can do example B by simplifying before I multiply." Can you explain Brian's method?

$\boxed{B}\ \frac{\cancel{5}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{\cancel{15}_{3}} = \frac{1}{6}$

4. Study the flow chart to understand Brian's shortcut.

Problem	→ Divide 5 and 15 by 5.	→ Divide 4 and 8 by 4.	→ Multiply to find the product.
$\frac{5}{8} \times \frac{4}{15}$	$\frac{\cancel{5}^{1}}{8} \times \frac{4}{\cancel{15}_{3}}$	$\frac{\cancel{5}^{1}}{\cancel{8}_{2}} \times \frac{\cancel{4}^{1}}{\cancel{15}_{3}}$	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Try the shortcut for these problems.

A $\frac{2}{3} \times \frac{3}{4}$ B $12 \times \frac{2}{3}$ C $\frac{8}{27} \times \frac{9}{20}$

5. Can the shortcut be used to find the product $\frac{2}{3} \times \frac{5}{7}$? What is the product?

1. Find the products. You can use the shortcut discussed on the previous page to make your work easier.

A $\frac{4}{9} \times \frac{3}{8}$	E $\frac{5}{6} \times \frac{12}{5}$	I $\frac{4}{15} \times \frac{10}{28}$	M $\frac{27}{32} \times \frac{4}{9}$
B $\frac{5}{12} \times \frac{3}{10}$	F $\frac{3}{4} \times \frac{20}{27}$	J $\frac{7}{12} \times \frac{8}{21}$	N $\frac{4}{5} \times \frac{15}{16}$
C $\frac{6}{5} \times \frac{15}{18}$	G $\frac{6}{9} \times \frac{18}{36}$	K $\frac{9}{10} \times \frac{25}{21}$	O $\frac{18}{25} \times \frac{15}{81}$
D $\frac{3}{8} \times \frac{8}{9}$	H $\frac{5}{8} \times \frac{6}{15}$	L $\frac{4}{5} \times \frac{15}{8}$	P $\frac{30}{33} \times \frac{11}{20}$

2. Find the products. Use the easiest possible method.

A $24 \times \frac{5}{24}$	C $100 \times \frac{3}{100}$	E $10 \times \frac{7}{100}$	G $24 \times \frac{5}{6}$	I $\frac{7}{8} \times \frac{12}{14}$
B $10 \times \frac{3}{10}$	D $100 \times \frac{3}{10}$	F $\frac{5}{10} \times \frac{20}{100}$	H $\frac{9}{15} \times \frac{7}{9}$	J $\frac{7}{10} \times \frac{100}{21}$

Complete the following function tables.

3. Function Rule: $\frac{3}{4} \times n$

	n	$f(n)$
A	$\frac{4}{27}$	
B	$\frac{20}{21}$	
C	24	
D	$\frac{4}{3}$	
E	$2\frac{2}{3}$	

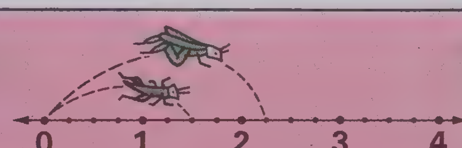
4. Function Rule: $n \times \frac{2}{3}$

	n	$f(n)$
A	$\frac{6}{5}$	
B	0	
C	9	
D	$6\frac{3}{4}$	
E	$\frac{3}{2}$	

5. Function Rule: $3\frac{1}{4} \times n$

	n	$f(n)$
A	8	
B	$\frac{1}{3}$	
C	$5\frac{5}{8}$	
D	$8\frac{4}{5}$	
E		$9\frac{3}{4}$

think



The picture shows the sizes of the jumps as a grasshopper and a cricket jump along a number line.

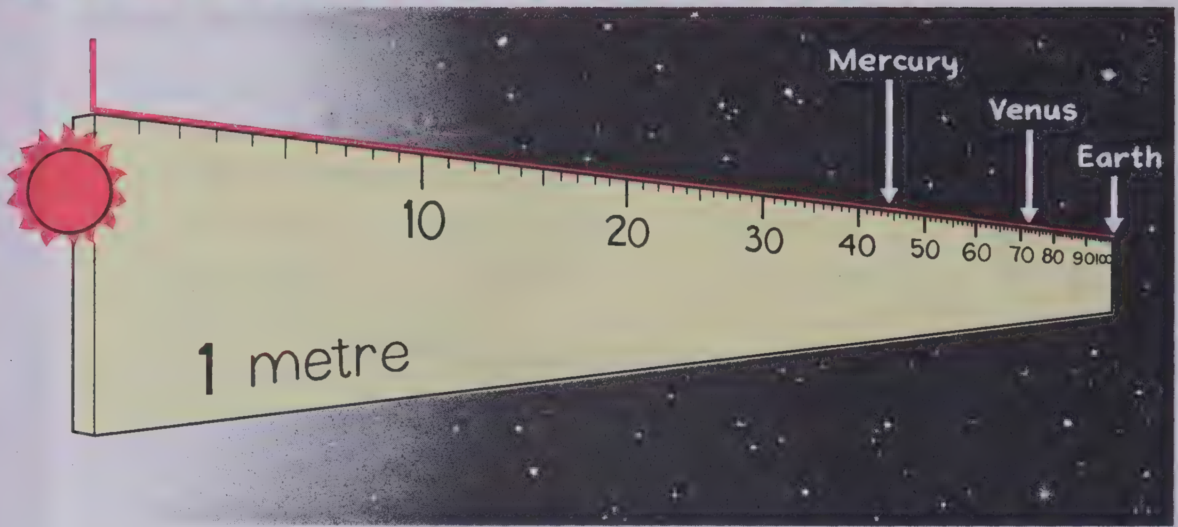
Grasshopper: 7 jumps.

Cricket: 11 jumps.

How far apart are they?

Can you find 3 different points on which both insects will land?

OUR SOLAR SYSTEM



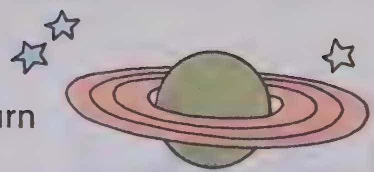
To give an idea of the size of our solar system, we will let one metre (100 cm) represent the distance from Earth to the sun.

Scale: 1 metre = 150 million kilometres
or 1 cm = 1.5 million kilometres

On this scale the sun would be about the size shown above. Mercury, the closest planet, would be a speck about 44 centimetres away from the sun. The table below gives the distances of the planets from the sun.

Planet	Average distance (kilometres) from the sun
Mercury	66 000 000
Venus	108 000 000
Earth	150 000 000
Mars	228 000 000
Jupiter	778 000 000
Saturn	1 427 000 000
Uranus	2 870 000 000
Neptune	4 500 000 000
Pluto	5 909 000 000

1. Use the scale and the table on page 198 for these exercises.



A Is Earth closer to Venus or to Mars ? How much closer ?

B On a scale where 1 centimetre = 1 500 000 km, about how many centimetres should Venus be shown from the sun ?

C Which planet would be nearly 10 metres from the sun ?

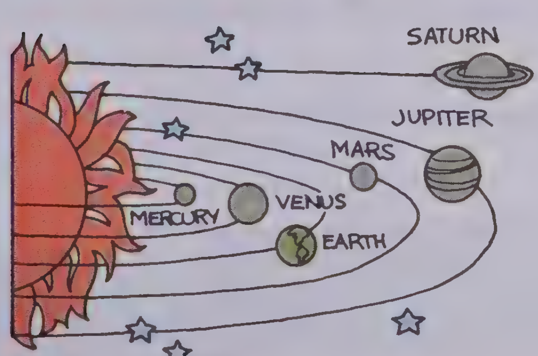
D How many times as far from the sun is the planet Pluto than Earth ?

E Using the scale on page 198, how many kilometres would Pluto be from the sun ?

F How much farther is Uranus from Neptune than from Saturn ?

2. Venus is about the same size as Earth. Mars is $\frac{1}{7}$ as large (in volume) as Earth. Mercury is $\frac{7}{16}$ as large as Mars. What fraction of Earth's size is Mercury ?

3. Saturn is 730 times as large as Earth. Neptune is about $\frac{1}{12}$ as large as Saturn. About how many times as large as Earth is Neptune ?



4. Some astronomers believe there is a tenth planet, Planet X, as yet undiscovered. In the scale drawing on page 198, this planet would be about 60 metres from the sun. About how many kilometres is Planet X from the sun ?

TRACK RECORDS IN SPACE???

Each planet in our solar system pulls objects toward it with a different amount of force. In most cases, the smaller the planet, the smaller this force and the easier it would be to jump away from the surface of the planet.

Near-Record Earth Jumps	
High jump	$2\frac{1}{4}$ m
Long jump	$8\frac{4}{5}$ m
Pole vault	$5\frac{1}{2}$ m

Planet Jumps Compared with Earth Jumps*
A person's jump on Earth would be about:
6 times as high on the **moon**
$2\frac{3}{5}$ times as high on **Mercury**
$2\frac{1}{2}$ times as high on **Mars**
$1\frac{1}{7}$ times as high on **Venus**
$\frac{5}{6}$ times as high on **Neptune**
$\frac{5}{12}$ times as high on **Jupiter**

1. If the holder of the pole-vault record were on the moon, about how high could he vault?

2. The largest planet is Jupiter. What would be a near-record pole vault on Jupiter?

3. If the pole-vault record holder were on each of the other planets listed in the table, about how high could he vault?

4. About how far could the long-jump record holder jump on each of the planets listed in the table?

5. About how high could the high-jump record holder jump on each of the planets listed in the table?

★ 6. If you could pole vault 8 metres on Mars, how high could you pole vault on Earth?

*For the purpose of these exercises, we assume that the multiples given in the table can be applied to the given Earth jumps.

Keeping in Touch with Computing Fractional numbers

1. Find the sums and differences.

A $\frac{3}{5} + \frac{1}{5}$ B $\frac{5}{8} + \frac{3}{8}$ C $1 - \frac{7}{100}$ D $1 - \frac{3}{10}$ E $\frac{3}{4} + 0$ F $1 + \frac{9}{10}$ G $5 - \frac{1}{5}$

2. Find the sums and differences. Give your answers in lowest terms.

A $\begin{array}{r} \frac{1}{5} \\ +\frac{3}{4} \\ \hline \end{array}$ B $\begin{array}{r} \frac{5}{8} \\ -\frac{1}{4} \\ \hline \end{array}$ C $\begin{array}{r} 2\frac{1}{3} \\ +5\frac{1}{4} \\ \hline \end{array}$ D $\begin{array}{r} 4\frac{3}{10} \\ +8\frac{4}{5} \\ \hline \end{array}$ E $\begin{array}{r} \frac{5}{6} \\ -\frac{1}{7} \\ \hline \end{array}$ F $\begin{array}{r} 9\frac{1}{5} \\ -4\frac{4}{10} \\ \hline \end{array}$ G $\begin{array}{r} 18\frac{1}{3} \\ +23\frac{4}{6} \\ \hline \end{array}$ H $\begin{array}{r} 4\frac{1}{2} \\ +19\frac{3}{8} \\ \hline \end{array}$

I $\frac{1}{5} + \frac{3}{15}$ J $4\frac{2}{5} + \frac{5}{3}$ K $5\frac{31}{50} - 4\frac{1}{100}$ L $6\frac{1}{10} - 4\frac{2}{5}$ M $32\frac{1}{4} - 8\frac{3}{4}$

3. Find the sums.

A $\begin{array}{r} 4\frac{1}{8} \\ 3\frac{1}{2} \\ +6\frac{1}{4} \\ \hline \end{array}$ B $\begin{array}{r} 5\frac{1}{10} \\ 8\frac{4}{5} \\ +9\frac{1}{4} \\ \hline \end{array}$ C $\begin{array}{r} 89\frac{3}{5} \\ 95\frac{7}{15} \\ +67\frac{2}{3} \\ \hline \end{array}$

D $5\frac{1}{10} + 8\frac{21}{100}$

E $138\frac{7}{10} + 57\frac{4}{5}$

F $5\frac{3}{4} + 9\frac{1}{5} + 7\frac{3}{20}$

G $7\frac{1}{3} + 8\frac{1}{2} + 3\frac{5}{6}$

4. Write an improper fraction in place of each mixed numeral.

A $2\frac{1}{2}$ B $3\frac{1}{3}$ C $4\frac{1}{4}$ D $5\frac{2}{5}$ E $9\frac{1}{6}$ F $12\frac{3}{7}$ G $8\frac{5}{9}$ H $15\frac{3}{10}$

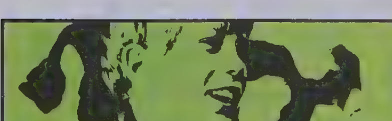
5. Write a mixed numeral in place of each improper fraction.

A $\frac{9}{2}$ B $\frac{13}{3}$ C $\frac{19}{4}$ D $\frac{39}{4}$ E $\frac{78}{5}$ F $\frac{139}{7}$ G $\frac{242}{10}$ H $\frac{167}{100}$

6. Write and solve an equation for each problem.

A Jack: $87\frac{1}{2}$ kilograms.
Jim: $96\frac{3}{4}$ kilograms.
On the scales together.
How many kilograms ?

B Gasoline tank holds $17\frac{1}{2}$ litres.
Takes $8\frac{7}{10}$ litres to fill it.
How many litres were
already in the tank ?

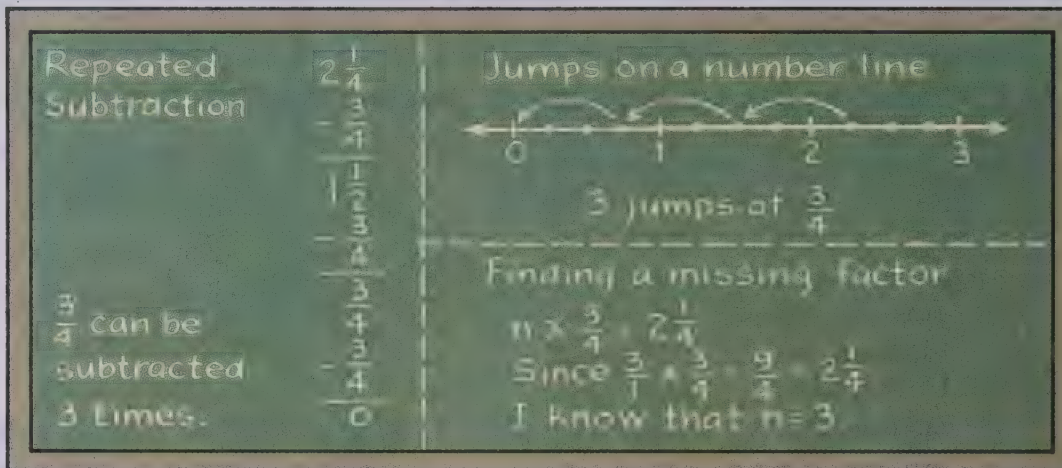


You are invited to explore

ACTIVITY CARD 10
Page 360

Investigating the Ideas

Here are three ways that some students used to find how many times $\frac{3}{4}$ is contained in $2\frac{1}{4}$.



? Can you use one of these methods to find how many times $\frac{5}{8}$ is contained in $2\frac{1}{2}$?

Discussing the Ideas

1. $\frac{3}{4}$ is contained 3 times in $2\frac{1}{4}$. You can write this fact as the division equation $2\frac{1}{4} \div \frac{3}{4} = 3$. What division equation can you write to answer the Investigation question?

2. You can think of division as finding a missing factor. Explain how you can find the number for n in the example.

   You find this quotient when you find this factor.

   $$\frac{8}{15} \div \frac{4}{5} = n$$
   $$n \times \frac{4}{5} = \frac{8}{15}$$

3. Jane showed how she solved this division problem.

   $$\frac{8}{15} \div \frac{4}{3} = \frac{8 \div 4}{15 \div 3} = \frac{2}{5}$$

   A Do you think Jane's method is correct? How can you check?

   B Would this method work for $\frac{3}{8} \div \frac{2}{5}$? Explain.

1. For each multiplication equation, write two division equations.
Example: $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15} \longrightarrow \frac{2}{15} \div \frac{1}{5} = \frac{2}{3};\ \frac{2}{15} \div \frac{2}{3} = \frac{1}{5}$

A $\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ **B** $\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$ **C** $3 \times \frac{2}{5} = \frac{6}{5}$

D $\frac{2}{3} \times \frac{3}{2} = 1$ **E** $\frac{1}{3} \times 5 = \frac{5}{3}$ **F** $\frac{1}{2} \times \frac{4}{5} = \frac{4}{10}$

2. In each exercise, when you find the **missing factor** in the first equation, you will have found the **quotient** in the second equation. Write the second equation with the correct quotient.

A $n \times \frac{1}{3} = \frac{1}{6}$ $\quad \frac{1}{6} \div \frac{1}{3} = n$

B $n \times \frac{1}{4} = \frac{1}{12}$ $\quad \frac{1}{12} \div \frac{1}{4} = n$

C $n \times \frac{3}{4} = \frac{9}{4}$ $\quad \frac{9}{4} \div \frac{3}{4} = n$

D $n \times \frac{2}{5} = \frac{14}{5}$ $\quad \frac{14}{5} \div \frac{2}{5} = n$

E $n \times \frac{3}{8} = 1$ $\quad 1 \div \frac{3}{8} = n$

F $n \times \frac{5}{3} = \frac{10}{27}$ $\quad \frac{10}{27} \div \frac{5}{3} = n$

G $n \times \frac{7}{5} = 1$ $\quad 1 \div \frac{7}{5} = n$

H $n \times 1 = \frac{4}{13}$ $\quad \frac{4}{13} \div 1 = n$

I $n \times \frac{1}{9} = \frac{5}{18}$ $\quad \frac{5}{18} \div \frac{1}{9} = n$

J $n \times \frac{6}{1} = \frac{12}{13}$ $\quad \frac{12}{13} \div \frac{6}{1} = n$

K $n \times \frac{1}{9} = 1$ $\quad 1 \div \frac{1}{9} = n$

L $n \times \frac{4}{7} = \frac{12}{14}$ $\quad \frac{12}{14} \div \frac{4}{7} = n$

3. Find each quotient. Check your work by multiplying.

A $\frac{10}{3} \div \frac{2}{3}$

B $\frac{9}{16} \div \frac{3}{4}$

C $\frac{3}{8} \div \frac{1}{4}$

D $\frac{20}{9} \div \frac{4}{3}$

E $\frac{8}{21} \div \frac{2}{3}$

F $\frac{3 \times 5}{4 \times 7} \div \frac{5}{7}$

G $\frac{6 \times 2}{5 \times 11} \div \frac{6}{11}$

H $\frac{2 \times 3 \times 5}{2 \times 5 \times 8} \div \frac{2 \times 5}{3 \times 8}$

think

Using each of this year's digits once, parentheses, and the signs $+$, $-$, $\times$, and $\div$, find as many more whole numbers in the list as you can.

$(7 + 3) - (9 + 1) = 0$
$1 \times (7 + 3) - 9 = 1$
$(3 + 1) - (9 - 7) = 2$
$(7 - 1) - (9 \div 3) = 3$
⋮

Discussing the Ideas

1. Answer the questions about the division problem $\frac{2}{3} \div \frac{5}{7}$.

A Can 2 be divided exactly by 5?

$\frac{2}{3} \div \frac{5}{7}$

B Can 2×5 be divided by 5?
Is 3×5 divisible by 7?

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5}$ $\quad \frac{2 \times 5}{3 \times 5} \div \frac{5}{7}$

C Is $2 \times 5 \times 7$ divisible by 5?
Is $3 \times 5 \times 7$ divisible by 7?

$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{2 \times 5 \times 7}{3 \times 5 \times 7}$ $\quad \frac{2 \times 5 \times 7}{3 \times 5 \times 7} \div \frac{5}{7}$

D Explain how you can use the inverse relation of division and multiplication to check this division problem.

$$\frac{2 \times \cancel{5} \times \cancel{7}}{3 \times 5 \times 7} \div \frac{5}{7} = \frac{2 \times 7}{3 \times 5}$$

E Explain these equations.

$$\frac{2}{3} \div \frac{5}{7} = \frac{2 \times 7}{3 \times 5} = \frac{2}{3} \times \frac{7}{5}$$

2. Patty said that she saw a shortcut for dividing. Can you use Patty's method to find the quotient and then check your work?

$$\frac{1}{3} \div \frac{4}{5} = \frac{1}{3} \times \frac{5}{4} = \square$$

Reciprocals

3. Follow the steps in the flow chart. Then give the quotient.

Division problem → Find the reciprocal of the divisor. → Multiply the dividend by the reciprocal of the divisor. → Quotient

$\frac{3}{4} \div \frac{2}{5}$ | The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$. | $\frac{3}{4} \times \frac{5}{2}$ | $\square$

4. Explain how you can use the flow chart in exercise 3 to find these quotients.

A $\frac{1}{2} \div \frac{2}{3}$ B $\frac{7}{8} \div \frac{2}{5}$ C $\frac{1}{2} \div \frac{1}{10}$ D $\frac{3}{4} \div \frac{5}{8}$

1. Solve the equations.

A $6 \div \frac{5}{9} = n \times \frac{9}{5}$ D $\frac{3}{5} \div \frac{1}{2} = n \times 2$ G $\frac{4}{5} \div 5 = \frac{4}{5} \times n$

B $12 \div \frac{3}{4} = n \times \frac{4}{3}$ E $\frac{1}{3} \div \frac{1}{4} = n \times 4$ H $\frac{7}{4} \div \frac{4}{5} = \frac{7}{4} \times n$

C $5 \div \frac{3}{2} = n \times \frac{2}{3}$ F $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times n$ I $\frac{5}{8} \div \frac{3}{2} = \frac{5}{8} \times n$

2. Complete this sentence: To divide a first number by a second number, multiply the first number by the __?__ of the second number.

3. Write each division problem as a multiplication problem. Then find the number for n.

Example: $\frac{4}{5} \div \frac{3}{2} = n$ Solution: $n = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

A $\frac{1}{2} \div \frac{3}{4} = n$ D $\frac{7}{8} \div \frac{1}{2} = n$ G $\frac{5}{12} \div \frac{1}{3} = n$

B $\frac{2}{3} \div \frac{3}{5} = n$ E $\frac{4}{5} \div \frac{1}{5} = n$ H $\frac{7}{10} \div \frac{2}{5} = n$

C $\frac{5}{6} \div \frac{3}{4} = n$ F $\frac{1}{3} \div \frac{5}{9} = n$ I $\frac{5}{8} \div \frac{5}{8} = n$

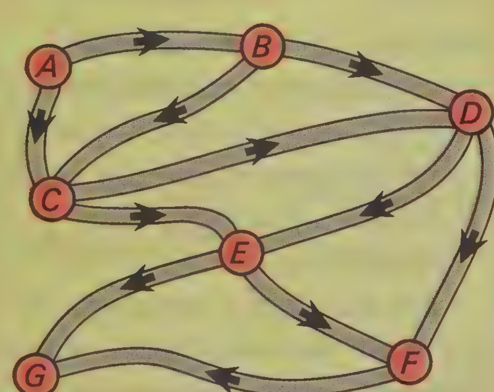
4. Find the quotients.

A $\frac{3}{8} \div \frac{1}{2}$ C $\frac{5}{3} \div \frac{5}{4}$ E $6 \div \frac{2}{3}$ G $7 \div \frac{1}{3}$ I $\frac{5}{8} \div 10$

B $\frac{1}{4} \div \frac{2}{3}$ D $\frac{4}{7} \div 8$ F $\frac{3}{10} \div \frac{7}{10}$ H $7 \div 3$ J $5 \div 10$

think

If you travel along each "one-way" street only in the direction shown by the arrow, how many different routes from A to G can you find?



More practice, page A-16, Set 31

Discussing the Ideas

1. Explain the steps used to find the quotient.

Problem:	$2\frac{1}{2} \div 1\frac{3}{4}$
Solution:	$\frac{5}{2} \div \frac{7}{4} = \frac{5}{\cancel{2}_1} \times \frac{\cancel{4}^2}{7} = \frac{10}{7}$, or $1\frac{3}{7}$.

2. The fractional number $\frac{3}{10}$ is the quotient for $3 \div 10$. Since it will be helpful in later work with fractional numbers, we shall define expressions such as

$$\frac{\frac{4}{5}}{\frac{7}{10}} \text{ to mean } \frac{4}{5} \div \frac{7}{10}.$$

You can think of $\frac{4}{5}$ as the numerator and $\frac{7}{10}$ as the denominator of this "fraction." These more complicated "fractions" have the same properties as the fractions with whole-number numerators and denominators.

Explain how you can use division to "simplify" this fraction: $\frac{\frac{2}{3}}{\frac{5}{6}}$

3. For any division problem you choose (other than division by 0), there is always a fractional-number quotient. The example below shows that the quotient for $29 \div 3$ is the fractional number $\frac{29}{3}$. $29 \div 3 = 29 \times \frac{1}{3} = \frac{29}{3}$

Quotients like $\frac{29}{3}$ are usually represented by a mixed numeral. You can use the division process shown in example A to represent $\frac{29}{3}$ as the mixed numeral $9\frac{2}{3}$. You can complete all this work and find the fractional-number quotient more easily by arranging your work as in example B. Explain how to find fractional-number quotients for each of the following.

A
$$\begin{array}{r} 9 \\ 3\overline{)29} \\ \underline{27} \\ 2 \end{array}$$

B
$$\begin{array}{r} 9\frac{2}{3} \\ 3\overline{)29} \\ \underline{27} \end{array}$$

A $12 \div 5$ B $38 \div 7$ C $53 \div 8$ D $77 \div 9$ E $237 \div 10$

1. Find the quotients.

A $1\frac{1}{2} \div \frac{3}{4}$ C $3\frac{1}{4} \div \frac{5}{6}$ E $\frac{4}{5} \div 2\frac{2}{3}$ G $3\frac{1}{2} \div 2\frac{1}{4}$ I $5\frac{2}{3} \div 6\frac{3}{4}$

B $2\frac{1}{3} \div \frac{1}{5}$ D $\frac{3}{8} \div 1\frac{1}{3}$ F $\frac{1}{6} \div 3\frac{1}{2}$ H $1\frac{3}{8} \div 4\frac{1}{3}$ J $4\frac{2}{3} \div 2\frac{1}{7}$

2. Simplify each expression. Example: $\frac{\frac{4}{5}}{\frac{7}{10}} = \frac{4}{5} \div \frac{7}{10} = \frac{40}{35} = 1\frac{1}{7}$

A $\frac{\frac{1}{3}}{\frac{1}{2}}$ B $\frac{\frac{2}{3}}{\frac{3}{4}}$ C $\frac{\frac{3}{10}}{\frac{1}{10}}$ D $\frac{\frac{3}{5}}{1}$ E $\frac{\frac{6}{5}}{\frac{6}{5}}$ F $\frac{1}{\frac{3}{10}}$ G $\frac{\frac{1}{2}}{100}$ H $\frac{8\frac{1}{3}}{100}$

3. Solve the equations.

A $\frac{\frac{1}{2} \times \frac{1}{3}}{\frac{2}{5} \times \frac{1}{3}} = n$ B $\frac{\frac{2}{3} \times n}{\frac{1}{4} \times n} = \frac{\frac{8}{3}}{1}$ C $\frac{8\frac{1}{3} \times n}{100 \times n} = \frac{25}{300}$ D $\frac{\frac{2}{3}}{\frac{4}{5}} \times \frac{\frac{3}{2}}{\frac{5}{6}} = n$

4. Find the quotients. Check your work.

A $54 \div 7$ C $168 \div 9$ E $495 \div 8$ G $569 \div 11$ I $467 \div 23$

B $72 \div 5$ D $3576 \div 5$ F $347 \div 3$ H $2372 \div 25$ J $896 \div 13$

think

Why is a 25-cent piece called a quarter? The answer may help you find an easy way to solve the two problems below.

1. A restaurant ordered the supplies shown on the right. What was the total bill ?
2. If hamburgers cost 75¢, how many could they buy for $3000 ?

BILL

2564 hot dogs	||||
720 steaks	||||
182 kg potatoes	||||
54 bottles ketchup	||||
Total	||||

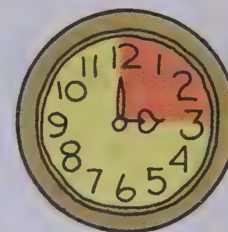
Hot dogs 25¢ Potatoes 15¢ kg

Steaks $1.60 Ketchup $1.50

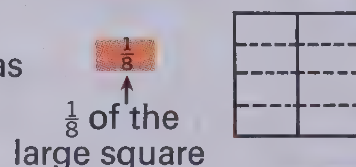
More practice, page A-16, Set 32

Using division to solve problems

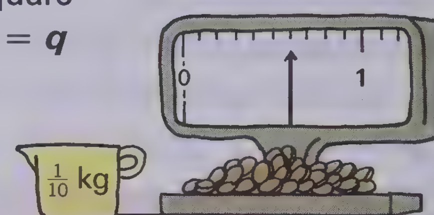
Each problem below can be solved by division of fractional numbers. For each problem, first use any method you wish to find the solution. Then choose the division equation that you can use to solve the problem. The solution to the division equation should agree with your solution to the problem.



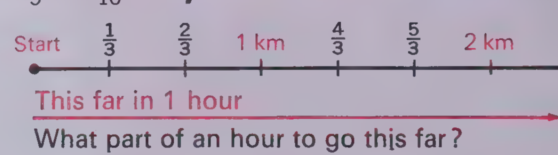
1. How many $\frac{1}{4}$-hour periods are there in 8 hours?
   A $\frac{1}{4} \div 8 = q$ **B** $8 \div \frac{1}{4} = q$ **C** $8 \div 4 = q$



2. How many pieces of red paper $\frac{1}{8}$ as large as the square does it take to cover $\frac{1}{4}$ of the square?
   A $\frac{1}{4} \div \frac{1}{8} = q$ **B** $\frac{1}{8} \div \frac{1}{4} = q$ **C** $4 \div 8 = q$



3. How many cups of peanuts, each weighing $\frac{1}{10}$ kg, are in a pile of peanuts weighing $\frac{3}{5}$ kg?
   A $\frac{3}{5} \div 10 = q$ **B** $\frac{1}{10} \div \frac{3}{5} = q$ **C** $\frac{3}{5} \div \frac{1}{10} = q$



4. Joe walks at a rate of $\frac{7}{3}$ km/h. How long does it take him to walk 1 km?
   A $1 \div \frac{3}{7} = q$ **B** $\frac{7}{3} \div 1 = q$ **C** $1 \div \frac{7}{3} = q$

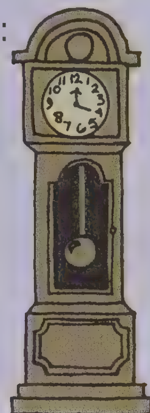
5. 32 candy bars. Same amount to each of 5 children. How many candy bars does each child get?
   A $5 \div 32 = q$ **B** $32 \div 5 = q$ **C** $32 \div \frac{1}{5} = q$

6. Jan walks $\frac{3}{4}$ km each hour. How many hours will it take her to go $\frac{5}{2}$ km?
   A $\frac{5}{2} \div \frac{3}{4} = q$ **B** $\frac{5}{2} \div \frac{4}{3} = q$ **C** $\frac{3}{4} \div \frac{5}{2} = q$

Solving Short Stories

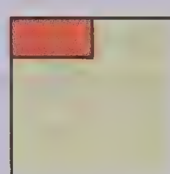
Write and solve a division equation to answer each exercise.

1. 5 hours. How many:
   - A $\frac{1}{2}$-hour periods
   - B $\frac{1}{3}$-hour periods
   - C $\frac{1}{5}$-hour periods



2. $\frac{3}{4}$ hour. How many:
   - A $\frac{1}{4}$-hour periods
   - B $\frac{1}{2}$-hour periods

3. Square piece of paper. Red piece is $\frac{1}{8}$ as large as the square. How many red pieces are needed to cover:
   - A $\frac{1}{2}$ of the square
   - B $\frac{3}{4}$ of the square
   - C 2 squares
   - D $\frac{1}{16}$ of the square



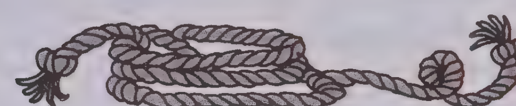
4. Nuts. A cup weighs $\frac{4}{10}$ kg. How many cups of nuts are in a pile weighing:
   A 1 kg B 2 kg C 10 kg
   D 100 kg E $\frac{4}{5}$ kg F $\frac{1}{5}$ kg

5. $\frac{2}{5}$ kg of nuts in each bag. How many bags of nuts from:
   A 1 kg B 30 kg C $1\frac{1}{5}$ kg

6. Walk 12 kilometres. How long does it take going:
   A $\frac{3}{2}$ km/h B $\frac{7}{3}$ km/h C 5 km/h

7. A strange coincidence. Six hens, each the same weight. Total weight, 35 kilograms. What does each hen weigh?

8. $\frac{4}{5}$ km to school and back. How many trips to go: A 15 km
   B 24 km C 1 km D $\frac{5}{4}$ km



9. Rope: $5\frac{1}{4}$ metres long. How many $\frac{3}{4}$-metre pieces can be cut from it?

10. Running track: $\frac{4}{10}$ km. How many laps in:
    A 1 km B 10 km

11. Area of rectangle: $\frac{7}{10}$ square centimetre. Width: $\frac{3}{5}$ cm. What is the length?

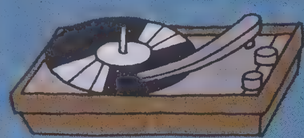
12. Takes $1\frac{3}{4}$ minutes to make a gadget. How many gadgets can be made in 14 minutes?

13. Total weight of 9 children: 329 kg. What is the average weight of the children?

DIAMONDS

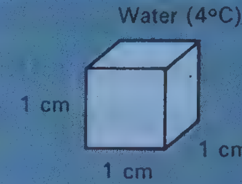
Uses of diamonds


Jewelry


Phonograph needles

Cutting and drilling

Weighing diamonds



One cubic centimetre of water weighs **1 gram.** Diamonds are weighed by using a unit called a **carat.**

A diamond weighing **1 carat** weighs $\frac{1}{5}$ **gram.**

1. A diamond this size  weighs about 4 carats. What part of a gram is this?

2. The largest blue diamond in the world is the Hope diamond, which weighs $44\frac{2}{5}$ carats. Another famous diamond was the Timken stone, which weighed $28\frac{3}{4}$ carats. How much more than the Timken stone does the Hope diamond weigh?

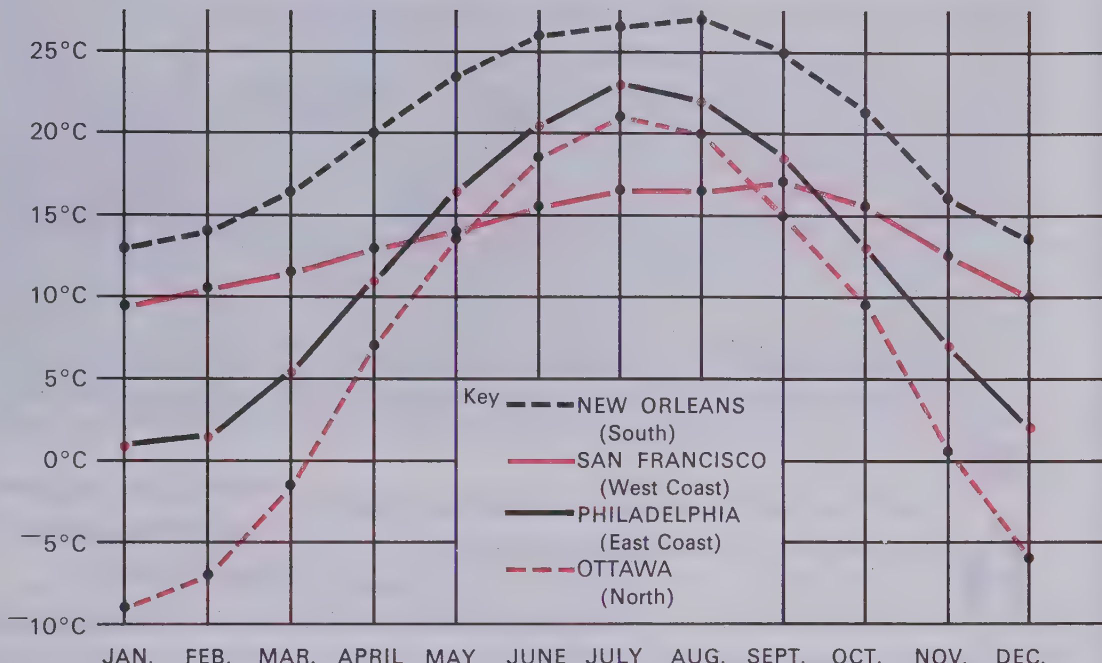
3. The Cullinan crystal, the world's largest, weighed about 585 grams. How many carats did it weigh?

4. The world's largest cut diamond, the Star of South Africa, was cut from the Cullinan crystal. The Star weighed about 530 carats. Did the Star of South Africa weigh closer to $\frac{1}{10}$kg, $\frac{2}{10}$kg, or $\frac{3}{10}$kg?

5. Diamonds, which are pure carbon, are the hardest natural substance. If measured for hardness, a fingernail might get a rating of $2\frac{1}{3}$; a copper coin, $2\frac{3}{4}$; a window glass, $5\frac{1}{2}$; and a diamond, 10. Since $5\frac{1}{2} \div 2\frac{3}{4} = 2$, we could say that glass is twice as hard as a copper coin. Diamonds are how many times as hard as
   A your fingernail? B a copper coin? C glass?

AVERAGE TEMPERATURES

Monthly normal temperatures (degrees Celsius)

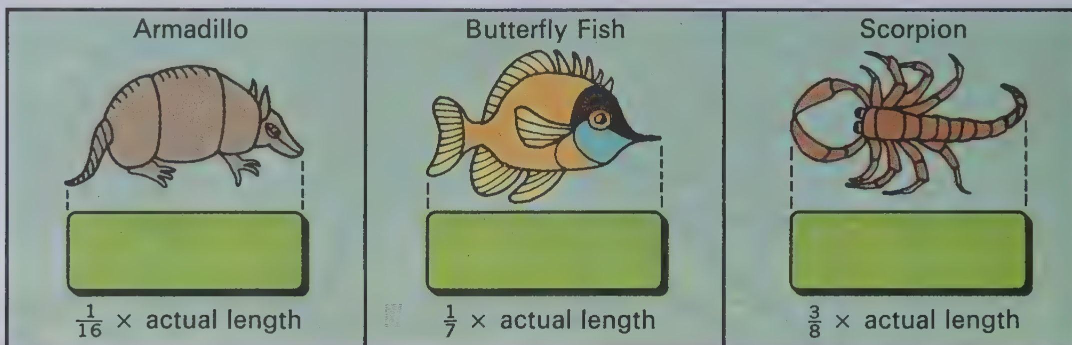


1. Give the normal July temperature for each city.
2. How much higher is the July temperature in New Orleans than in:
   A Ottawa B San Francisco
3. Give the average summer (June, July, August) temperature for:
   A San Francisco B Philadelphia C Ottawa D New Orleans
4. How much greater is the average summer temperature in Ottawa than San Francisco ?
5. Give the average winter (December, January, February) temperature for each of the four cities.
6. How much greater is the average winter temperature in San Francisco than in Ottawa ?
7. Give the average yearly temperature for San Francisco.

More practice, page A-17, Set 33

Investigating the Ideas

The picture of each creature has been reduced until it is just as long as the light green strip.



?		
?	Can you lay down strips to show the actual length of each creature?	Record the number of strips you used for each part.

Discussing the Ideas

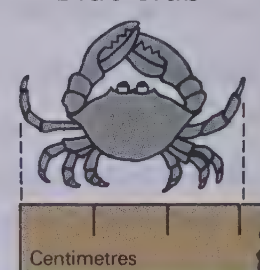
1. When objects are drawn to scale, you can find their actual lengths by division. For example, the picture of the blue crab is $\frac{1}{12}$ the width of a real crab. You can write the multiplication equation

$$\frac{1}{12} \times w = 3.$$

   A Explain how to find the missing factor.

   B What is the actual width of a blue crab?

Blue crab

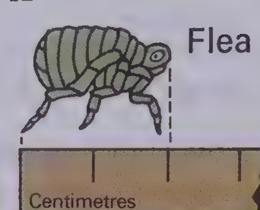


$\frac{1}{12}$ × actual width

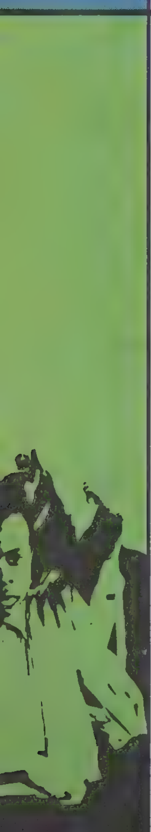
2. A Is a flea larger or smaller than the picture?

   B Explain how to find the actual length of a flea.

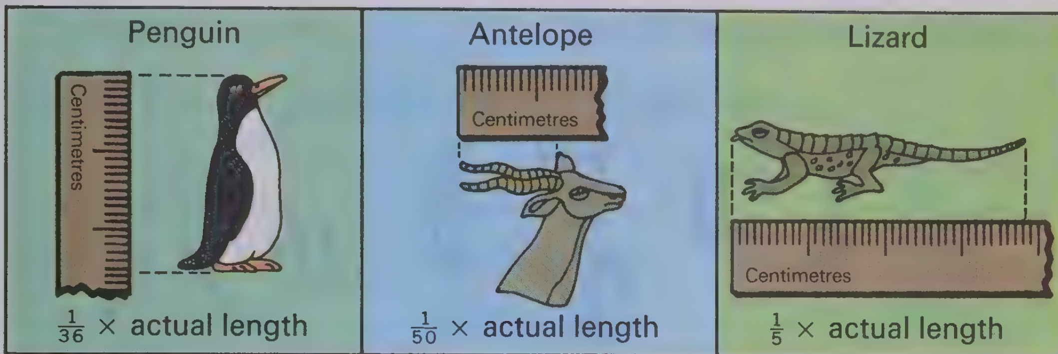
Flea



8 × actual length



ANIMAL LENGTHS

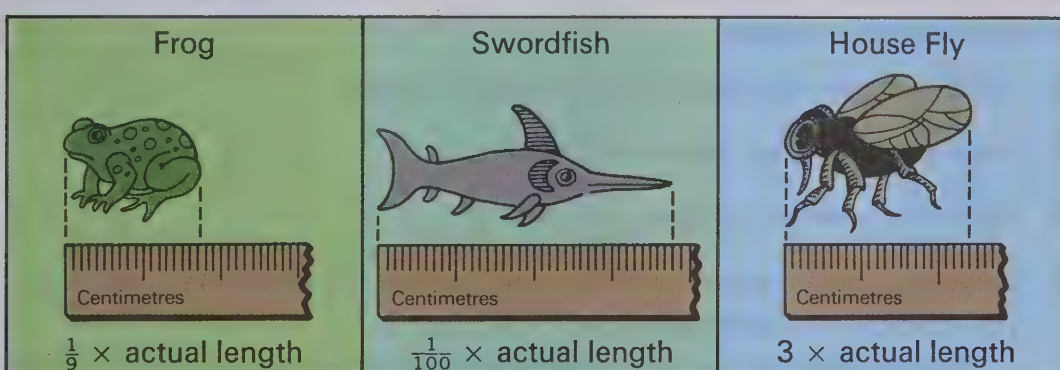


1. How tall is a penguin
   A in centimetres? B in metres?

2. Find the length of an antelope's horns
   A in centimetres. B in metres.

3. One of the longest elephant tusks is about $4\frac{3}{5}$ times as long as the antelope's horn. How long is the tusk?

4. Find the length of a lizard.



5. How long is a frog's body?

6. How long is the swordfish?

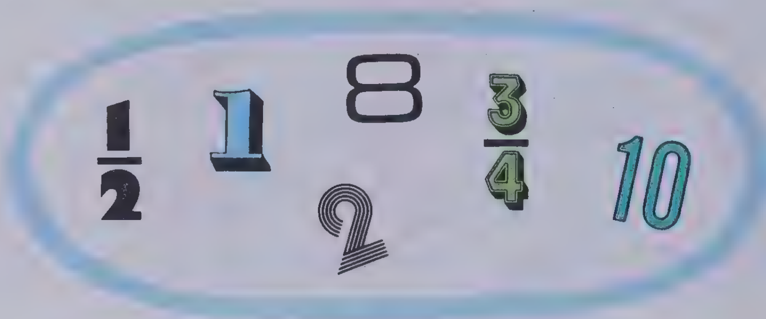
7. How long is the house fly?

8. If a frog can jump $17\frac{1}{2}$ times as far as the length of his body, how many metres can he jump?

★ 9. The lizard's tail is $\frac{3}{4}$ the length of his body. How long is the lizard's tail?

Investigating the Ideas

Use only the numbers in this set.



Can you find the two different numbers in the set that give

- **A** the largest product?
- **B** the smallest product?
- **C** the largest quotient?
- **D** the smallest quotient?

Discussing the Ideas

1. Explain how you found the largest product.
2. How can you find the smallest product?
3. Which quotient is larger, $2 \div \frac{1}{2}$ or $2 \div \frac{1}{4}$?
4. Which phrase (**greater than, less than,** or **equal to**) would you choose to complete each sentence?
   - **A** The product of two fractional numbers (each less than 1) is __?__ either of the factors.
   - **B** The product of two fractional numbers (each greater than 1) is __?__ either of the factors.
   - **C** When dividing by a fractional number less than 1, the missing factor is __?__ the product.
   - **D** When dividing by a fractional number greater than 1, the missing factor is __?__ the product.

These exercises will help you compare fractional-number products and factors.

1. Give the symbol ($<$, $=$, $>$) for each $\bigcirc$.

A $8 \times 8 \bigcirc 8$
B $8 \times 4 \bigcirc 8$
C $8 \times 2 \bigcirc 8$
D $8 \times 1 \bigcirc 8$
E $8 \times \frac{1}{2} \bigcirc 8$
F $8 \times \frac{1}{4} \bigcirc 8$
G $8 \times \frac{1}{8} \bigcirc 8$

H $\frac{1}{2} \times 8 \bigcirc \frac{1}{2}$
I $\frac{1}{2} \times 4 \bigcirc \frac{1}{2}$
J $\frac{1}{2} \times 2 \bigcirc \frac{1}{2}$
K $\frac{1}{2} \times 1 \bigcirc \frac{1}{2}$
L $\frac{1}{2} \times \frac{1}{2} \bigcirc \frac{1}{2}$
M $\frac{1}{2} \times \frac{1}{4} \bigcirc \frac{1}{2}$
N $\frac{1}{2} \times \frac{1}{8} \bigcirc \frac{1}{2}$

O $5 \times 1000 \bigcirc 5$
P $5 \times 100 \bigcirc 5$
Q $5 \times 10 \bigcirc 5$
R $5 \times 1 \bigcirc 5$
S $5 \times \frac{1}{10} \bigcirc 5$
T $5 \times \frac{1}{100} \bigcirc 5$
U $5 \times \frac{1}{1000} \bigcirc 5$

2. Give the symbol ($<$, $>$, $=$) for each $\bigcirc$.

A $24 \div 4 \bigcirc 24$
B $24 \div 3 \bigcirc 24$
C $24 \div 2 \bigcirc 24$
D $24 \div 1 \bigcirc 24$
E $24 \div \frac{1}{2} \bigcirc 24$
F $24 \div \frac{1}{3} \bigcirc 24$
G $24 \div \frac{1}{4} \bigcirc 24$

H $6 \div 12 \bigcirc 6$
I $6 \div 6 \bigcirc 6$
J $6 \div 3 \bigcirc 6$
K $6 \div 1 \bigcirc 6$
L $6 \div \frac{1}{3} \bigcirc 6$
M $6 \div \frac{1}{6} \bigcirc 6$
N $6 \div \frac{1}{12} \bigcirc 6$

O $1 \div 1000 \bigcirc 1$
P $1 \div 100 \bigcirc 1$
Q $1 \div 10 \bigcirc 1$
R $1 \div 1 \bigcirc 1$
S $1 \div \frac{1}{10} \bigcirc 1$
T $1 \div \frac{1}{100} \bigcirc 1$
U $1 \div \frac{1}{1000} \bigcirc 1$

3. Give the symbol ($<$, $>$, $=$) for each $\bigcirc$.
Do as little computing as possible.

A $128 \div 4 \bigcirc 128 \div 2$
B $128 \div 2 \bigcirc 128 \div 1$
C $128 \div 1 \bigcirc 128 \div \frac{1}{2}$

D $\frac{5}{6} \times \frac{3}{8} \bigcirc \frac{5}{6}$
E $\frac{5}{6} \times \frac{3}{8} \bigcirc \frac{3}{8}$
F $1 \div \frac{1}{3} \bigcirc 1$

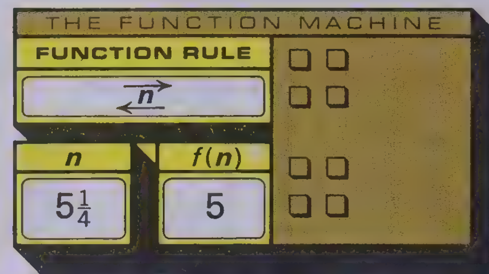
G $24 \div \frac{5}{4} \bigcirc 24$
H $\frac{1}{2} \div \frac{1}{2} \bigcirc \frac{4}{7} \div \frac{4}{7}$
I $\frac{2}{3} \times \frac{5}{7} \bigcirc \frac{5}{7} \times \frac{2}{3}$

4. A If, in the equation $a \times b = 1$, the number for a is less than 1, what do you know about the number for b?
B Can the numbers for both a and b be less than 1?

Let's explore a special function.

Investigating the Ideas

Tim thought of a special function. Since he could not use addition, subtraction, multiplication, or division to describe the function rule, Tim used the symbol $\overset{\rightarrow}{\underset{\leftarrow}{n}}$. Study the tables below to understand Tim's function.



A Function Rule $\overset{\rightarrow}{\underset{\leftarrow}{n}}$

n	$f(n)$
5	5
$5\frac{1}{8}$	5
$5\frac{3}{8}$	5
$5\frac{5}{8}$	6
$5\frac{7}{8}$	6

B Function Rule $\overset{\rightarrow}{\underset{\leftarrow}{n}}$

n	$f(n)$
$8\frac{1}{4}$	8
$8\frac{1}{2}$	9
$8\frac{3}{4}$	9
9	9
$9\frac{1}{4}$	9

C Function Rule $\overset{\rightarrow}{\underset{\leftarrow}{n}}$

n	$f(n)$
$\frac{8}{3}$	3
$\frac{7}{2}$	4
$\frac{1}{4}$	0
$\frac{17}{2}$	9
$23\frac{5}{6}$	24

? Can you discover Tim's function rule and use the rule to make a table of your own?

Discussing the Ideas

1. Describe Tim's rule in your own words.
2. Explain how to find the missing numbers.
   A $\overset{\rightarrow}{\underset{\leftarrow}{3}} = ▒$ B $\overset{\rightarrow}{\underset{\leftarrow}{3\frac{1}{4}}} = ▒$ C $\overset{\rightarrow}{\underset{\leftarrow}{3\frac{1}{2}}} = ▒$ D $\overset{\rightarrow}{\underset{\leftarrow}{3\frac{3}{4}}} = ▒$ E $\overset{\rightarrow}{\underset{\leftarrow}{4\frac{2}{5}}} = ▒$
3. Explain how to find the sums.
   A $\overset{\longrightarrow}{\underset{\longleftarrow}{1\frac{1}{3} + 2\frac{1}{3}}} = ▒$ B $\overset{\rightarrow}{\underset{\leftarrow}{2\frac{3}{4}}} + \overset{\rightarrow}{\underset{\leftarrow}{3\frac{1}{2}}} = ▒$
4. Can you explain why the two sums below are not the same?
   A $\overset{\rightarrow}{\underset{\leftarrow}{5\frac{3}{8}}} + \overset{\rightarrow}{\underset{\leftarrow}{4\frac{3}{8}}} = 9$ B $\overset{\longrightarrow}{\underset{\longleftarrow}{5\frac{3}{8} + 4\frac{3}{8}}} = 10$

Complete the function tables in exercises 1 through 3.

1. Function Rule: $\overrightarrow{\underleftarrow{n}}$

	n	$f(n)$
A	$\frac{7}{8}$	$\square$
B	$1\frac{2}{3}$	$\square$
C	$\frac{3}{2}$	$\square$
D	$\frac{8}{2}$	$\square$
E	$\frac{22}{7}$	$\square$

2. Function Rule: $\overrightarrow{\underleftarrow{n}}$

	n	$f(n)$
A	$\frac{1562}{1561}$	$\square$
B	$\frac{1562}{1500}$	$\square$
C	$\frac{287}{288}$	$\square$
D	$\frac{49}{10}$	$\square$
E	$\frac{65}{7}$	$\square$

3. Function Rule: $\overrightarrow{\underleftarrow{n}}$

	n	$f(n)$
A	$7\frac{3}{8}$	$\square$
B	$13\frac{7}{16}$	$\square$
C	$1\frac{8}{5}$	$\square$
D	$5\frac{33}{65}$	$\square$
E	$\frac{66}{12}$	$\square$

4. Give the missing numbers.

A $\overrightarrow{\underleftarrow{\frac{9}{2}}} = \square$ **D** $\overrightarrow{\underleftarrow{\frac{324}{10}}} = \square$ **G** $\overrightarrow{\underleftarrow{\frac{25}{2}}} = \square$ **J** $\overrightarrow{\underleftarrow{5\frac{13}{27}}} = \square$

B $\overrightarrow{\underleftarrow{\frac{7}{8}}} = \square$ **E** $\overrightarrow{\underleftarrow{\frac{546}{100}}} = \square$ **H** $\overrightarrow{\underleftarrow{18\frac{3}{4}}} = \square$ **K** $\overrightarrow{\underleftarrow{9\frac{23}{45}}} = \square$

C $\overrightarrow{\underleftarrow{\frac{73}{10}}} = \square$ **F** $\overrightarrow{\underleftarrow{\frac{494}{10}}} = \square$ **I** $\overrightarrow{\underleftarrow{\frac{5500}{1000}}} = \square$ **L** $\overrightarrow{\underleftarrow{\frac{6852}{100}}} = \square$

5. Give the missing numbers. Do as little computing as possible.

A $\overrightarrow{\underleftarrow{2\frac{3}{4}}} + \overrightarrow{\underleftarrow{3\frac{1}{2}}} = \square$

B $\overrightarrow{\underleftarrow{2\frac{3}{4} + 3\frac{1}{2}}} = \square$

C $\overrightarrow{\underleftarrow{7\frac{5}{11}}} + \overrightarrow{\underleftarrow{6\frac{5}{8}}} = \square$

D $\overrightarrow{\underleftarrow{7\frac{5}{10} + 6\frac{5}{8}}} = \square$

E $\overrightarrow{\underleftarrow{68\frac{1}{3}}} + \overrightarrow{\underleftarrow{99\frac{1}{16}}} = \square$

F $\overrightarrow{\underleftarrow{58\frac{1}{2} + 69\frac{8}{17}}} = \square$

G $\overrightarrow{\underleftarrow{9\frac{1}{2}}} \times \overrightarrow{\underleftarrow{6\frac{3}{8}}} = \square$

H $\overrightarrow{\underleftarrow{12\frac{5}{6}}} \times \overrightarrow{\underleftarrow{12\frac{1}{3}}} = \square$

I $\overrightarrow{\underleftarrow{35\frac{6}{13}}} \times \overrightarrow{\underleftarrow{7\frac{7}{13}}} = \square$

J $\overrightarrow{\underleftarrow{4\frac{3}{4} \times 3}} = \square$

K $\overrightarrow{\underleftarrow{9 \times 7\frac{3}{8}}} = \square$

L $\overrightarrow{\underleftarrow{32\frac{1}{8}}} - \overrightarrow{\underleftarrow{19\frac{3}{4}}} = \square$

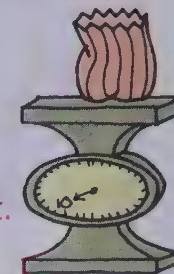
M $\overrightarrow{\underleftarrow{58\frac{3}{4} - 8\frac{1}{2}}} = \square$

N $\overrightarrow{\underleftarrow{24}} \div \overrightarrow{\underleftarrow{\frac{7}{8}}} = \square$

O $\overrightarrow{\underleftarrow{35\frac{1}{4}}} \div \overrightarrow{\underleftarrow{7\frac{3}{8}}} = \square$

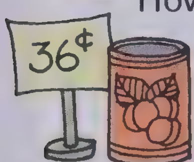
ESTIMATION

For each exercise the answer given may or may not be correct. You are to: **A** Choose whole numbers nearest to the numbers in the problem and **estimate** the answer. **B** Then tell whether or not the given answer is correct. Use your estimate to determine this if possible. Exercise 1 is completed correctly.



1 $3\frac{5}{8}$¢ for each gram. 10 grams. Costs how much ? ($42\frac{1}{8}$¢)
A Estimate: 10 × 4, or 40¢ **B** The answer is not correct.

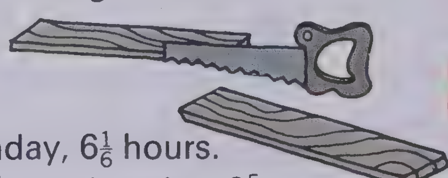
2 Packet costs 36¢.
Contains $5\frac{9}{10}$ grams.
How much per gram ? ($4\frac{3}{4}$¢)



3 Drove 165 km.
$3\frac{3}{4}$ hours. Speed ? (38 km/h)

4 Man weighs $93\frac{1}{5}$ kg. Another weighs $69\frac{3}{4}$ kg.
What is the difference of their weights ? (13 kg)

5 Cut a board. One piece, $5\frac{4}{5}$ m long. Other piece, $8\frac{1}{4}$ m long. How long before cut ? ($14\frac{1}{20}$ m)



6 Ribbon, $\frac{9}{10}$ metre long.
Cut into 5 equal pieces.
How long is each piece ? ($\frac{1}{4}$ m)

7 Sleep: Monday, $6\frac{1}{6}$ hours. Tuesday, $7\frac{1}{3}$. Wednesday, $8\frac{5}{6}$. Thursday, $10\frac{2}{3}$. Friday, $8\frac{1}{2}$. Saturday, $9\frac{2}{3}$. Sunday, $7\frac{5}{6}$. What is the average number of hours slept each night ? ($6\frac{3}{7}$ h)

8 Length of field, $39\frac{2}{5}$ metres. Width of field, 75 metres. What is the area of the field ? (2955 square metres)

9 $\frac{9}{20}$ km to school.
To school and back for 30 days. How far ? ($31\frac{7}{20}$ km)

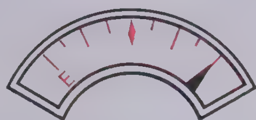
10 Walk $\frac{9}{4}$ km in an hour. Walked $20\frac{1}{4}$ km. How many hours ? (11 h)

★**11** Satellite: Around Earth in $1\frac{2}{5}$ hours. How many trips in one week ? (165)

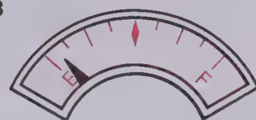
ESTIMATION FOR FUN

1. The gasoline gauge on a car looked like this when there were 2 litres of gasoline in the tank. Estimate the number of litres of gasoline in the tank for each gauge-reading below.

A



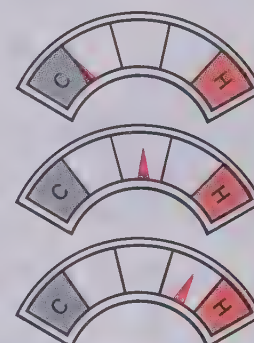
B



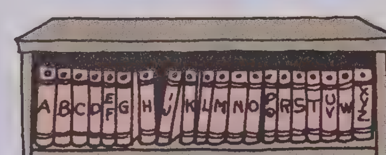
C

D

2. When the temperature gauge on the car looked like this, the temperature was 60 °C. ⟶
When the gauge looked like this, the temperature was 75 °C. ⟶
What was the temperature when the gauge looked like this? ⟶

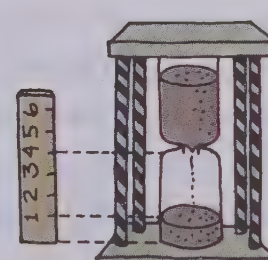


3. The thickest book in a set of encyclopedias is $4\frac{3}{4}$ cm. The thinnest book is $1\frac{3}{4}$ cm. Estimate the length of shelf needed to hold the complete set of 20 books.



4. The small can holds $19\frac{3}{4}$ grams. Estimate the number of grams the large can holds.

5. A If it has taken $1\frac{3}{4}$ hours for the sand shown to go into the bottom part of the time glass, about how much longer will it take for all the sand to go down?
   B Suppose it takes only 1 hour for all the sand to go down. Estimate the number of minutes that have passed when the sand has reached the point midway between 2 and 3 on the ruler.



1. Find the products.

A $9 \times \frac{1}{5}$ B $\frac{1}{10} \times 4$ C $\frac{1}{2} \times \frac{1}{5}$ D $\frac{1}{3} \times \frac{1}{8}$ E $\frac{3}{4} \times \frac{4}{5}$ F $\frac{2}{3} \times \frac{1}{6}$ G $4 \times 3\frac{3}{4}$ H $5 \times 6\frac{2}{3}$ I $5\frac{2}{3} \times 3\frac{1}{5}$ J $7\frac{1}{6} \times 8\frac{1}{4}$

2. Find the quotients.

A $1 \div \frac{5}{6}$ B $1 \div 3$ C $5 \div \frac{3}{4}$ D $8 \div 3$ E $6 \div 7$ F $\frac{1}{9} \div \frac{1}{3}$ G $\frac{8}{5} \div \frac{2}{5}$ H $\frac{3}{10} \div \frac{4}{5}$ I $\frac{3}{5} \div 4$ J $3\frac{3}{4} \div 1\frac{2}{3}$

3. Match each quotient with a product that is equal to it.

A $\frac{5}{6} \div \frac{5}{6}$ $\boxed{1}$ $\frac{5}{6} \times \frac{5}{6}$
B $\frac{6}{5} \div \frac{5}{6}$ $\boxed{2}$ $\frac{5}{6} \times \frac{6}{5}$
C $\frac{5}{6} \div \frac{6}{5}$ $\boxed{3}$ $\frac{6}{5} \times \frac{5}{6}$
D $\frac{6}{5} \div \frac{6}{5}$ $\boxed{4}$ $\frac{6}{5} \times \frac{6}{5}$

4. Simplify each expression.

A $\dfrac{\frac{1}{2}}{\frac{2}{3}}$ B $\dfrac{\frac{3}{5}}{\frac{1}{10}}$ C $\dfrac{\frac{3}{10}}{\frac{1}{10}}$ D $\dfrac{12\frac{1}{2}}{100}$

E $\dfrac{\frac{1}{4}}{\frac{1}{5}}$ F $\dfrac{\frac{3}{2}}{\frac{2}{3}}$ G $\dfrac{\frac{3}{4}}{\frac{3}{4}}$ H $\dfrac{\frac{1}{3}}{100}$

5. Find the quotients. Write them as mixed numerals.

A $49 \div 7$ B $85 \div 3$ C $104 \div 9$ D $276 \div 5$ E $5963 \div 24$

6. Give the symbol ($<$, $>$, $=$) for each ●.

A $5 \div 2$ ● $5 \div \frac{1}{2}$ B $8 \div \frac{1}{4}$ ● $8 \div \frac{1}{2}$ C $\frac{3}{4} \div \frac{2}{3}$ ● $\frac{2}{3} \div \frac{3}{4}$

7. Suppose a can contains $67\frac{1}{2}$ grams of nuts.

A How many grams of nuts are in 5 such cans ?

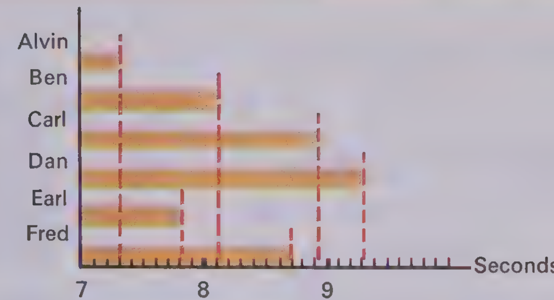
B How many grams in $\frac{1}{2}$ can ?

C How many $6\frac{3}{4}$-gram paper cups can you fill from 1 can of nuts ?

think

Twinkles is $\frac{2}{3}$ as old as Taffy. The difference in their ages is 4 years. How old is each cat ?

8. The graph gives the time it took each boy to run the 50-metre dash. Give the average (arithmetic mean) of these times.

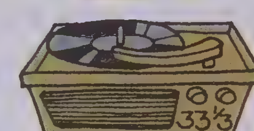


9. Write and solve an equation to answer each question.

A An average walker can cover 1 kilometre in about $\frac{1}{6}$ hours. How long would it take him to reach Geneva ?

B A cubic centimetre of gold weighs $2\frac{2}{5}$ as much as a cubic centimetre of iron, which weighs about $4\frac{6}{10}$ g. What is the weight of a cubic centimetre of gold ?

C A $33\frac{1}{3}$-rpm (revolutions per minute) phonograph record turns around $33\frac{1}{3}$ times each minute. If it takes 21 minutes to play the record, how many times does it turn around ?



D Each $\frac{3}{10}$ on a map represents 1 km on land. The distance between two cities on the map is $2\frac{4}{10}$ cm. How many kilometres apart are these cities ?

E On a scale drawing, a room is 9 cm long. If $\frac{2}{5}$ cm represents 1 metre, how long is the room ?

F A diamond about this size  weighs $\frac{1}{3}$ carat. An object that weighs 2500 carats weighs 500 g. How many diamonds of the size shown does it take to make 1 kilogram ?

10. Estimate the number of $4\frac{7}{10}$-metre pieces of rope you can cut from a piece of rope $89\frac{3}{10}$ metres long.

1. Solve the equations.

A $9 \times n = 56$ B $27 + n = 34$ C $48 \div n = 6$ D $98 - 29 = n$

2. Solve the equations.

A $(9 \times 6) + n = 60$ C $(4 \times 8) + t = 36$ E $(3 \times n) + 5 = 29$

B $(8 \times 7) + s = 59$ D $(9 \times 7) + r = 70$ F $(5 \times p) + 7 = 42$

3. Find the sums, differences, products, and quotients.

A $\begin{array}{r} 9672 \\ +4865 \\ \hline \end{array}$ B $\begin{array}{r} 7004 \\ -879 \\ \hline \end{array}$

C $\begin{array}{r} 598 \\ \times 63 \\ \hline \end{array}$ D $\begin{array}{r} 529 \\ \times 304 \\ \hline \end{array}$

E $9\overline{)6347}$ F $58\overline{)429}$

think

Suppose you snap your fingers 2 minutes from now; wait 4 minutes, do it again; wait twice as long (8 minutes), do it again. If you continue this, will you snap your fingers about 2000, 200, or 20 times during the next year?

4. Find the totals. Change each answer so that you have the greatest number of the larger unit.

A 6 h 54 min
8 h 10 min

B 6 min 30 sec
9 min 40 sec

C 8 yr 7 mo
23 yr 9 mo

D 12 wk 2 days
9 wk 6 days

5. Find the differences.

A 15 wk 4 days
6 wk 5 days

B 5 h 1 min
3 h 2 min

C 24 min 17 sec
15 min 35 sec

D 39 yr 7 mo
19 yr 10 mo

You are invited to explore

ACTIVITY CARD 11
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Short Stories

TIME

1 2044 seconds. How many minutes?

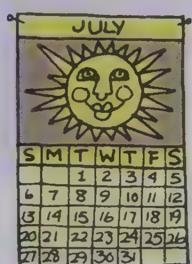
2 $\frac{1}{2}$ hour. How many seconds?

3 9 hours. How many minutes?

4 $5\frac{1}{4}$ days. How many hours?

5 1 week. How many minutes?

6 3312 minutes. How many days?



7 11 years old. How old
A in days? B in hours?
C in minutes? D in seconds?

8 324 months. How many years?

9 Semimonthly means twice a month. Meet a friend semimonthly for 12 years. How many meetings?

10 100 years: 1 century. How many full centuries have passed since the year 1 A.D.?

11 10 years: 1 decade. How many decades of the 20th century have passed?

12 Bimonthly means once very 2 months. Play golf bimonthly for 12 years. How many games?

13 How long is twice as long as: 3 weeks, 4 days, 8 hours, 35 minutes, 40 seconds? Use the smallest number of each unit.

14 1000 years: 1 millenium. Write a mixed numeral that describes the milleniums that have passed since the year 1 A.D.

15 1 millenium, 9 centuries, 7 decades, 6 years, and 8 months. How many years?

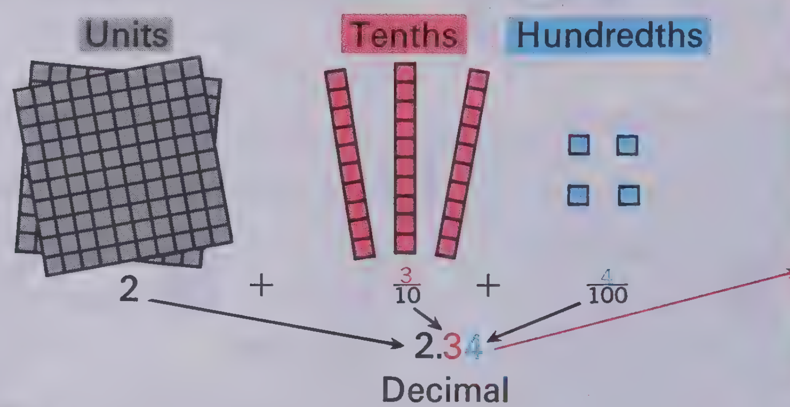
16 The Great Sphinx in Egypt was built in 2900 B.C. How many years ago?

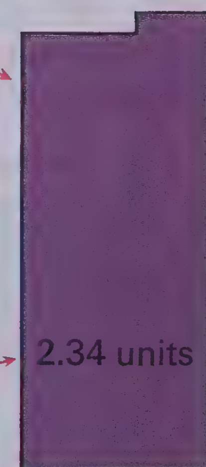
10 Decimals

- *Let's explore decimal names for fractional numbers.*

Investigating the Ideas

The pieces below will just cover this region.
We can show the region's area as a decimal.



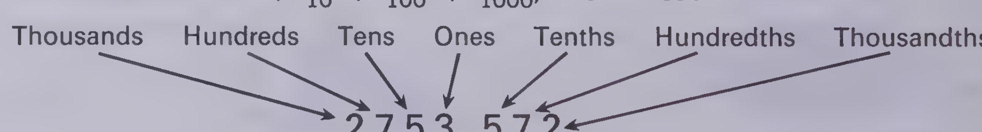


? Can you cut from graph paper some pieces like those above, put them together to form an interesting region, and give the region's area as a decimal ?

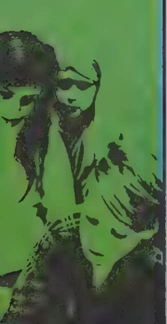
Discussing the Ideas

1. The decimal 2.34 is another way of writing $2 + \frac{3}{10} + \frac{4}{100}$.
   What decimal would you write for $3 + \frac{5}{10} + \frac{7}{100}$?

2. Study the diagram. Give the missing fractions.
   For the sum $2753 + \frac{5}{10} + \frac{7}{100} + \frac{2}{1000}$, we write:



   A The 5 in the tenths' place means __?__.
   B The 7 in the hundredths' place means __?__.
   C The 2 in the thousandths' place means __?__.



1. Copy the sentence. Give the missing word and number.
   - A 537.29: The 3 in the __?__ place means ▒.
   - B 537.29: The 2 in the __?__ place means ▒.
   - C 537.29: The 5 in the __?__ place means ▒.
   - D 537.29: The 9 in the __?__ place means ▒.
   - E 8462.157: The 8 in the __?__ place means ▒.
   - F 8462.157: The 7 in the __?__ place means ▒.
   - G 8462.157: The 5 in the __?__ place means ▒.

2. Write each fractional number as in the example.

   $54.21 = 54 + \frac{2}{10} + \frac{1}{100}$

A 356.3	D 89.05	G 836.4	J 0.999	M 5.490
B 49.86	E 3.008	H 83.64	K 8.364	N 8.603
C 2.475	F 50.7	I 5.607	L 0.836	O 29.07

3. Write the correct decimal for each sum.

A $9 + \frac{4}{10}$	F $5 + \frac{6}{10}$	K $8 + \frac{3}{10} + \frac{9}{1000}$
B $9 + \frac{4}{10} + \frac{3}{100}$	G $5 + \frac{0}{10} + \frac{6}{100}$	L $17 + \frac{4}{100} + \frac{6}{1000}$
C $9 + \frac{4}{10} + \frac{3}{100} + \frac{7}{1000}$	H $5 + \frac{0}{10} + \frac{0}{100} + \frac{6}{1000}$	M $80 + \frac{8}{10}$
D $50 + \frac{7}{10}$	I $5 + \frac{3}{100}$	N $600 + \frac{6}{100}$
E $50 + \frac{7}{10} + \frac{2}{100}$	J $5 + \frac{7}{1000}$	O $9000 + \frac{9}{1000}$

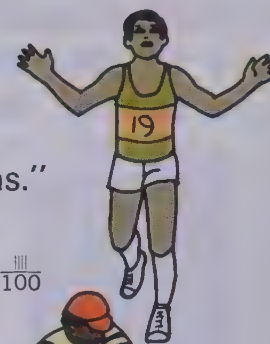
★ 4. Use what you know about place value to answer questions about the decimal 5 496 000.007281 Give the number of:
   - A thousands
   - B thousandths
   - C ten thousands
   - D ten thousandths
   - E hundred thousands
   - F hundred thousandths
   - G millions
   - H millionths

think

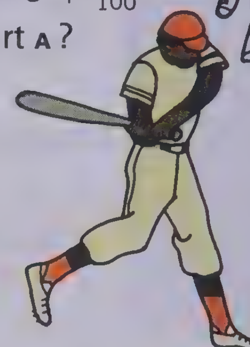
Give the missing numbers so this will be a magic square.

▒	$\frac{9}{12}$	$\frac{4}{12}$
▒	▒	$\frac{11}{12}$
$\frac{10}{12}$	▒	$\frac{6}{12}$

Discussing the Ideas

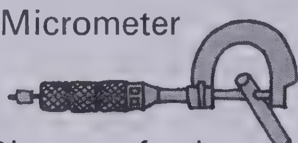


1. Jack ran a kilometre in 2.54 minutes.
$2.54 = 2 + \frac{5}{10} + \frac{4}{100} = 2 + \frac{50}{100} + \frac{4}{100} = 2\frac{54}{100}$
For 2.54 we read "two and fifty-four hundredths."
   A Give the missing numerators.
   $8.34 = 8 + \frac{3}{10} + \frac{4}{100} = 8 + \frac{\square}{100} + \frac{4}{100} = 8 + \frac{\square}{100}$
   B How would you read the decimal in part **A**?



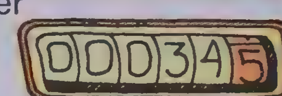
2. Tony's batting average is 0.355
$0.355 = \frac{3}{10} + \frac{5}{100} + \frac{5}{1000}$
$= \frac{300}{1000} + \frac{50}{1000} + \frac{5}{1000} = \frac{355}{1000}$
For 0.355 we read "three hundred fifty-five thousandths."
   A Give the missing numerators.
   $0.317 = \frac{3}{10} + \frac{1}{100} + \frac{7}{1000} = \frac{\square}{1000} + \frac{\square}{1000} + \frac{7}{1000} = \frac{\square}{1000}$
   B How would you read the decimal in part **A**?

3. The examples show some uses of decimals. Read each decimal.
   A Micrometer



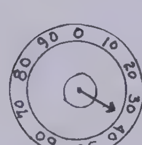
Diameter of rod: 1.241 cm

   B Odometer



Distance: 34.5 kilometres

   C Thermometer



Temperature: 37.46°C

   D Racing car

Speedway record: 257.735 km/h

4. Read these decimals.

A 27.8	**E** 6.38	**I** 4.07	**M** 125.7	**Q** 7.08	**U** 0.007
B 8.76	**F** 19.56	**J** 0.36	**N** 32.09	**R** 0.023	**V** 0.654
C 5.123	**G** 35.872	**K** 0.743	**O** 4.008	**S** 0.08	**W** 0.054
D 4.364	**H** 2.654	**L** 96.04	**P** 16.017	**T** 3.64	**X** 0.09

1. Copy each exercise and give the missing numerator or denominator.

A $5.7=5\frac{\square}{10}$	E $9.06=9\frac{6}{\square}$	I $0.407=\frac{\square}{1000}$	M $5.060=5\frac{60}{\square}$
B $6.54=6\frac{\square}{100}$	F $30.43=30\frac{\square}{100}$	J $0.560=\frac{\square}{1000}$	N $0.024=\frac{\square}{1000}$
C $8.37=8\frac{37}{\square}$	G $8.276=8\frac{\square}{1000}$	K $0.560=\frac{\square}{100}$	O $3.004=3\frac{4}{\square}$
D $6.50=6\frac{\square}{100}$	H $3.495=3\frac{495}{\square}$	L $0.078=\frac{78}{\square}$	P $14.07=14\frac{7}{\square}$

2. Find the correct decimal.

A $\frac{13}{100}=\square.\square\square$	D $\frac{17}{1000}=\square.\square\square\square$	G $1\frac{5}{10}=\square.\square$
B $\frac{63}{100}=\square.\square\square$	E $6\frac{25}{100}=\square.\square\square$	H $\frac{15}{10}=\square.\square$
C $\frac{572}{1000}=\square.\square\square\square$	F $8\frac{56}{100}=\square.\square\square$	I $\frac{125}{100}=\square.\square\square$

3. Copy and complete each exercise.

A $\frac{1}{2}=\frac{\square}{10}=\square.\square$	E $\frac{7}{8}=\frac{\square}{1000}=\square.\square\square\square$	H $\frac{1}{4}=\frac{\square}{100}=\square.\square\square$
B $\frac{3}{25}=\frac{\square}{100}=\square.\square\square$	F $\frac{4}{5}=\frac{\square}{10}=\square.\square$	I $\frac{5}{8}=\frac{\square}{1000}=\square.\square\square\square$
C $\frac{8}{20}=\frac{\square}{100}=\square.\square\square$	G $\frac{17}{50}=\frac{\square}{100}=\square.\square\square$	J $\frac{3}{4}=\frac{\square}{100}=\square.\square\square$
D $\frac{1}{8}=\frac{\square}{1000}=\square.\square\square\square$		

4. Give the correct number for n.

A n thousandths $=1$ hundredth

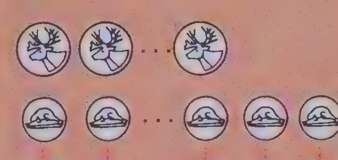
B n hundredths $=1$ tenth

C n tenths $=1$

5. Write a decimal for each number.

A $8\frac{7}{10}$	G $49\frac{76}{1000}$
B $53\frac{27}{100}$	H $\frac{7}{20}$
C $\frac{75}{100}$	I $\frac{9}{50}$
D $54\frac{357}{1000}$	J $\frac{6}{25}$
E $76\frac{7}{100}$	K $\frac{3}{5}$
F $873\frac{9}{1000}$	L $\frac{17}{25}$

think

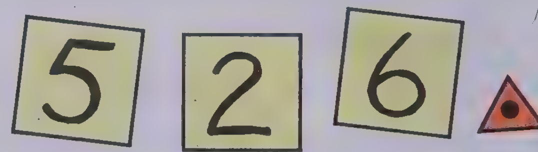


$30 + 30 \ \ldots \ 30 + 5 + 5 = 280$

In a collection of quarters and nickels, there are 2 more nickels than quarters. Use the diagram to tell how many of each coin if the collection is worth \$2.80.

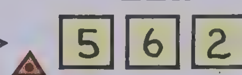
Investigating the Ideas

Make 4 slips of paper like these.



How many ways can you place all your slips on the spaces so that the inequality is true?	Record your results.

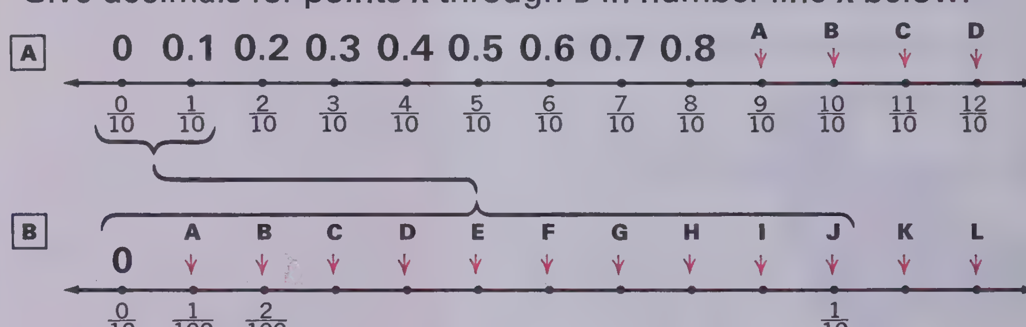
Example: 6.52 >



Discussing the Ideas

1. Explain how to tell if each inequality below is true. One of them is false.
   A $7.82 < 8.72$ B $7.82 > 7.62$ C $7.82 > 0.872$ D $7.82 > 27.8$

2. Give decimals for points A through D in number line A below.



3. Number line B shows part of number line A magnified 10 times.
   A Give fractions with denominator 100 that could be used to label the points at 0 and $\frac{1}{10}$.
   B Give a decimal for each point in number line B.

1. Give the correct sign (<, =, or >) for each ▒.

A 0.5 ▒ 0.3	F 28.9 ▒ 29.1	K 0.004 ▒ 0.04
B 18.7 ▒ 18.9	G 36.08 ▒ 36.80	L 0.680 ▒ 0.68
C 47.6 ▒ 46.6	H 0.3 ▒ 0.30	M 0.080 ▒ 0.08
D 0.28 ▒ 0.38	I 0.30 ▒ 0.300	N 1.010 ▒ 0.999
E 12.54 ▒ 12.45	J 0.4 ▒ 0.04	O 4.009 ▒ 4.090

2. Give the number that is **one tenth** more than:
A 0.3 B 0.9 C 1.5 D 0.38 E 0.004 F 5.95

3. Give the number that is **one hundredth** more than:
A 0.05 B 0.09 C 0.29 D 0.037 E 0.6 F 0.99 G 99.99

4. Give the number that is **one thousandth** more than:
A 0.003 B 0.009 C 0.569 D 0.6 E 0.38 F 0.099 G 0.999

5. Give the correct sign (<, =, >) for each ▒.

A 54.237 ▒ $54+\frac{3}{10}+\frac{3}{100}+\frac{7}{1000}$
B 3.058 ▒ $3+\frac{5}{10}+\frac{8}{1000}$
C 9.7042 ▒ $9+\frac{7}{10}+\frac{4}{1000}+\frac{2}{10{,}000}$
D 61.359 ▒ $60+\frac{13}{10}+\frac{5}{100}+\frac{9}{1000}$
E 5.070 ▒ $50+\frac{70}{1000}$
F 7.008 ▒ $7+\frac{8}{100}$
G 5.280 ▒ $5+\frac{2}{10}+\frac{8}{100}$
H 8.064 ▒ $8+\frac{6}{100}$
I 4.000 ▒ $3+\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}$
J 7.999 ▒ 8
K 6.204 ▒ $6+\frac{200}{1000}$
L 5.832 ▒ $5+\frac{800}{1000}+\frac{32}{1000}$

think

▒ ▒ ▒ ▒ . ▒ ▒ ▒

Each digit in this decimal is covered by a red screen. Give the decimal if
A it names the largest number possible when no two digits are alike.
B it names the smallest number possible when no digits are alike.
C it names a number that is closer to 10,000 than any other such decimal.

Investigating the Ideas

Find these sums and differences.

$\begin{array}{r} \frac{8}{10} \\ +\frac{7}{10} \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \frac{7}{10} \\ -\frac{5}{10} \\ \hline \end{array}$ $\quad$ $\begin{array}{r} \frac{18}{100} \\ +\frac{27}{100} \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 4\frac{6}{10} \\ -1\frac{7}{10} \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 2\frac{9}{10} \\ +5\frac{17}{100} \\ \hline \end{array}$

? Can you use the problems above to help you find the sums and differences below?

$\begin{array}{r} 0.8 \\ +0.7 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 0.7 \\ -0.5 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 0.18 \\ +0.27 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 4.6 \\ -1.7 \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 2.9 \\ +5.17 \\ \hline \end{array}$

Discussing the Ideas

Explain the steps in the two examples.

A

Adding thousandths	Adding hundredths	Adding tenths
$\begin{array}{r} \scriptstyle 1\;\; \\ 0.563 \\ +0.749 \\ \hline 2 \end{array}$	$\begin{array}{r} \scriptstyle 1\;1\;\; \\ 0.563 \\ +0.749 \\ \hline 12 \end{array}$	$\begin{array}{r} \scriptstyle 1\;1\;\;\; \\ 0.563 \\ +0.749 \\ \hline 1.312 \end{array}$
$\frac{3}{1000} + \frac{9}{1000} = \frac{12}{1000}$	$\frac{1}{100} + \frac{6}{100} + \frac{4}{100} = \frac{11}{100}$	$\frac{1}{10} + \frac{5}{10} + \frac{7}{10} = 1\frac{3}{10}$

B

Subtracting thousandths	Subtracting hundredths	Subtracting tenths
$\begin{array}{r} \scriptstyle 2\;14 \\ 0.7\cancel{3}\cancel{4} \\ -0.578 \\ \hline 6 \end{array}$	$\begin{array}{r} \scriptstyle 6\;12\;14 \\ 0.\cancel{7}\cancel{3}\cancel{4} \\ -0.578 \\ \hline 56 \end{array}$	$\begin{array}{r} \scriptstyle 6\;12\;14 \\ 0.\cancel{7}\cancel{3}\cancel{4} \\ -0.578 \\ \hline 0.156 \end{array}$
$\frac{14}{1000} - \frac{8}{1000} = \frac{6}{1000}$	$\frac{12}{100} - \frac{7}{100} = \frac{5}{100}$	$\frac{6}{10} - \frac{5}{10} = \frac{1}{10}$

1. Find the sums and differences.

A $\begin{array}{r} 0.65 \\ +0.86 \\ \hline \end{array}$	B $\begin{array}{r} 5.67 \\ +8.4 \\ \hline \end{array}$	C $\begin{array}{r} 0.68 \\ -0.23 \\ \hline \end{array}$	D $\begin{array}{r} 29.6 \\ +8.93 \\ \hline \end{array}$	E $\begin{array}{r} 29.37 \\ -5.62 \\ \hline \end{array}$
F $\begin{array}{r} 5.37 \\ -1.28 \\ \hline \end{array}$	G $\begin{array}{r} 34.96 \\ +59.45 \\ \hline \end{array}$	H $\begin{array}{r} 8.763 \\ -0.045 \\ \hline \end{array}$	I $\begin{array}{r} 17.078 \\ +6.974 \\ \hline \end{array}$	J $\begin{array}{r} 206.4 \\ -53.32 \\ \hline \end{array}$
K $\begin{array}{r} 0.067 \\ +0.836 \\ \hline \end{array}$	L $\begin{array}{r} 57.38 \\ +94.073 \\ \hline \end{array}$	M $\begin{array}{r} 102.35 \\ -41.17 \\ \hline \end{array}$	N $\begin{array}{r} 0.876 \\ -0.199 \\ \hline \end{array}$	O $\begin{array}{r} 700.3 \\ -267.5 \\ \hline \end{array}$
P $\begin{array}{r} 5.004 \\ -1.333 \\ \hline \end{array}$	Q $\begin{array}{r} 3.142 \\ +7.128 \\ \hline \end{array}$	R $\begin{array}{r} 80.637 \\ +109.85 \\ \hline \end{array}$	S $\begin{array}{r} 0.054 \\ -0.026 \\ \hline \end{array}$	T $\begin{array}{r} 55.62 \\ -7.99 \\ \hline \end{array}$

U $0.3 + 0.5 + 0.9 + 0.6 + 0.8$

V $8.6 + 7.4 + 9.3 + 6.9$

W $0.34 + 0.63 + 0.79 + 0.64$

X $0.521 + 0.467 + 0.816$

Y $4.37 + 8.56 + 7.38$

Z $5.6 + 3.48 + 29.6 + 0.387$

2. Find the missing amounts.
   A Since 9.67 + 5.38 = 15.05, we know that \$9.67 and \$5.38 is ▒.
   B Since 20.00 − 7.35 = 12.65, we know that \$20.00 less \$7.35 is ▒.

3. Solve each money problem.
   A Find the total cost of the items shown in this budget book.

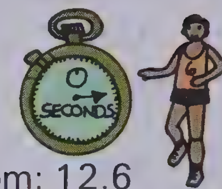
DATE	DESCRIPTION	AMOUNT	
1/17	Gas	16	13
1/17	Electricity	9	63
1/17	Water	4	17
1/20	Telephone	7	68
1/21	Other	2	60

   B Tom bought a pair of gym shoes for \$6.98. How much change did he get back from a \$20 bill?
   C Jan received a cheque for \$10.55. She spent \$4.98 for a record and \$2.75 for a book. How much did she have left?
   D Mr. Smith bought two adult tickets and one child's ticket for a play. He paid \$3.25. Mr. Jones bought two adult tickets and two children's tickets. He paid \$4.00. What is the price of an adult ticket?

More practice, page A-17, Set 34

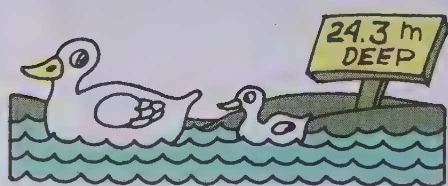
1. April: 4.7 cm of rain. May: 3.9 cm of rain. How much rain ?

2. Travel. Monday: 238.4 kilometres
Tuesday: 467.5 kilometres. Wednesday:
537.8 kilometres. How far ?



3. 100-metre dash. Tom: 12.6 seconds. Joe: 10.9 seconds. How much longer did it take Tom ?

4. 18.7 km per litre.
2 litres. How many km ?



5. Average depth of river: 15.9 m.
Average depth of lake: 24.3 m.
How much deeper is the lake ?

6. Pour in 25.35 cubic centimetres. Then pour in 37.68 cubic centimetres more. How much liquid ?

7. Expenses. Monday: \$3.95. Tuesday: \$6.23. Wednesday: \$5.67.
Thursday: \$3.94. Friday: \$4.58. Total ?

8. Micrometer reading: 1.53 centimetres. Polishing removes 0.67 centimetres. What does the micrometer read now ?

9. Old speedway record:
256.867 km/h.
New record:
257.735 km/h.
How much greater is the new record ?

10. Normal body temperature: 37 °C.
High fever: 2.7°C higher.
How many degrees ?

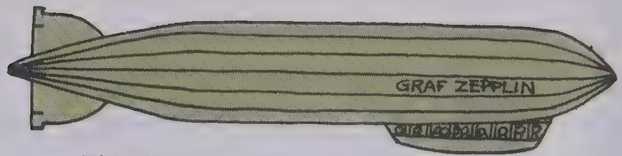
11. Magic square. Row, column, and diagonal sums the same. Give the missing numbers.

A	0.9	1.0
0.7	1.1	**B**
1.2	**C**	0.8

12. Highest recorded temperature: 58.2 °C. Lowest recorded temperature: 118.3°C **below zero.** Find the change in degrees from one temperature to the other.

DENSITY

A cubic metre of balsa wood weighs much less than a cubic metre of copper. We say that the density of copper is greater than the density of balsa wood.



This famous airship rose above the ground because it contained hydrogen gas. We say the density of hydrogen gas is less than the density of air.

Since oil rises to the top when mixed with water, it must be lighter. We say the density of oil is less than the density of water.

1. How much more does a litre of sea water weigh than a litre of pure water ?
2. Add to find the weight of 5 litres of gasoline.
3. Which weighs more, 2 millilitres of acid or 3 millilitres of oil ? How much more ?
4. Find the difference in the weights of a litre of mercury and a litre of kerosene.
5. Which is heavier, a cubic metre of cork or a cubic metre of glass ? How much heavier ?
6. How much more or less does a cubic centimetre of gold weigh than a cubic centimetre of iron and a cubic centimetre of lead ?

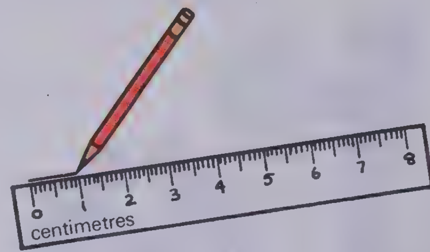
★ 7. Estimate:
   A The number of litres of water in a cubic metre of water.
   B The number of litres of water needed to weigh about the same as a cubic decimetre of gold.

Liquid	Grams per litre
Water	1000.0
Gasoline	681.7
Kerosene	820.7
Mercury	13 546.2
Oil	1082.41
Sulfuric acid	1821.13
Sea water	1025.6

Material	Grams per cubic centimetre
Water	1.0
Balsa wood	0.112
Copper	8.92
Cork	0.225
Glass	2.542
Gold	19.31
Iron	7.86
Lead	11.288

Investigating the Ideas

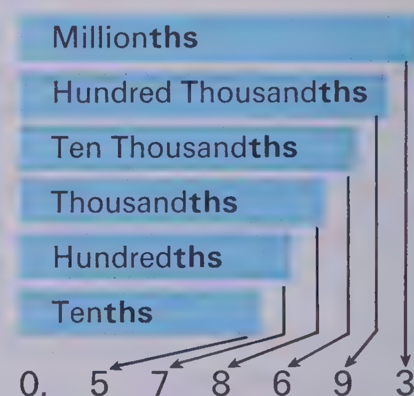
> **?** Can you draw line segments that are very close to
> 4.52 centimetres long ?
> 6.017 centimetres long ?
> 5.249 centimetres long ?



Discussing the Ideas

1. Explain how you thought about each decimal before you drew your segment.

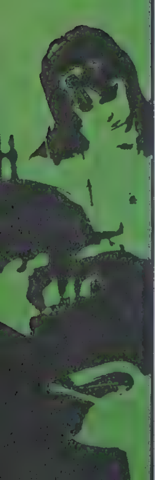
2. The chart shows place-value names through millionths. Give each of these decimals rounded to the nearest hundredth.



   A 0.5372
   Answer: 0.54
   B 0.6211
   C 0.7849
   D 0.6954
   E 0.2886
   F 0.2549
   G 0.2550
   H 0.6247
   I 0.62476
   J 3.5914
   K 7.8265
   L 17.96501
   M 38.0095
   N 99.9999

3. The rules you learned for rounding whole numbers are also used for rounding decimals. Give each decimal above rounded to the nearest:
   A tenth
   B thousandth

4. Study each example. Then read the decimals in exercise 2.
   A For 0.5786, we read "five thousand seven hundred eighty-six ten thousandths."
   B For 0.57869, we read "fifty-seven thousand eight hundred sixty-nine hundred thousandths."
   C For 0.578693, we read "five hundred seventy-eight thousand, six hundred ninety-three millionths."



1. Give the place value of each red digit.

A 3.57
Answer: $\frac{7}{100}$

B 8.7642

C 3.5876

D 8.76953

E 0.00058

F 3.7045

G 0.00617

H 4.82378

I 2.700

J 3.07610

K 8.00026

L 3.479

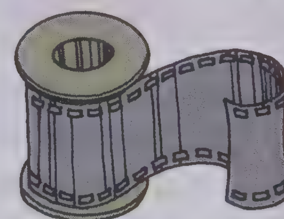
M 2.3604

N 0.0083

2. Give the missing word for each of these interesting decimals.

A Thin metallic film: 0.00002 cm
We read: two ___ ? ___ cm

B Human hair: about 0.005 cm
We read: five ___ ? ___ cm



3. Give the missing numbers.

A 0.28 rounded to the nearest tenth is ____.

B 0.745 rounded to the nearest hundredth is ____.

C 0.57362 rounded to the nearest hundredth is ____.

D 39.654 rounded to the nearest whole number is ____.

4. Give each decimal:

A rounded to the nearest hundredth

B rounded to the nearest tenth

C rounded to the nearest whole number

2.540005
30.48006
91.44018
0.3937

5. Balloons that rise into the air contain a gas that is lighter than air. Hydrogen, the lightest gas, weighs 0.089882 grams per litre.
Give the weight of hydrogen gas:

A rounded to the nearest hundred thousandth

B rounded to the nearest ten thousandth

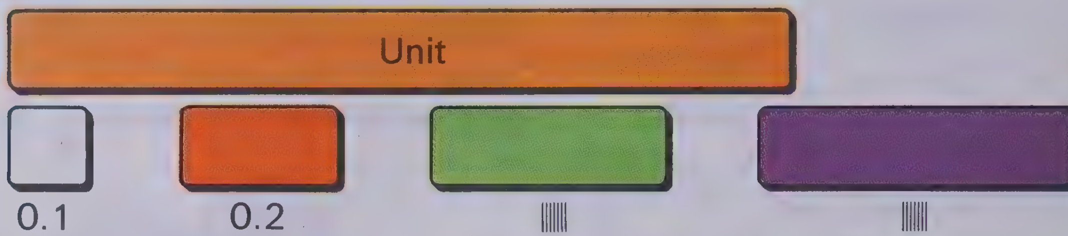
C rounded to the nearest thousandth

D rounded to the nearest hundredth

More practice, page A-18, Set 35

Investigating the Ideas

Suppose the orange strip is the unit.



? Can you find the length of a train of 10 strips for each of the four colors?

Discussing the Ideas

1. You can find these products by thinking about fractions. Give your answer by using a whole number or the simplest possible decimal.

A	B	C	D
$10 \times 0.1 = a$	$100 \times 0.01 = a$	$10 \times 0.01 = a$	$100 \times 0.1 = a$
$10 \times 0.2 = b$	$100 \times 0.02 = b$	$10 \times 0.02 = b$	$100 \times 0.2 = b$
$10 \times 0.3 = c$	$100 \times 0.03 = c$	$10 \times 0.04 = c$	$100 \times 0.6 = c$
$10 \times 0.8 = d$	$100 \times 0.07 = d$	$10 \times 0.09 = d$	$100 \times 0.9 = d$

2. The distributive principle is useful in finding the product of 10 or 100 times a fractional number named by a decimal. Explain the examples and give the products.

A $10 \times 4.6 = (10 \times 4) + (10 \times 0.6)$

Since $10 \times 4 = 40$, $10 \times 0.6 = 6$ we know that $10 \times 4.6 = n$.

B $100 \times 3.07 = (100 \times 3) + (100 \times 0.07)$

Since $100 \times 3 = 300$, $100 \times 0.07 = 7$ we know that $100 \times 3.07 = n$

1. Find the products.

A 10×0.4	E 100×0.05	I 0.2×10	M 10×0.06
B 10×0.7	F 100×0.08	J 0.07×100	N 100×0.09
C 10×0.04	G 100×0.3	K 0.05×10	O 10×0.5
D 10×0.08	H 100×0.6	L 0.5×100	P 100×0.03

2. Copy each exercise and give the number for each letter.

A $10 \times 5 = a$
$10 \times 0.3 = b \rightarrow 10 \times 5.3 = c$

B $10 \times 0.7 = a$
$10 \times 0.04 = b \rightarrow 10 \times 0.74 = c$

C $100 \times 6 = a$
$100 \times 0.2 = b \rightarrow 100 \times 6.2 = c$

D $100 \times 0.5 = a$
$100 \times 0.04 = b \rightarrow 100 \times 0.54 = c$

3. Find the products.

A 10×6.4	C 100×9.3	E 10×8.56	G 10×51.3
B 10×0.87	D 100×0.67	F 100×9.73	H 100×26.4

4. Complete each sentence correctly.

A To write the product of 10 times a number named by a decimal, write the decimal point __ ? __ place farther to the right.

B To write the product of 100 times a number named by a decimal, write the decimal point __ ? __ places to the right.

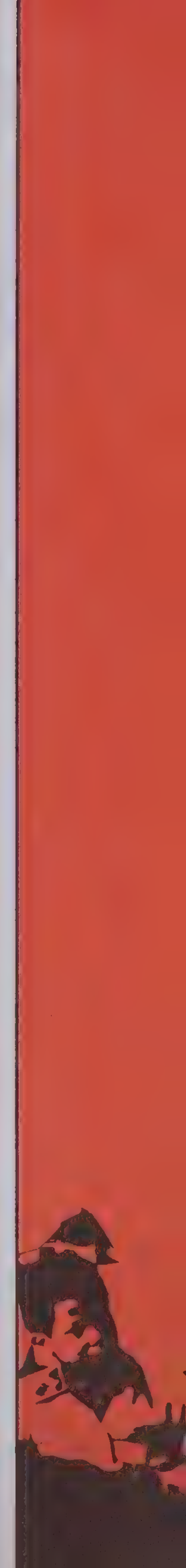
5. Find the products.

A 10×8.43	K 100×0.08
B 10×29.3	L 10×97.2
C 100×1.97	M 100×9.72
D 100×5.8	N 100×97.2
E 100×53.7	O 100×0.56
F 10×0.36	P 100×0.056
G 100×0.862	Q 10×0.056
H 100×34.23	★R 1000×0.001
I 10×27.64	★S 1000×0.01
J 100×54.8	★T 1000×0.1

think

Suppose you have these pails.

There are no markings on either pail. How can you use these pails to get 4 litres of water in the largest pail ?



More practice, page A-18, Set 36

Discussing the Ideas

1. Study each example. Then give the product *P*.

A $\frac{3}{10}$ $\frac{4}{10}$

$0.3 \times 0.4 = P$

To find this product, think:

1 $3 \times 4 = 12$

2 tenths × tenths = hundredths

B $\frac{4}{10}$ $\frac{6}{100}$

$0.4 \times 0.06 = P$

To find this product, think:

1 $4 \times 6 = 24$

2 tenths × hundredths = thousandths

C $\frac{5}{100}$ $\frac{13}{100}$

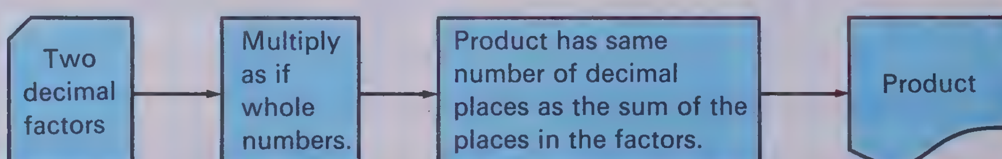
$0.05 \times 0.13 = P$

To find this product, think:

1 $5 \times 13 = 65$

2 hundredths × hundredths = ten thousandths

2. Study the flow chart below.



$$\begin{array}{r} 5.6 \\ \times 0.3 \\ \hline \end{array} \qquad \begin{array}{r} 56 \\ \times 3 \\ \hline 168 \end{array} \qquad \begin{array}{rl} 5.6 & \leftarrow \text{1-place decimal} \\ \times 0.3 & \leftarrow \text{1-place decimal} \\ \hline 1\ 68 & \leftarrow \text{2-place decimal} \end{array} \qquad \begin{array}{r} 5.6 \\ \times 0.3 \\ \hline 1.68 \end{array}$$

A Use the flow chart to help you find the product 7.2×0.6

B Explain how you would place the decimal in the product for 0.72×0.6.

C How many decimal places are there in the product 0.72×0.06?

3. How would you complete each sentence?

A When each factor is named by a 1-place decimal, the product is named by a __ ? __ -place decimal.

B When each factor is named by a 2-place decimal, the product is named by a __ ? __ -place decimal.

C To find the number of decimal places in the product, we __ ? __ the number of decimal places in each factor.

1. Do not compute the product. Simply tell whether the product will be written as tenths, hundredths, thousandths, or ten thousandths.

A 0.2×0.4
Answer: hundredths
B 0.5×0.3
C 4×0.08
D 0.23×0.9
E 0.12×0.25
F 4×0.2
G 0.07×0.38
H 0.49×0.87
I 0.26×0.3
J 0.4×0.04
K 0.7×0.39

2. Find the products.

A 0.5×0.7
B 0.9×0.6
C 0.8×0.5
D 0.3×0.52
E 0.2×3
F 0.49×0.2
G 0.12×0.3
H 0.5×0.24
I 0.68×0.7
J 0.9×0.8
K 0.59×0.67
L 0.41×0.09
M 0.03×0.12
N 0.09×0.07
O 0.8×0.06
P 0.57×0.40

3. Find the products.

A $\begin{array}{r} 3.7 \\ \times 0.06 \\ \hline \end{array}$
C $\begin{array}{r} 24.3 \\ \times 9 \\ \hline \end{array}$
E $\begin{array}{r} 6.32 \\ \times 0.8 \\ \hline \end{array}$
G $\begin{array}{r} 2.64 \\ \times 0.057 \\ \hline \end{array}$
I $\begin{array}{r} 6.308 \\ \times 9 \\ \hline \end{array}$

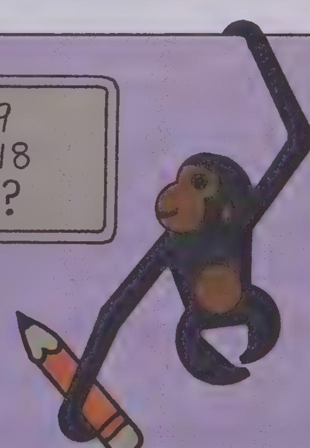
B $\begin{array}{r} 5.28 \\ \times 0.31 \\ \hline \end{array}$
D $\begin{array}{r} 61.5 \\ \times 4.3 \\ \hline \end{array}$
F $\begin{array}{r} 70.2 \\ \times 0.08 \\ \hline \end{array}$
H $\begin{array}{r} 93.04 \\ \times 0.033 \\ \hline \end{array}$
J $\begin{array}{r} 6.275 \\ \times 2.6 \\ \hline \end{array}$

4. Find the products.

A 83.6×0.5
B 97.1×3.14
C 72.1×0.004
D 0.007×5.98
E 0.023×0.24
F 53.4×2.6
G 6.38×24.5
H 9.76×8.06
I 2.03×4.04

think

$a + b = 0.9$
$a \times b = 0.18$
$a = ?\quad b = ?$



The sum of two decimals is 0.9.
The product of the same two decimals is 0.18.
Can you find both decimals?

More practice, page A-19, Set 37

1. Earth travels around the sun at 29.77 km/s. Mars travels around the sun at 0.81 times Earth's speed. How fast?

2. Planets. The closer to the sun, the faster they travel around it. Pluto's orbital speed: 4.8 km/s. Venus: 7.25 times as fast. How fast?

3. Earth's diameter: 12 755 km. Diameter of the moon: 0.27 times that of Earth. What is the diameter of the moon?

4.

Planet	Diameter
Earth	12 755 kilometres.
Mercury	0.38 × Earth's diameter
Venus	0.91 × Earth's diameter
Mars	0.52 × Earth's diameter
Jupiter	10.97 × Earth's diameter
Saturn	9.03 × Earth's diameter
Uranus	3.73 × Earth's diameter
Neptune	3.38 × Earth's diameter
Pluto	0.45 × Earth's diameter

A Find the diameter of each planet.

B Round each diameter to the nearest whole number.

5. Day on Mars: 24.6 Earth hours. 10 days on Mars. How many Earth hours?

6. Year on Mars: 1.9 Earth years. 8.3 years on Mars. How many Earth years?

7. Satellite. Takes 95.57 minutes to orbit Earth.
   A How many minutes for 9.5 orbits?
   B How many minutes for 100 orbits?
   C About how many hours for 100 orbits?
   D About how many days for 100 orbits?

8. Rocket. To leave Mercury, must go 4.2 km/s. To leave Earth, must go 1.28 km/s less than 3 times that for Mercury. How fast?

think

The weight of an object on Mars is $\frac{2}{5}$ of its weight on Earth. Give the total Earth weight of these objects. (The Mars weights are given.)

Girl: 13.6 kg Poodle: 4.6 kg
Purse: 0.28 kg Package: 3.75 kg

MONEY AROUND THE WORLD

Country	Monetary unit	Value in Canadian dollars
U.S.A.	Dollar	1.003
Great Britain	Pound	2.50
India	Rupee	0.13
Japan	Yen	0.0038
Mexico	Peso	0.08
Switzerland	Franc	0.31

1. One Mexican peso is worth 0.08 of a dollar or \$0.08. Two pesos are worth 2×0.08, or \$0.16. Find the value in dollars of:
   A 3 pesos B 4 pesos C 5 pesos D 7 pesos E 10 pesos
2. One Swiss franc is worth 0.31 of a dollar. Find the value in dollars of:
   A 2 francs B 3 francs C 5 francs D 8 francs
   E 10 francs F 100 francs G 1000 francs
3. Suppose you bought these articles in Switzerland: watch, 128 francs; music box, 48 francs; Swiss chocolate, 16 francs. What would be the total cost in dollars ?
4. In Canadian dollars give the value of:
   A 2 U.S. dollars B 3 U.S. dollars
   C 10 U.S. dollars D 100 U.S. dollars
5. How many Canadian dollars can you get for 378 Mexican pesos ?
6. Suppose you had 5000 yen. How many Canadian dollars would they be worth ?

★ 7. If you had 12 Mexican pesos, about how many Swiss francs would they be worth ?

★ 8. Which will buy most, 1000 yen, 1 pound, or 10 rupees ?

*These money values are subject to change because of political and economic factors.

Can decimals be divided by whole numbers?

Investigating the Ideas

Use multiplication to check this quotient.

$$\begin{array}{r} 243 \\ 6\overline{)1458} \end{array}$$

$$\begin{array}{r} 24.3 \\ 6\overline{)145.8} \end{array} \qquad \begin{array}{r} 2.43 \\ 6\overline{)145.8} \end{array}$$

$$\begin{array}{r} 2.43 \\ 6\overline{)14.58} \end{array} \qquad \begin{array}{r} 24.3 \\ 6\overline{)14.58} \end{array}$$

$$\begin{array}{r} 0.243 \\ 6\overline{)14.58} \end{array} \qquad \begin{array}{r} 0.243 \\ 6\overline{)1.458} \end{array}$$

? Can you use what you know about multiplying decimals to find which quotients on the paper are correct?

Discussing the Ideas

1. How could you use estimation to help you decide which quotients above are correct?

2. Can you give a shortcut for placing the decimal point in the quotient?

3. Explain each example.

A

Dividing **ones**	Dividing **tenths**	Dividing **hundredths**
$\begin{array}{r} 7 \\ 8\overline{)58.{}^{2}48} \end{array}$	$\begin{array}{r} 7.3 \\ 8\overline{)58.{}^{2}48} \end{array}$	$\begin{array}{r} 7.31 \\ 8\overline{)58.{}^{2}48} \end{array}$

B

Dividing **hundredths**	Dividing **thousandths**	Dividing **ten thousandths**
$\begin{array}{r} 0.02 \\ 34\overline{)0.9214} \\ 68 \\ \hline 24 \end{array}$	$\begin{array}{r} 0.027 \\ 34\overline{)0.9214} \\ 68 \\ \hline 241 \\ 238 \\ \hline 3 \end{array}$	$\begin{array}{r} 0.0271 \\ 34\overline{)0.9214} \\ 68 \\ \hline 241 \\ 238 \\ \hline 34 \\ 34 \\ \hline \end{array}$

1. Estimating products will help you check your quotients when dividing decimals. Give the missing numbers.

To estimate this product: →	Round to the nearest whole number. →	Find or estimate the whole-number product.
3.2×9	3×9	27
4.76×58	5×58	5×60, or 300
9.8×37	A $\times 37$	B
28.6×8	C $\times 8$	D
3.954×62	E $\times 62$	F
53.51×4	G $\times 4$	H

2. Estimate each product. Then find the product and compare it with your estimate.

A 35.6×8 C 0.72×47 E 0.059×8 G 0.0628×34
B 7.85×62 D 0.835×6 F 0.043×67 H 0.742×5

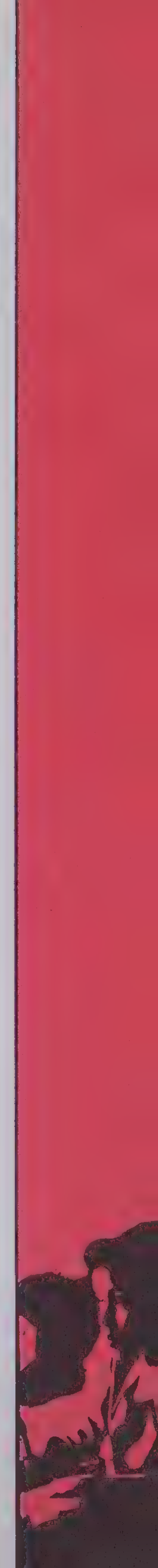
3. The correct digits are given for the quotient. $\begin{array}{r} 8\,2 \\ 6\overline{)49.2} \end{array}$
   A Estimate the products 82×6, 8.2×6, and 0.82×6.
   B Give the correct quotient for $49.2 \div 6$.

4. The correct digits, other than zero, are given for each quotient. Find the correct quotient.

A $\begin{array}{r} 43 \\ 8\overline{)34.4} \end{array}$ B $\begin{array}{r} 34 \\ 28\overline{)9.52} \end{array}$ C $\begin{array}{r} 437 \\ 6\overline{)2.622} \end{array}$ D $\begin{array}{r} 28 \\ 76\overline{)2.128} \end{array}$ E $\begin{array}{r} 523 \\ 9\overline{)47.07} \end{array}$

5. Find the quotients. Check your work.

A $4\overline{)34.0}$ B $43\overline{)30.1}$ C $5\overline{)1.285}$ D $67\overline{)361.8}$
E $8\overline{)198.08}$ F $20\overline{)174.0}$ G $50\overline{)3.350}$ H $79\overline{)2.686}$



More practice, page A-19, Set 38

Discussing the Ideas

1. Explain how the diagram shows that the quotient for $0.4\overline{)3.2}$ is the same as the quotient for $4\overline{)32}$. Find this quotient.

$$0.4\overline{)3.2} \rightarrow \boxed{\frac{3.2}{0.4} = \frac{3.2 \times 10}{0.4 \times 10} = \frac{32}{4}} \rightarrow 4\overline{)32}$$

2. Explain how the diagram shows that you can find the quotient for $0.21\overline{)6.741}$ by finding the quotient for $21\overline{)674.1}$. Find the quotient.

$$0.21\overline{)6.741} \rightarrow \boxed{\frac{6.741}{0.21} = \frac{6.741 \times 100}{0.21 \times 100} = \frac{674.1}{21}} \rightarrow 21\overline{)674.1}$$

3. Explain how to find the missing number so that the two problems each have the same quotient.

   A $0.4\overline{)2.4}$ $\quad 4\overline{)\square}$

   B $0.5\overline{)0.35}$ $\quad 5\overline{)\square}$

   C $0.06\overline{)3.42}$ $\quad 6\overline{)\square}$

   D $0.19\overline{)0.874}$ $\quad 19\overline{)\square}$

   E $1.1\overline{)693}$ $\quad 11\overline{)\square}$

   F $0.08\overline{)58.4}$ $\quad 8\overline{)\square}$

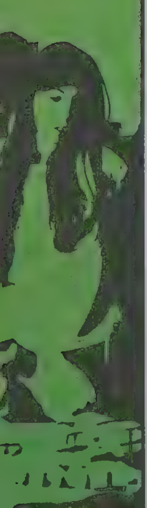
4. Explain steps 1 and 2. Then copy step 3 and complete the dividing.

A

1	2	3
Multiply the divisor by 100. Multiply the dividend by 100.	Think about the problem: $21\overline{)674.1}$	Complete the dividing.
$0.21_{\wedge}\overline{)6.74_{\wedge}1}$	$0.21_{\wedge}\overline{)6.74_{\wedge}1}$	$\begin{array}{r} 32.1 \\ 0.21_{\wedge}\overline{)6.74_{\wedge}1} \end{array}$

B

1	2	3
10 × divisor 10 × dividend	Think about the problem: $83\overline{)5.81}$	Complete the dividing.
$8.3\overline{)0.5\ 81}$	$8.3_{\wedge}\overline{)0.5_{\wedge}81}$	$\begin{array}{r} 0.07 \\ 8.3_{\wedge}\overline{)0.5_{\wedge}81} \\ \underline{5\ 81} \end{array}$



1. Copy the second part of each exercise. Give the missing number so that both division problems have the same quotient.

A $0.5\overline{)15.5}$ $\quad 5\overline{)\blacksquare}$

B $0.8\overline{)3.28}$ $\quad 8\overline{)\blacksquare}$

C $6.3\overline{)504}$ $\quad 63\overline{)\blacksquare}$

D $0.23\overline{)92.23}$ $\quad 23\overline{)\blacksquare}$

E $0.72\overline{)3.6}$ $\quad 72\overline{)\blacksquare}$

F $0.07\overline{)0.224}$ $\quad 7\overline{)\blacksquare}$

2. Find the quotients.

A $\begin{array}{r}\blacksquare\\ 0.03\overline{)0.12}\end{array}$ B $\begin{array}{r}\blacksquare\\ 0.3\overline{)1.2}\end{array}$ C $\begin{array}{r}\blacksquare\\ 3\overline{)12}\end{array}$ D $\begin{array}{r}\blacksquare\\ 30\overline{)120}\end{array}$

E $6\overline{)30}$ F $0.6\overline{)3}$ G $60\overline{)300}$ H $0.06\overline{)0.3}$

3. Each quotient is given without a decimal point.
Copy the problem and put in the decimal point correctly.

A $\begin{array}{r}35\\ 0.7\overline{)245}\end{array}$ B $\begin{array}{r}312\\ 0.04\overline{)0.1248}\end{array}$ C $\begin{array}{r}36\\ 1.2\overline{)4.32}\end{array}$ D $\begin{array}{r}231\\ 0.62\overline{)1.4322}\end{array}$

4. Find the quotients.
Check your work.

A $0.4\overline{)29.6}$ B $0.08\overline{)30.56}$

C $0.7\overline{)58.10}$ D $4.1\overline{)36.9}$

E $0.03\overline{)8.562}$ F $5.8\overline{)0.406}$

G $0.09\overline{)47.7}$ H $0.67\overline{)5.36}$

I $8.4\overline{)16.80}$ J $0.25\overline{)0.625}$

K $0.7\overline{)308.0}$ L $0.11\overline{)0.638}$

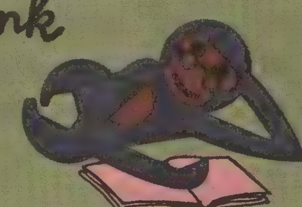
M $9.6\overline{)1824}$ N $0.71\overline{)326.6}$

O $6.5\overline{)351.0}$ P $0.008\overline{)3.568}$

Q $0.029\overline{)0.493}$ R $0.84\overline{)3.5868}$

S $3.45\overline{)24.15}$ T $61.8\overline{)5.562}$

think



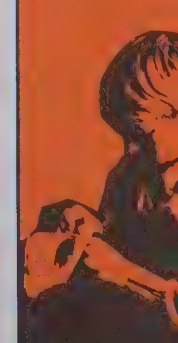
Study the number pattern.
Then solve the equations.

$$1^2 + 1 = 2^2 - 2$$
$$2^2 + 2 = 3^2 - 3$$
$$3^2 + 3 = 4^2 - 4$$
$$4^2 + 4 = 5^2 - 5$$

A $5^2 + 5 = 6^2 - n$

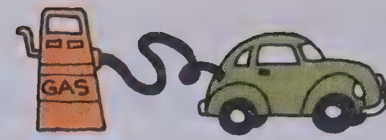
B $9^2 + a = 10^2 - 10$

C $19^2 + 19 = t^2 - 20$



More practice, page A-20, Set 39

Short Stories



1 Large wire: 0.58-cm diameter.
Cable: 3.48-cm diameter.
How many times as thick ?

2 Drove 16.1 km. Used 0.9 litres of gas. How many km/ℓ?

3 Drove 193.8 km in 3.4 hours. How many km/h ?

4 World's smallest motor: weighs about 0.000001 kg. About how many needed to weigh 1 kg ?

5 1 shoelace: 39.37 cm.
100 shoelaces.
How many centimetres ?

6 Divide $58.86 equally among 4 people. How much for each person ?

7 Gasoline record: Mon., 17.3 litres; Tues., 18.7 litres; Wed., 16.4 litres; Thurs., 19.5 litres; Fri., 17.6 litres.

A Give the average number of litres per day.

B Find the total cost at 22.9 cents a litre.

C Travelled 12.6 km/ℓ? How many kilometres in all ?

8 Litre of gasoline: about 660 g.
43.56 kg of gasoline.
How many litres ?

9 Litre of milk: about 1.56 times as heavy as a litre of gasoline. How heavy ?

10 Box of cereal: contains 315 grams. Costs 39¢.
How many cents per gram ?

11 Total cost: $38.34.

A Bought 9 items, each costing the same amount. How much ?

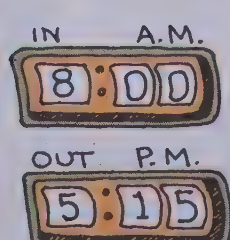
B How much change received from a 50-dollar bill ?

12 Square: each side 4.69 cm.

A What is the area ?

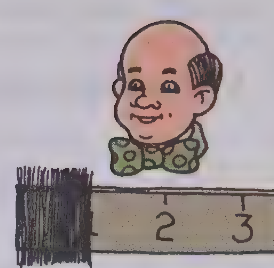
B What is the perimeter ?

★ **13** Salary: $2.32 per hour.
Time clock:

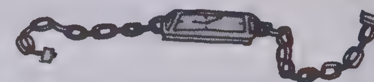


Deduct 1 hour for lunch and then find the exact pay for the day.

HOW SMALL IS SMALL



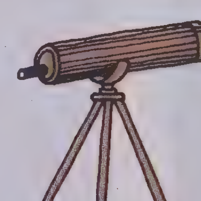
1. It is estimated that it would take about 200 "average" human hairs laid side by side to cover 1 cm. Assuming that this is true, what part of a centimetre is the width of a human hair ? Give this as a decimal.



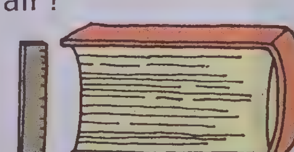
2. The diameter of the wire used to make coat hangers is about 0.036 cm. How many times the diameter of a human hair is this ?

3. One of the smallest watches is about 1 cm long and 0.5 cm wide and weighs 7.08 g.
   A How many times as wide is this watch as the thickness of a human hair ?
   B How many of these watches would it take to weigh 1 kilogram ?
   C One of the largest clocks in the world has a width of 18.28 m. How many times the width of the small watch is this ?

4. One of the smallest insects is the minute beetle, which is 0.02 cm long. One of the largest insects is the goliath beetle, which is 14.86 cm long. How many minute beetles laid end to end would be the same length as the goliath beetle ?



5. The human eye can see a bright light coming from an opening as small as 0.00036 cm in diameter. The diameter of this opening is about 6 times the diameter of the fine platinum wire used in a telescope.
   A What is the diameter of the platinum wire ?
   B How many times as thick as this wire is a human hair ?



6. One of the largest books is 30.6 cm thick (not including the cover). If each page is 0.006 cm thick, how many pages does the book have ?

Investigating the Ideas

Find the whole number for ***n***. ⟶ $\frac{1}{4} = \frac{n}{100}$

There is **no** whole number for ***m***. ⟶ $\frac{1}{3} = \frac{m}{100}$

? Can you find a number of hundredths that is **very close to** $\frac{1}{3}$?

Discussing the Ideas

1. The fact that $\frac{3}{8} = 3 \div 8$ suggests that you use the dividing process to find a decimal for $\frac{3}{8}$.

1 $8\overline{)3}$	2 $\begin{array}{r}0.3\\8\overline{)3.0}\end{array}$	3 $\begin{array}{r}0.3\ 7\\8\overline{)3.0^{6}0}\end{array}$	4 $\begin{array}{r}0.3\ 7\ 5\\8\overline{)3.0^{6}0^{4}0}\end{array}$

Now divide to find the decimal for $\frac{5}{8}$.

2. Study the example for finding a decimal for $\frac{2}{3}$.

1 Dividing tenths	2 Dividing hundredths	3 Dividing thousandths
$\begin{array}{r}0.6\ \text{R}2\\3\overline{)2.0}\end{array}$	$\begin{array}{r}0.6\ 6\ \text{R}2\\3\overline{)2.0^{2}0}\end{array}$	$\begin{array}{r}0.6\ 6\ 6\ \text{R}2\\3\overline{)2.0^{2}0^{2}0}\end{array}$

Explain these sentences about each step in the example above.

1 $\frac{2}{3}$ is **approximately** 0.7 (to the nearest tenth).
2 $\frac{2}{3}$ is approximately 0.67 (to the nearest hundredth).
3 $\frac{2}{3}$ is approximately 0.667 (to the nearest thousandth).

3. The decimal for $\frac{2}{3}$ is called a repeating decimal. We write $\frac{2}{3} = 0.66\ldots$ Explain how 0.66... is different from 0.66.

1. Fractions which have denominators that are factors of 10 or 100 are easily written as decimals. Study the example. Then find decimals for the fractions given.

   Examples: $\frac{4}{5} = \frac{4 \times 2}{5 \times 2} = \frac{8}{10} = 0.8$

   $\frac{11}{25} = \frac{11 \times 4}{25 \times 4} = \frac{44}{100} = 0.44$

   A $\frac{1}{5}$ **B** $\frac{3}{20}$ **C** $\frac{3}{4}$ **D** $\frac{3}{5}$ **E** $\frac{19}{25}$ **F** $\frac{43}{50}$

2. Use division to find decimals for these fractions.

   A $\frac{1}{8}$ **B** $\frac{7}{8}$ **C** $\frac{3}{16}$ **D** $\frac{7}{16}$ **E** $\frac{15}{32}$

3. The repeating decimal for $\frac{2}{3}$ repeats one digit at a time.

   $\frac{2}{3} = 0.66 \ldots$

   Other repeating decimals may repeat blocks of two or more digits. Study the example for $\frac{13}{33}$. Then find the repeating decimal for $\frac{3}{11}$.

$$
\begin{array}{r}
0.3939\ \ldots \\
33\overline{)13.0000\ \ldots} \\
\underline{9\,9} \\
3\,10 \\
\underline{2\,97} \\
130 \\
\underline{99} \\
310 \\
\underline{297} \\
13
\end{array}
$$

4. Find the repeating decimal for each of these.

   A $\frac{1}{3}$ **B** $\frac{1}{6}$ **C** $\frac{5}{6}$

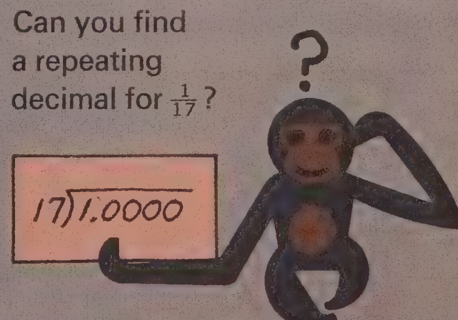
   D $\frac{1}{9}$ **E** $\frac{2}{9}$ **F** $\frac{5}{9}$

   G $\frac{5}{11}$ **H** $\frac{9}{11}$ **I** $\frac{10}{11}$

5. Some decimals repeat in "large" blocks. Find the repeating decimal for $\frac{1}{7}$.

think

Can you find a repeating decimal for $\frac{1}{17}$?



More practice, page A-20, Set 40

Discussing the Ideas

1. Study the examples below to see how you can write **mixed-decimal numerals** for $\frac{1}{3}$. Give the missing names in the chart.

Dividing process	0.3 3)1.0 9 1	0.33 3)1.00 9 10 9 1	0.333 3)1.000 9 10 9 10 9 1
Mixed-decimal numeral	$0.3\frac{1}{3}$	$0.33\frac{\square}{\square}$	$\square$

2. We read $0.3\frac{1}{3}$ as "3 and $\frac{1}{3}$ tenths." How would you read the other mixed-decimal numerals in exercise 1?

3. **A** When we use **A**, what is the mixed numeral in **tenths** for $\frac{5}{6}$?
   B When we use **B**, what is the mixed numeral in **hundredths** for $\frac{5}{6}$?

   A
   0.8
   6)5.0
   4 8
   2

   B
   0.83
   6)5.00
   4 8
   20
   18
   2

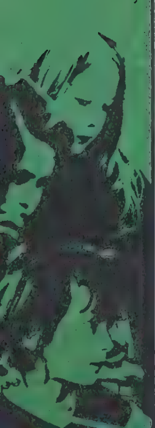
4. The examples show two ways to express the quotient for problems such as $0.3\overline{)0.136}$. Explain the examples.

A
0.453
0.3‸)0.1‸360
1 2
16
15
10
9
1

We say, "The quotient, rounded to the nearest hundredth, is 0.45."

B
0.45
0.3‸)0.1‸36
1 2
16
15
1

The quotient contains 45 hundredths. The remainder is 1 hundredth. We say, "The quotient is $45\frac{1}{3}$ hundredths $(0.45\frac{1}{3})$."



1. Use division to find a mixed-decimal numeral in **tenths** for each fraction.

   A $\frac{1}{6}$ B $\frac{2}{3}$ C $\frac{5}{9}$ D $\frac{3}{8}$ E $\frac{4}{7}$

2. Find a mixed-decimal numeral in **hundredths** for each fraction.

   A $\frac{5}{8}$ B $\frac{7}{12}$ C $\frac{9}{16}$ D $\frac{2}{7}$ E $\frac{4}{15}$

3. Find a mixed-decimal numeral in **thousandths** for each fraction.

   A $\frac{5}{6}$ B $\frac{5}{11}$ C $\frac{3}{16}$ D $\frac{2}{3}$ E $\frac{1}{99}$

4. Give each quotient rounded to the nearest hundredth.

   A $9\overline{)4}$ B $0.4\overline{)0.53}$ C $6\overline{)1.33}$ D $9\overline{)8.243}$

   E $1.3\overline{)0.967}$ F $3.5\overline{)0.56342}$ G $0.07\overline{)3.8764}$ H $0.63\overline{)6.4798}$

5. Give each quotient expressed as a mixed-decimal numeral in hundredths.

   A $3\overline{)2}$ B $0.4\overline{)5.426}$ C $0.6\overline{)0.874}$ D $0.9\overline{)0.657}$

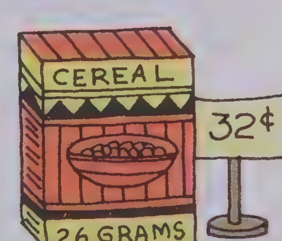
   E $3\overline{)4.63}$ F $0.04\overline{)0.5638}$ G $8\overline{)3.61}$ H $0.05\overline{)4.634}$

6. Solve the problems.

   A A 5-centimetre rod is cut into 11 pieces of the same size. How long is each piece (to the nearest hundredth)?

   B Give the cost of 1 gram of this cereal (to the nearest tenth of a cent).

   C A car is driven 96.4 km and uses 8.1 litres of gasoline. Give the average number of km for 1 litre of gasoline.

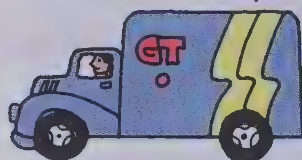
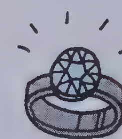


More practice, page A-21, Set 41

How are decimals used in scientific notation?

Investigating the Ideas

? Can you answer these questions?

How many times as heavy?	How many times as high?	How many times as much?
		
Truck: 9000 kg	Jet: 12 000 m	Diamond: \$1250
		
Toy truck: 9 kg	Bee: 1.2 m	Glass: \$1.25

Discussing the Ideas

1. Solve the equations.
   A $9000 = 9 \times n$ B $35\,000 = 3.5 \times r$ C $1250 = 1.25 \times a$

2. Give the exponent.
   A $9000 = 9 \times 10^n$ B $35\,000 = 3.5 \times 10^n$ C $1250 = 1.25 \times 10^n$

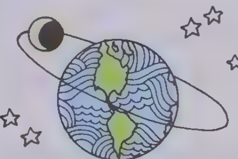
3. To name a number in scientific notation, we write it as the product of a number between 1 and 10 and a power of ten. Study the table and give the missing numbers.

Number		Number between 1 and 10		Power of ten
593	=	5.93	×	10^2
64	=	6.4	×	10
4627	=	4.627	×	______
895	=	______	×	10^2
1634.7	=	1.6347	×	______

1. Find the number for n in each row of the table.

	Number		Number between 1 and 10		Power of 10
A	472.3	=	n	×	10^2
B	46	=	n	×	10^1
C	383	=	3.83	×	n
D	5378	=	n	×	10^3
E	4769	=	4.769	×	n
F	n	=	5.9	×	10^3
G	n	=	6.49	×	10^2
H	65 000	=	6.5	×	n

2. Use scientific notation to represent each of these numbers.
   - A Speed of sound in cool air in km/h: **1231**
   - B 1971 school population, Northwest Territories: **10 006**
   - C Estimated age of Earth in millions of years: **4950**
   - D Height in metres of tallest redwood tree: **113**
   - E Orbital speed (km/h) of moon around Earth: **3792**
   - F Equatorial diameter of Earth in kilometres: **12 753.77**



3. Study the paper. Then use scientific notation to help you estimate these products.
   - A 5280×789
   - B 317.5×59.38
   - C 8926.52×314.16

To estimate the product
593.7 × 68.42:

1. Write in scientific notation.
   $5.937 \times 10^2 \times 6.842 \times 10^1$
2. Round to the nearest whole number.
   $6 \times 10^2 \times 7 \times 10^1$
3. Find this product.
   $42 \times 10^2 \times 10 =$ **42 000**

4. Express each of these numbers in scientific notation.
   - A Distance to sun: 149 600 000 kilometres
   - B Speed of light: 9 174 000 000 000 000 metres per year
   - C Sun's mass: 2 220 000 000 000 000 000 000 000 000 000 kg

Reviewing the Ideas

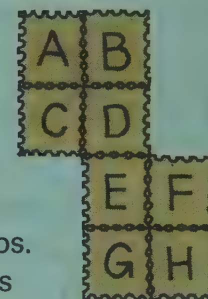
1. 2 837 465.156427

For the decimal above, give the number of:

A tens B tenths C hundreds
D hundredths E thousands
F thousandths G ten thousands
H ten thousandths
I hundred thousands
J hundred thousandths
K millions L millionths

think

Suppose you have 8 stamps left in this shape on a sheet. You are to tear off 3 connected stamps. How many choices do you have?



2. Give a decimal for each fractional number.

A $8\frac{9}{10}$ B $67\frac{5}{100}$ C $3\frac{76}{1000}$ D $\frac{32}{10}$ E $\frac{563}{100}$ F $\frac{3}{5}$ G $\frac{6}{25}$

3. Give the correct sign ($<$, $>$, $=$) for each ▒.

A 34.06 ▒ 34.60 B 0.070 ▒ 0.0700 C $\frac{1}{7}$ ▒ 0.142 D $\frac{1}{3}$ ▒ 0.3334

4. Find the sums and differences.

A $\begin{array}{r} 5.47 \\ +7.63 \\ \hline \end{array}$ B $\begin{array}{r} 27.34 \\ -9.27 \\ \hline \end{array}$ C $\begin{array}{r} 407.6 \\ -29.83 \\ \hline \end{array}$

D $9.7+0.35+0.867+17$
E $354.6 + 98.67 + 3.704$

5. Estimate each product. Then find the correct product.

A $\begin{array}{r} 4.5 \\ \times 6 \\ \hline \end{array}$ B $\begin{array}{r} 0.79 \\ \times 9 \\ \hline \end{array}$ C $\begin{array}{r} 0.063 \\ \times 32 \\ \hline \end{array}$ D $\begin{array}{r} 3.14 \\ \times 6.7 \\ \hline \end{array}$

E 0.057×8
F 5.6×7.3

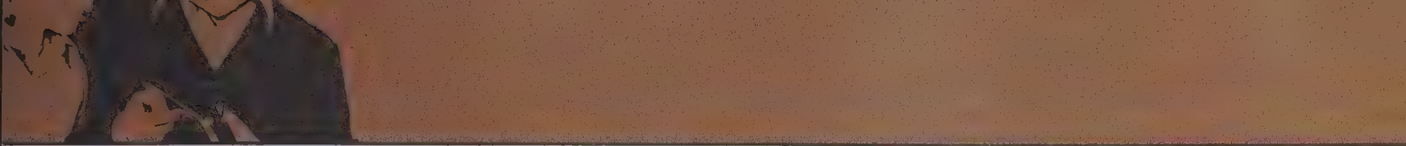
6. Find the quotients.

A $5\overline{)32.5}$ B $28\overline{)0.588}$ C $0.6\overline{)3246}$ D $5.3\overline{)371}$ E $8\overline{)7}$

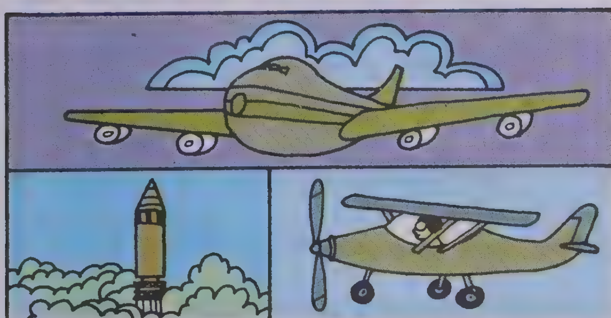
7. Represent $\frac{7}{9}$ as a repeating decimal.
Then give this decimal rounded to the nearest hundredth.

★ **8.** Use scientific notation to represent each number.

A 736 B 8459 C 37.5



SPEED AND SOUND



Aircraft	Speed
Small plane	Mach 0.21
Propeller airliner	Mach 0.52
Jet airliner	Mach 0.9
Jet fighter	Mach 1.9
Supersonic transport	Mach 2.2
Rocket	Mach 5.75

At sea level (0 °C temperature) sound travels at a speed of about 1194 kilometres per hour. At higher altitudes, where the air is thinner and cooler, sound travels more slowly. The speeds of aircraft are often given according to the speed of sound. A speed of Mach 1 is the speed of sound, Mach 0.5 is half the speed of sound, Mach 2 is twice the speed of sound, Mach 2.2 is 2.2 times the speed of sound, and so on. Mach is the name of the scientist who made some important discoveries about sound.

For the exercises below use 1078 km/h for the speed of sound at the cruising altitude of the aircraft.

1. Give the speed in kilometres per hour of each aircraft listed in the table.

2. To find the Mach number for a certain aircraft speed, you divide its speed by the speed of sound. Find the Mach number, to the nearest tenth, for these speeds.
   A 2156 km/h B 3234 km/h C 2575 km/h D 805 km/h

★ 3. The speed of sound at sea level is 1194 km/h. Find this speed of sound to the nearest tenth in:
   A kilometres per minute B kilometres per second C metres per second

★ 4. On a very warm day, sound might travel 362.2 metres per second.
   A How much more is this than the speed you found in exercise 3C?
   B If you blow a whistle, how far will the sound travel in 5.4 seconds?

1. Solve the equations.

A $97 + 0 = n$ C $97 \times 1 = n$ E $97 \div 97 = n$ G $0 + 0 = n$

B $97 \times 0 = n$ D $0 \div 97 = n$ F $0 \times 83 = n$ H $1 + 83 = n$

2. Solve the equations.

A $5 \times 2\frac{1}{2} = (5 \times 2) + (5 \times n)$ B $\frac{1}{2} \times 7\frac{1}{5} = (n \times 7) + (\frac{1}{2} \times \frac{1}{5})$

3. Find the products in the blue screens in exercise 2.

4. Find the sum and difference for each pair of numbers.

A $\frac{3}{4}, \frac{1}{6}$ B $\frac{2}{3}, \frac{1}{4}$ C $\frac{5}{4}, \frac{1}{10}$ D $\frac{5}{6}, \frac{1}{8}$ E $\frac{4}{9}, \frac{1}{6}$

5. Give each fraction in lowest terms.

A $\frac{6}{8}$ Answer: $\frac{3}{4}$ B $\frac{12}{16}$ C $\frac{18}{24}$ D $\frac{75}{100}$ E $\frac{45}{100}$ F $\frac{27}{300}$ G $\frac{24}{800}$

6. Draw an angle. Use ruler and compass to copy the angle.

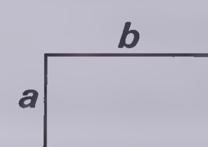
7. Find the missing numbers.

Function Rule

$(\frac{1}{3} \times n) + \frac{3}{4}$

n	$f(n)$
$\frac{1}{4}$	A
$\frac{1}{2}$	B
9	C
8	D
★ E	$\frac{3}{4}$

★ **8.** This table refers to rectangles. Give the number pair for each row.



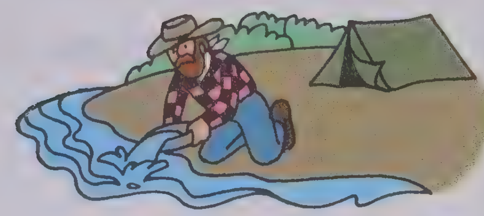
	Area		$\times$ / $+$		Half the perimeter
A	11	a	$\times$ $+$	b	12
B	27	a	$\times$ $+$	b	12
C	35	a	$\times$ $+$	b	12
D	36	a	$\times$ $+$	b	12
E	32	a	$\times$ $+$	b	12

You are invited to explore

ACTIVITY CARD 12
Page 361

GOLD

Trying to make gold



Trying to find gold

Because of its value, men have sought gold for thousands of years. It is valuable because it is rare, beautiful, and (to some extent) useful. There are at least 15 other elements that are less plentiful than gold, but gold has been accepted as a standard value for money. Some of the uses of gold, other than for money, are for decoration and dental work.

1. Gold was discovered in the Klondike in 1896. How many years ago was this ?
2. Gold is a soft metal. It can be hammered into sheets as thin as 0.0000102 cm. How many such sheets would make a pile 1 cm thick ?
3. One gram of gold can be stretched into 3.28 km of fine wire. How many kilometres of wire could be made from 0.5 kg of gold ?
4. Gold is 7.6 times as heavy as glass. A cubic centimetre of glass weighs 2.54 g. What is the weight of a cubic centimetre of gold ?
5. Gold is stored in standard-sized bars. Each bar is worth $14 000. The dimensions of a bar are 17.15 by 8.90 by 4.45 cm. Find the volume of a bar.
6. ★ One of the largest gold nuggets, the Welcome Nugget found in Australia, weighed 112.5 kg. About what was its volume ?
7. ★ All the gold mined in the world in the 469 years after the discovery of North America would form a cube with edges each about 14.9 m long.
   - A Between what two dates was the gold for this cube mined ?
   - B Estimate the volume of this cube.

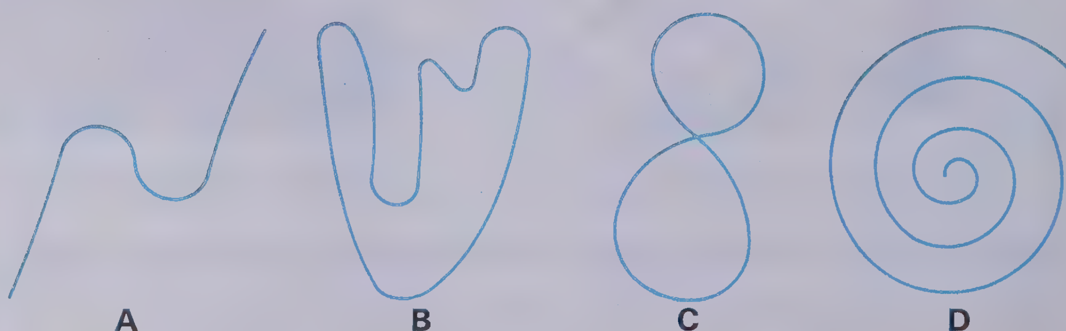
11

Geometry and Measurement II

Can you find approximate lengths of curves?

Investigating the Ideas

Guess the length of each curve.



? Can you find some way to measure each of the curves to check your guess?

Discussing the Ideas

1. A What unit did you use to measure the curves?
   B How accurate do you think your measurements are? How close were your lengths to those of your classmates?

2. Can you think of any other way to measure the lengths of the curves?

3. You can think of a **simple closed curve** as a loop of string that is on a flat surface and does not cross itself. **Polygons** are special simple closed curves that are made up entirely of segments.

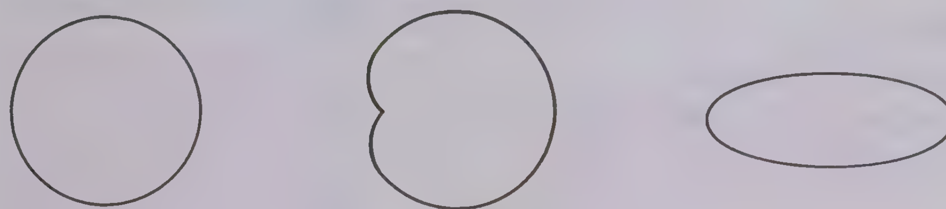
A simple closed curve

A polygon

   A Which curve in the Investigation is a simple closed curve?
   B Draw a simple closed curve on the chalkboard and estimate its length by looking. How can you get a closer estimate?

1. The curves named below have no segments in them. Find the length of each curve as accurately as you can.

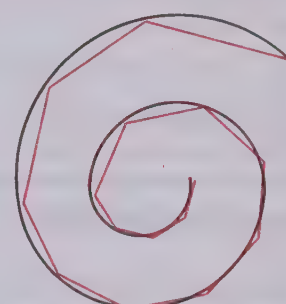
A B C



circle cardioid ellipse

2. A The approximate length of a curve can be found by measuring segments. Find the length of this red path to the nearest centimetre.

   B Is it longer or shorter than the spiral?



3. Draw some curves of your own and find their lengths.

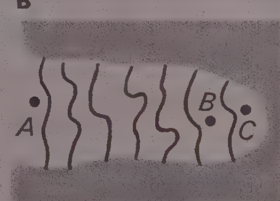
think

Only a small part of a simple closed curve is shown in each picture. Can you answer the questions?

A

If the bug is outside the curve and crosses it an odd number of times, will he end up outside or inside?

B



If point *A* is outside the curve, where is *B*? Where is *C*?

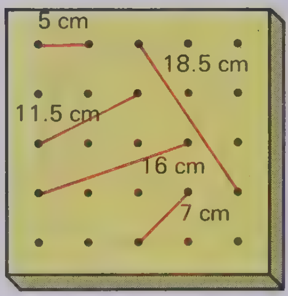
C

Draw a picture of how the polygon might look if
(1) *D* is inside.
(2) *D* is outside.

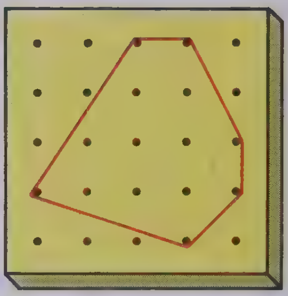
Investigating the Ideas

Picture A shows approximate lengths of certain segments on the geoboard.

A



B



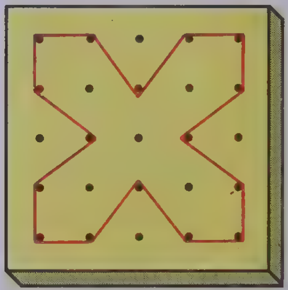
?

Can you find the distance around the polygon on geoboard B ?

Discussing the Ideas

1. The sum of the lengths of the sides of a polygon is called the **perimeter** of the polygon. Show each of these polygons on your geoboard or dot paper and explain how you would find its perimeter.

   A square B triangle C quadrilateral
   D pentagon E hexagon



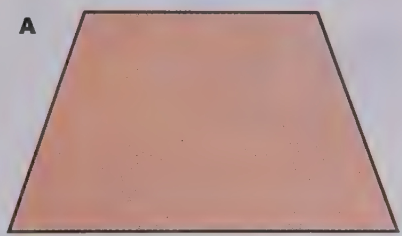
2. Here is a polygon with 16 sides.
   A What is its perimeter ?
   B Can you find a polygon with more sides and greater perimeter ?

3. Show how you could use your ruler to find the perimeter of a polygon.

1. Show the largest right triangle you can on the geoboard or dot paper and find its perimeter in centimetres.

2. Use your centimetre ruler to find each perimeter.

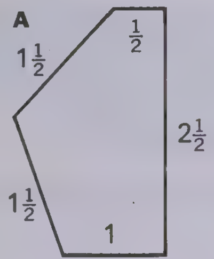
A



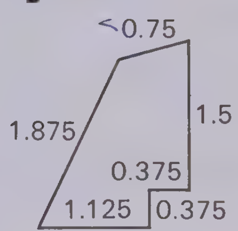
B

3. Find the perimeter of each figure.

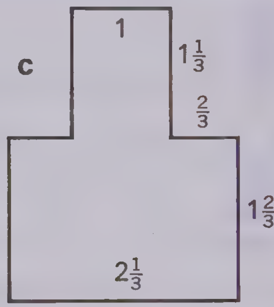
A



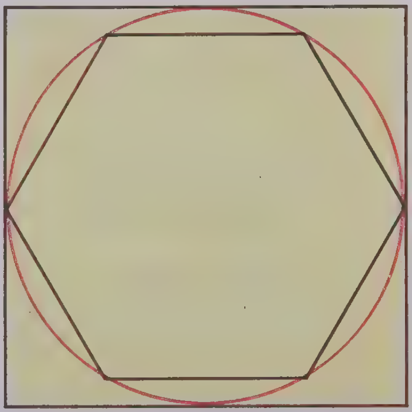
B



C



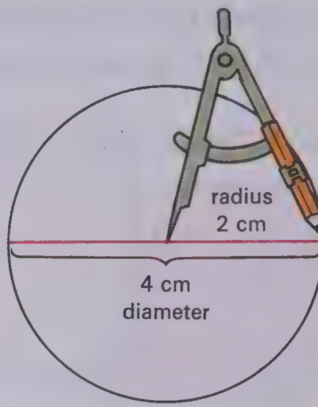
4. A A square has a perimeter of 36. What is the length of each side?
   B A rectangle has a perimeter of 36 and one side of 4. What is the length of each side?
   C A triangle has two sides that have lengths of 17. It has a perimeter of 50. What is the length of the third side?



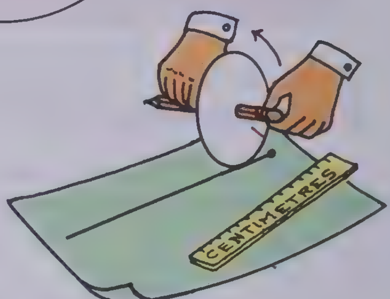
5. A What is the perimeter of the square in centimetres?
   B What is the perimeter of the hexagon?
   C Estimate the "perimeter" of the circle.

Investigating the Ideas

If you open your compass to a **radius** of 2 centimetres, you can draw a circle with a **diameter** 4 centimetres. Draw circles with diameters 3, 4, 5, 6, 7, 8, and 9 centimetres on cardboard and cut them out.



? Can you use the idea suggested by this picture to find the "distance around" each circle to the nearest one-tenth centimetre?



Discussing the Ideas

1. The "distance around" a circle is called the **circumference** of the circle. Copy the table below on the chalkboard and write in the circumference you found for each circle. For each circle, which whole number times the diameter will give a number closest to the circumference?

Special factor		Diameter (cm)		Circumference
	×	3	≈	___?___
	×	4	≈	___?___
	×	5	≈	___?___
	×	6	≈	___?___
	×	7	≈	___?___
	×	8	≈	___?___
	×	9	≈	___?___

2. A special factor times the diameter gives the circumference. Do you think this special factor is more or less than the whole number you found in exercise 1?

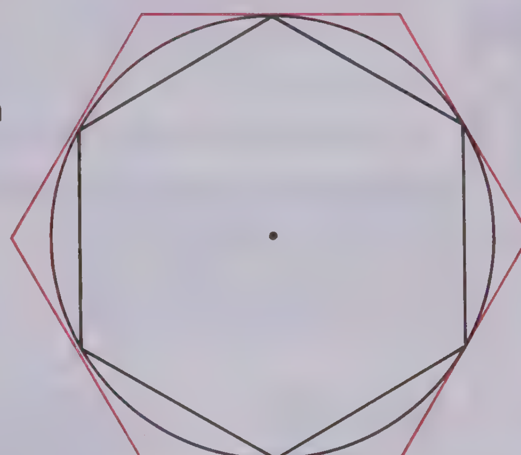
Using the Ideas

1. The experiments on page 262 suggest that the circumference of any circle is found by multiplying the diameter by a number slightly greater than 3. This number is the same for all circles. It is named by the Greek letter π (pronounced "pie").
We write:

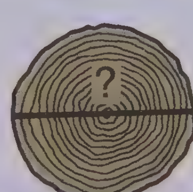
> Circumference = $\pi \times$ diameter,
> where $\pi = 3.14$ (to the nearest hundredth).

Find the circumference of a circle with diameter:
A 3 cm B 7 cm C 10 cm D 2 cm E 100 cm F 12 cm

2. A Is the circumference of the circle greater or less than the perimeter of the polygon shown in black? in red?
B Estimate the circumference of the circle by finding the average of the perimeters of the polygons. (Use centimetres.)
C Check your estimate by measuring to find the diameter (in centimetres) and multiplying to find the circumference.



★ 3. A rope 49.682 m long was needed to encircle one of the world's largest trees. What was the diameter of a cross section of the tree trunk?



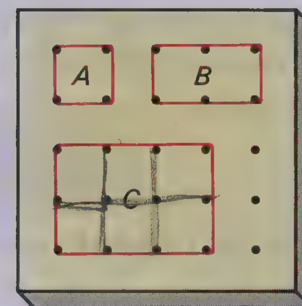
think

Use notebook paper to help you color a circle pattern.

More practice, page A-21, Set 42

Investigating the Ideas

The area of square *A* is 1. Count squares to find the areas of rectangles *B* and *C*.

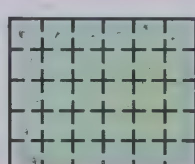


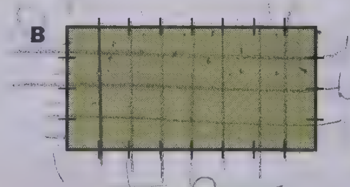
? Can you find squares or rectangles on the geoboard with areas equal to each of the numbers from 1 through 16?

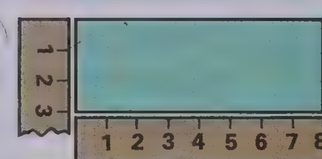
Show each figure you find on dot paper.

Discussing the Ideas

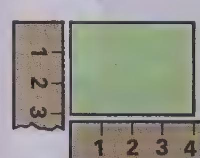
1. Find the area of each figure. Can you find a way that is shorter than counting squares?

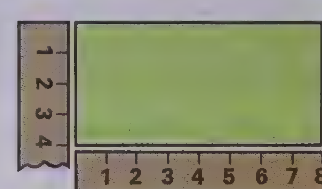
A


B


C


D


E


F


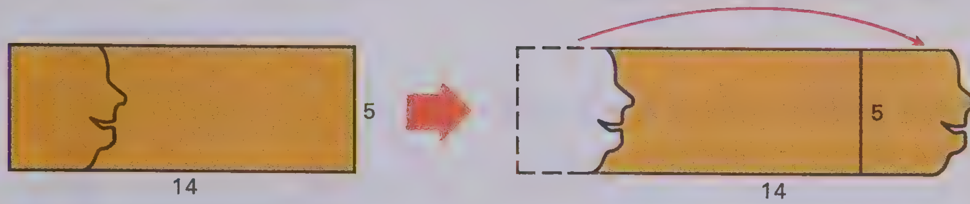
2. A Copy and complete a table like this for each rectangle in exercise 1.

B Explain this formula:
$A = \ell \times w$

Figure	ℓ (length)	w (width)	A (area)
A	6	5	
B	8		

1. Find the area of each region.

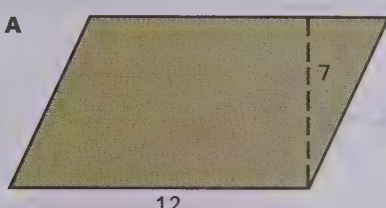
A



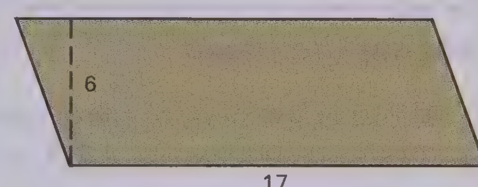
B

2. Find the area of each of these **parallelogram** regions.

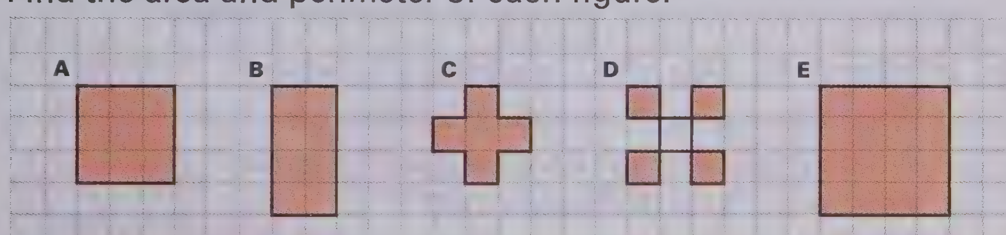
A



B

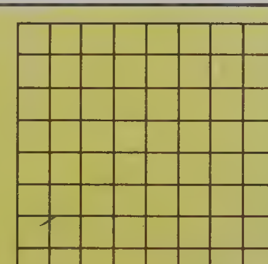


3. Find the area and perimeter of each figure.



think

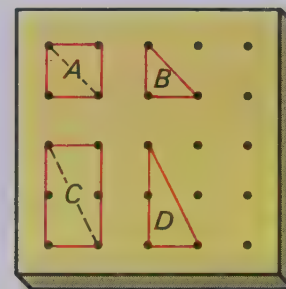
Draw an 8-by-8 figure like this.
Color 8 small square regions so that no 2 colored regions lie in the same row (↔), column (↕), or diagonal (⤧).
There is more than one way to do this.



Investigating the Ideas

If the area of square *A* is 1, the area of right triangle *B* is __?__.

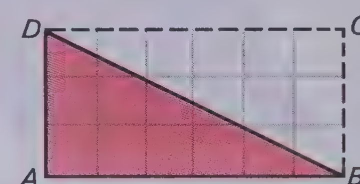
If the area of rectangle *C* is 2, the area of right triangle *D* is __?__.



? Can you find right triangles with areas $1\frac{1}{2}$, 2, 3, 4, $4\frac{1}{2}$, 6, and 8 on a geoboard?

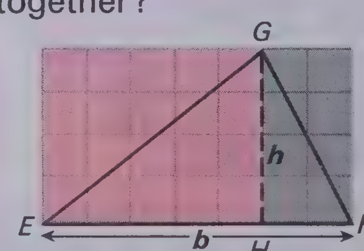
Show each triangle you find on dot paper.

Discussing the Ideas



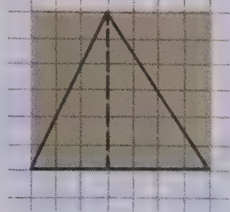
1. **A** What is the area of rectangle *ABCD*?
   B The area of triangle *ABD* is what fractional part of the area of *ABCD*?
   C What is the area of triangle *ABD*?
   D Explain how to find the area of any right triangle.

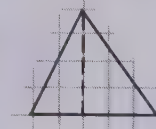
2. **A** What is the area of the region shaded pink?
   B What is the area of the region shaded gray?
   C What is the area of the two regions together?
   D What is the area of triangle *EHG*?
   E What is the area of triangle *FHG*?
   F What is the area of triangle *EFG*?
   G The area of triangle *EFG* is what part of the entire shaded region?

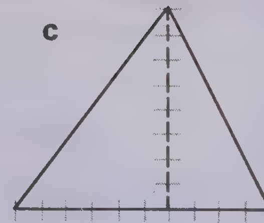


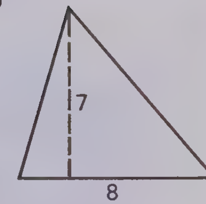
3. In exercise 2, the length of segment *EF* is called the base (***b***) of the triangle. The length of segment *GH* is called the height (***h***). Can you give a formula for finding the area of a triangle when you know its base and height?

1. Find the area of each large triangle.

A


B


C


D


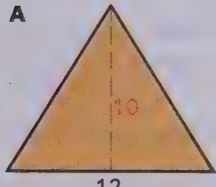
2. Use the flow chart to help you find the areas of the following triangles with the given bases and heights.

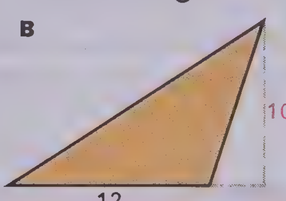
Base b and height h of a triangle → Find $b \times h$. → Multiply by $\frac{1}{2}$. → Area $= \frac{1}{2} \times b \times h$

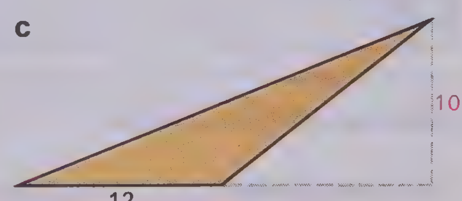
A $b = 12$, $h = 5$ B $b = 10$, $h = 8$ C $b = 14$, $h = 7$ D $b = 27$, $h = 8$ E $b = 54$, $h = 9$

F $b = 8$, $h = 4\frac{1}{2}$ G $b = 9$, $h = 2\frac{1}{3}$ H $b = 12.4$, $h = 6.5$ I $b = 16$, $h = 7\frac{1}{4}$ J $b = 128$, $h = 46\frac{3}{4}$

3. The height of each triangle is given in red. Find the area of each triangle.

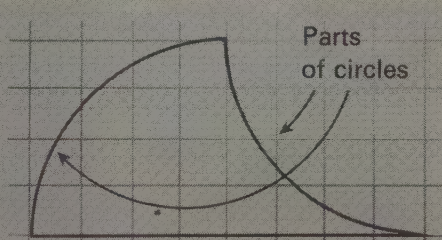
A


B


C


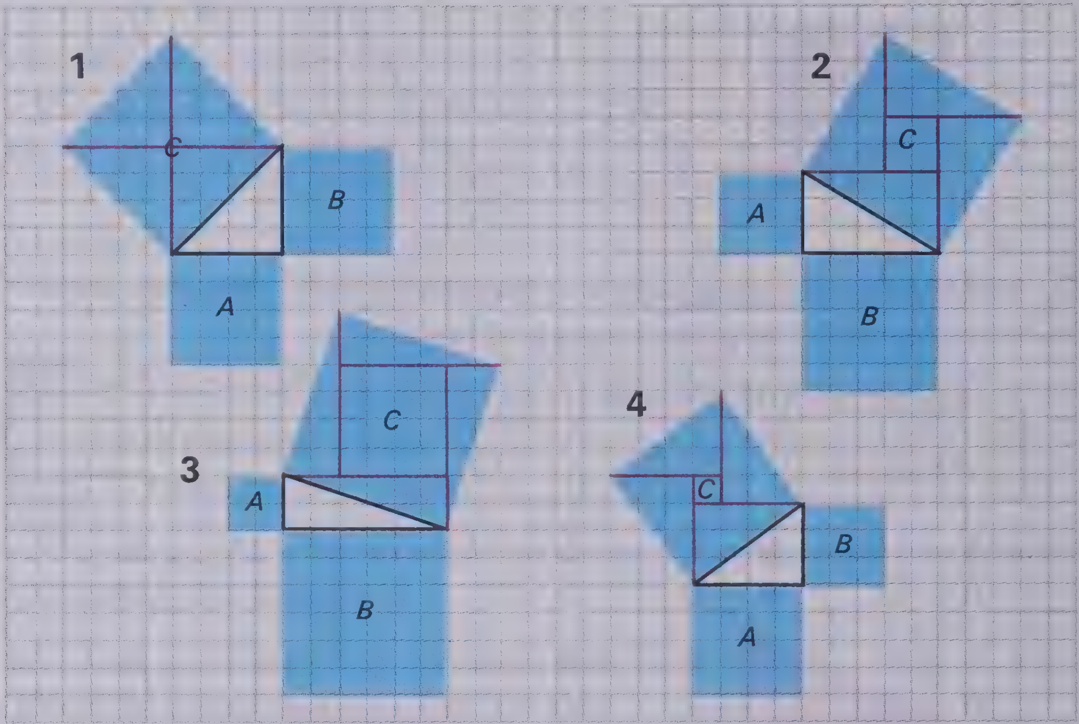
think

Make a larger figure like this on graph paper. Can you make one straight cut that separates the figure into two parts that will fit together to form a square?



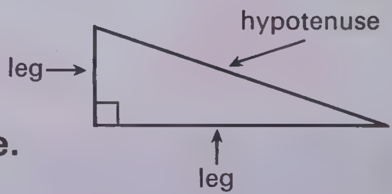
More practice, page A-22, Set 43

Investigating the Ideas



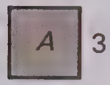
? Can you find the area of the smaller squares *A* and *B* and the larger square *C* for each triangle above?

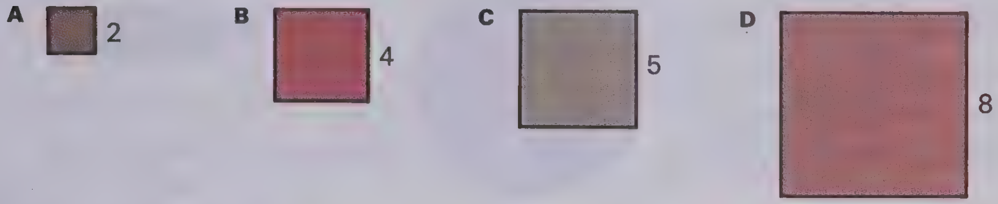
Discussing the Ideas



1. In a right triangle, the side opposite the right angle is called the **hypotenuse.** The other two sides are called the **legs.**
   A What is the area of the square on the shortest leg of the right triangle in figure 4 above? What is the area of the square on the other leg?
   B What is the area of the square on the hypotenuse in figure 4?
2. What pattern do you see in your answers to the Investigation question? Do you think this is true for all right triangles?

1. If square A has a side 3 units long, the **area of square A** is written **3×3** or **3^2** or simply **9**. Write the area of these squares in three ways.

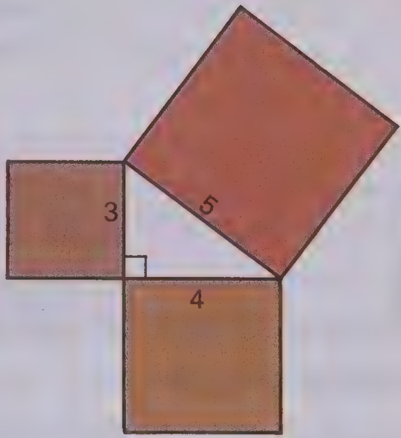




2. Over 2000 years ago a famous mathematician named Pythagoras proved this important theorem:

   The sum of the areas of the squares on the legs of a right triangle is the same as the area of the square on the hypotenuse.

   Write an equation to show that this theorem is true for this right triangle.

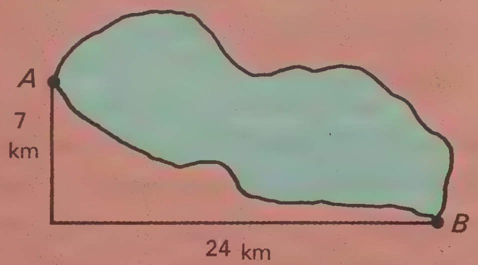


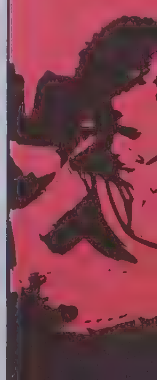
★ 3: Draw a picture and write an equation to show the Pythagorean theorem for a right triangle with legs 5 and 12 cm long.

★ 4. If a right triangle has legs 6 and 8 cm long, how long is the hypotenuse?

think

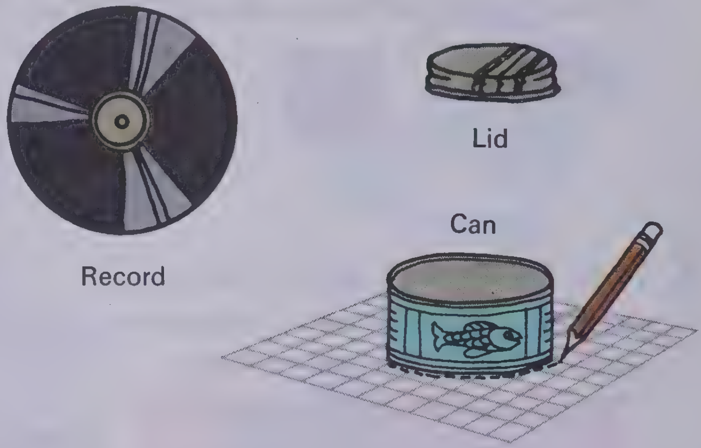
How far is it from point A on the shore to point B, straight across the lake?





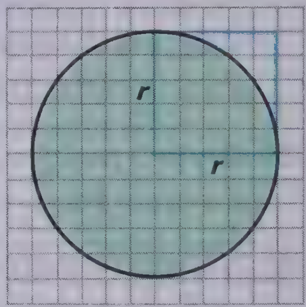
Investigating the Ideas

Choose one or more circular objects. Trace around each object on graph paper.



? Can you find about how many squares each circular object covers?

Discussing the Ideas



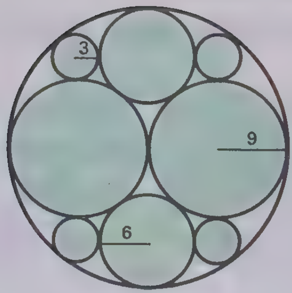
1. Estimate the area of the circle shown at the right by counting squares.

2. The radius (r) of the circle is the side of the blue square. What is the area of this square?

3. A Is your estimate for the area of the circle less than $4 \times r^2$ (four times the area of the square)?
   B Is your estimate more than $3 \times r^2$ (three times the area of the square)?

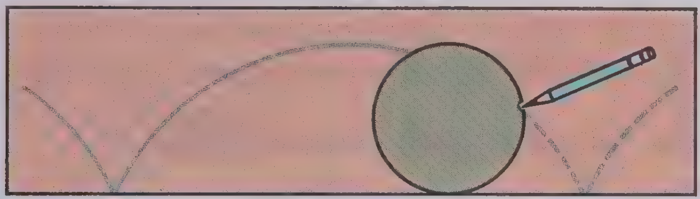
4. As you can see, the area of a circle is slightly more than $3 \times r^2$. More detailed mathematical methods have shown that the area of a circle (to the nearest hundredth) is $\mathbf{3.14} \times r^2$. Using A for **area**, we write: $A = \pi \times r^2$. Can you use this formula to check your estimates of the areas of the circles in the Investigation?

1. A Find the area of each different-sized circle. (Use $\pi = 3.14$.)
   B Find the total area of the colored part.
   C What is the area of the inside part not colored?



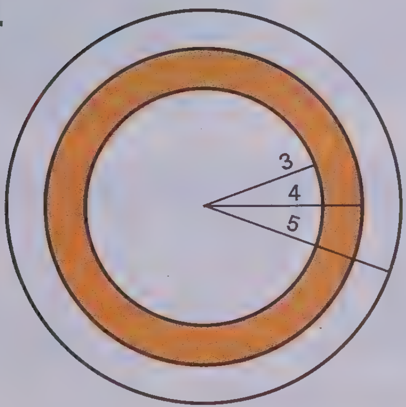
(The radius is 18.)

2. When a pencil fastened to the edge of a cardboard circle marks the wall as the circle rolls along, it makes a curve called a **cycloid.**



Mathematicians have proved that the area of region *A* (the area of region *B*, also) is the same as the area of the circle. What is the total area of the three colored regions under the cycloid?

★ 3.

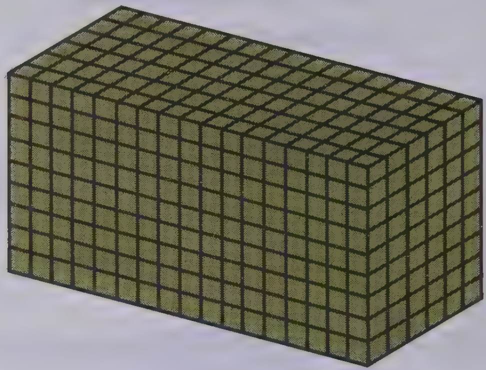


Is the area of the outside white ring less than, equal to, or more than the area of the inside white circle?

More practice, page A-22, Set 44

Investigating the Ideas

Suppose this rectangular prism was made by gluing many small cubes together.

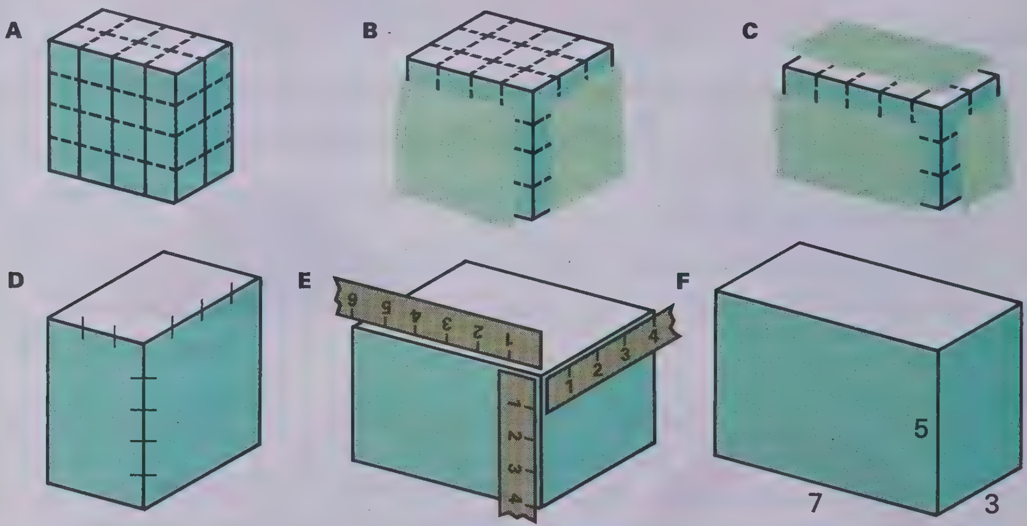


> Can you think of an easy way to find how many cubes were used?

Discussing the Ideas

1. The **volume** of a figure is the number of unit cubes it takes to "make" the figure. What is the volume of the figure above?

2. What is the volume of each of these figures?



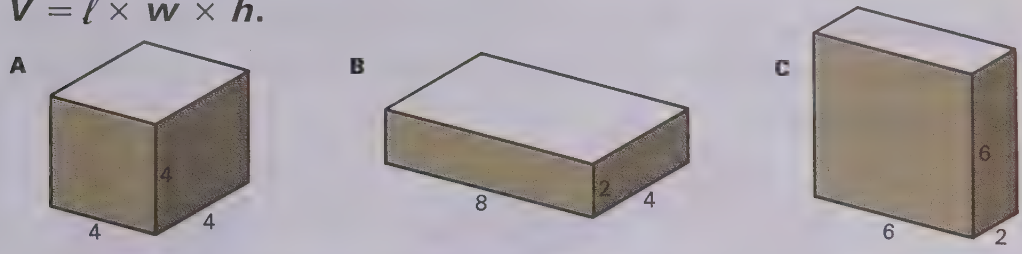
3. A On the chalkboard, copy and complete a table like this for each figure in exercise 2.

Figure	ℓ(↔)	w(↗)	h(↕)	V(volume)
A	5	2	4	▒
B	4	3	▒	▒

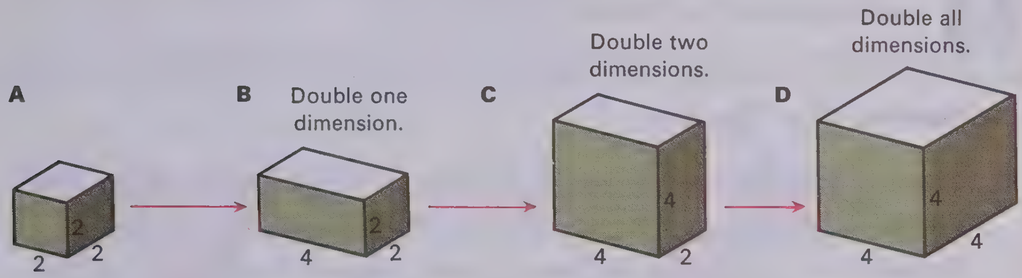
ℓ = length; w = width; h = height

B Can you give a formula for finding the volume of a rectangular prism?

1. Find the volume of each figure by using the volume formula, $V = \ell \times w \times h$.

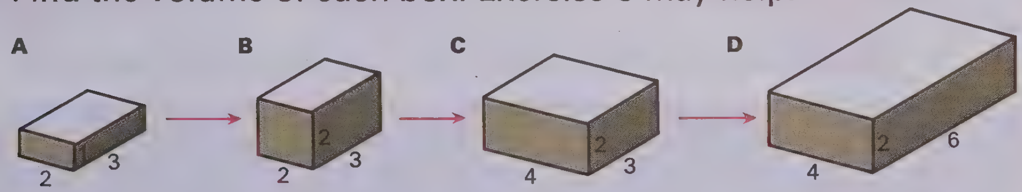


2. Length, width, and height are sometimes called the three **dimensions** of a figure. Find the volume of each figure.

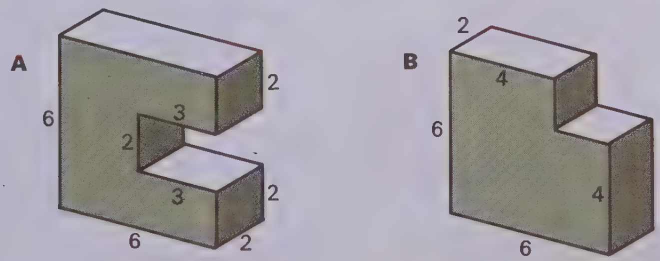


3. Use exercise 2 to help you complete each sentence.
   A If two dimensions of a box are doubled, the volume is __?__ times as large.
   B If each dimension of a box is doubled, the volume is __?__ times as large.

4. Find the volume of each box. Exercise 3 may help.



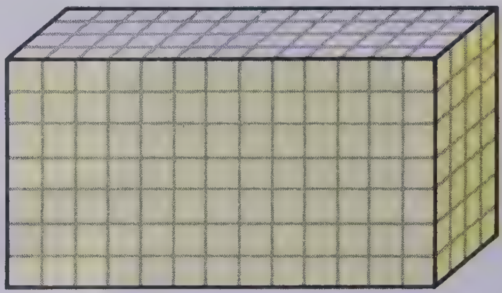
★ 5. Find the volume of each figure.



More practice, page A-23, Set 45

Investigating the Ideas

This box has been covered on all sides with graph paper.

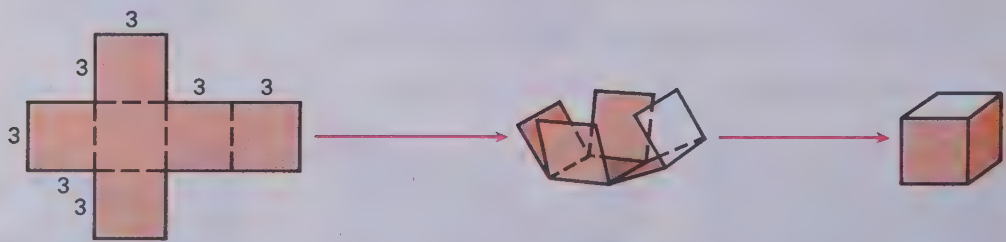


? Can you think of an easy way to find how many unit squares ☐ it took to cover it?

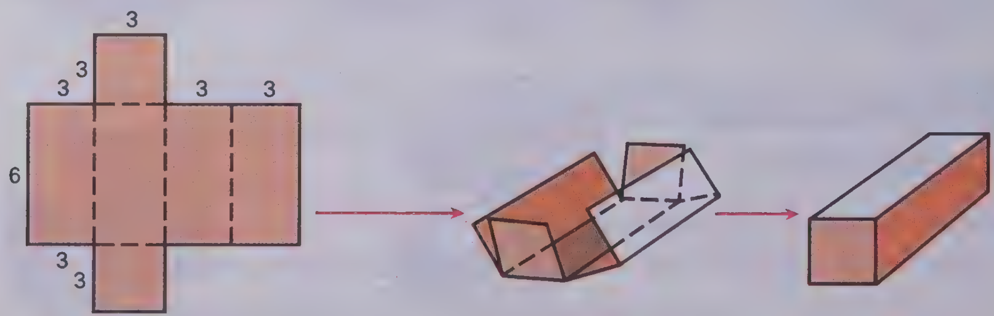
Discussing the Ideas

1. The number of unit squares it takes to completely cover the surface of a figure is called the **surface area** of the figure. Explain how you found the surface area of the "box" above.

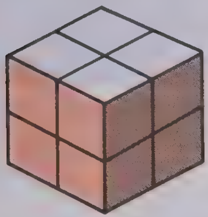
2. A What is the area of the region in the first figure?
   B Explain why the surface area of the cube is 54.

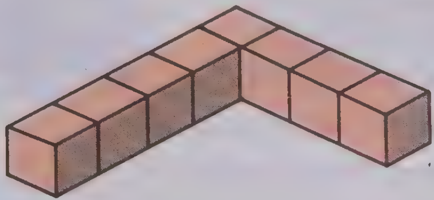


3. What is the surface area of the "box" below?



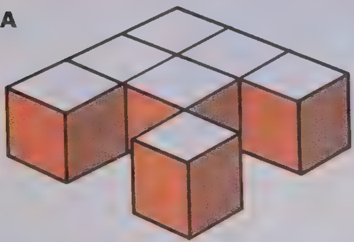
1. The two figures below have the same volume. Tell whether the first or the second figure has the greater surface area. Then tell how much greater. Use the units indicated.



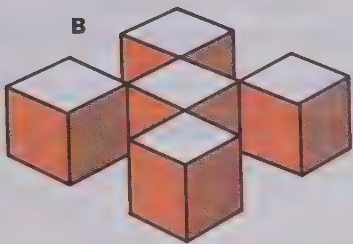


2. Give the volume and surface area of each figure. Use the indicated unit.

A

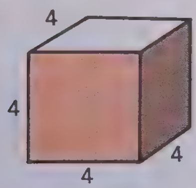


B

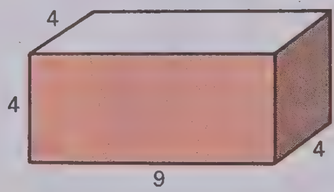


3. Give the volume and surface area of each figure. Use the lengths given.

A



B



★ 4. Give the surface areas of the figures in exercise 5, page 273.

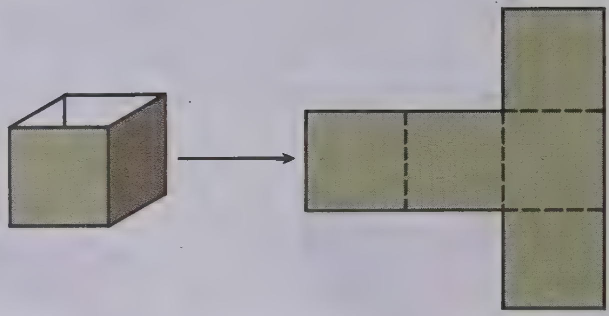
think

FOR BASEBALL FANS ONLY:

It was near the end of the season. Jones and Smith were tied for the National League batting title. In the remaining games, Jones went 7 for 8 and Smith went 9 for 12. Who finished with the higher batting average?

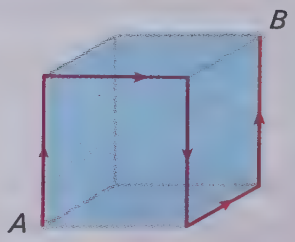
Solving Cube Problems

1. A cubical box without a lid was cut along certain edges and opened to make the pattern shown.



   A Can you show another pattern that could have been made?
   B How many different patterns can you show?

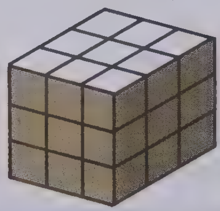
2. The picture shows a "5-edge trip" from A to B. Can you draw a picture of a cube to show



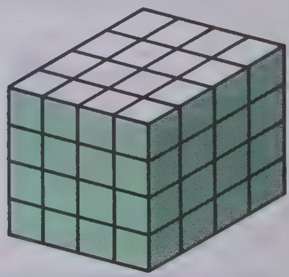
   A a "3-edge trip" from A to B?
   B a "7-edge trip" from A to B?

3. How many cubes are hidden (have no faces showing even if you could pick the stack up and look at all sides)

   A in a 2 × 2 × 2 stack?
   B in a 3 × 3 × 3 stack?
   C in a 4 × 4 × 4 stack?

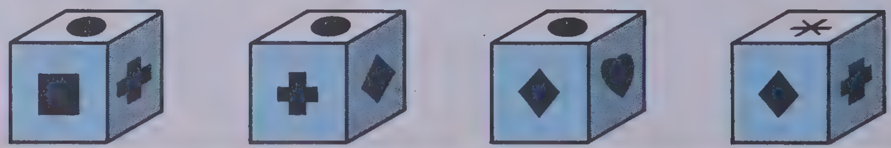


3 × 3 × 3 stack



4 × 4 × 4 stack

★ 4. Here are 4 views of the same cube. Which designs are opposite each other on the cube?

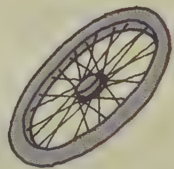


Solving Practical Geometry Problems

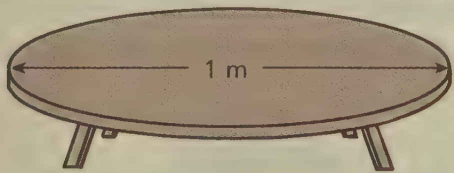
1. Mrs. Alvarez saw a carpet that is 3.5 metres wide and 6.5 metres long on sale for $204.75.
   - A How many square metres ?
   - B Does the carpet sell for more than ten dollars per square metre ?

2. Bill measured his school classroom. It is 7.5 metres wide, 9 metres long, and 3.25 metres high.
   - A How many cubic metres of volume are in the room ?
   - B In Bill's class there are 26 children, one teacher, and a teacher's aide. How many cubic metres of room (to the nearest tenth) are there for each person ?

3. How much wire is needed to fence a rectangular field 80 metres wide and 120 metres long ?



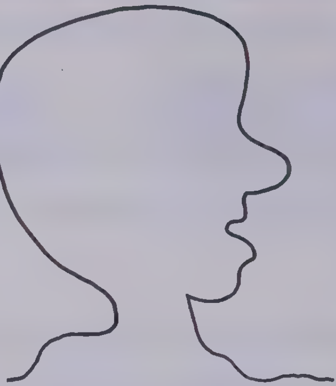
4. A bicycle wheel has a 66-cm diameter.
   - A What is the circumference of the wheel ?
   - B About how many metres will the bicycle travel in one turn of the wheel ?
   - C About how many times must the wheel turn in one kilometre ?



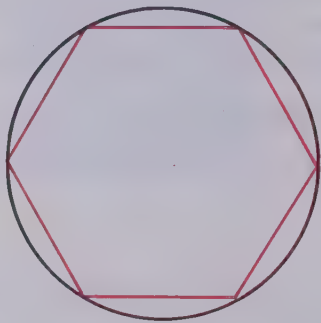
5. A circular table has a top that is 1 metre in diameter. What is the area of the table top ?

Reviewing the Ideas

1. Guess the length of this curve in centimetres. Then measure it as accurately as you can.

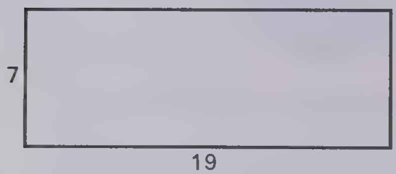


2. A Which is larger, the circumference of the circle or the perimeter of the polygon? (Answer before measuring.)
   B Find the circumference of the circle and the perimeter of the polygon in centimetres.

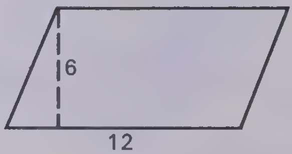


3. Find the area of these figures.

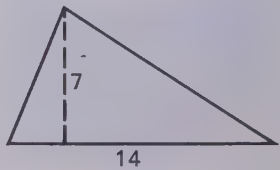
A



B

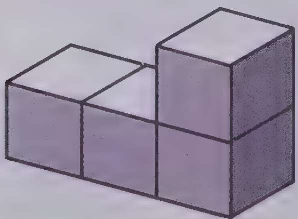


C

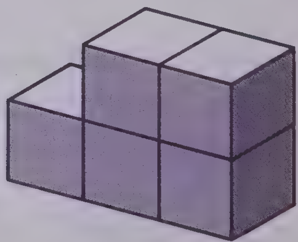


4. Find the volume and surface area of each figure. Use the units indicated by the figures.

A

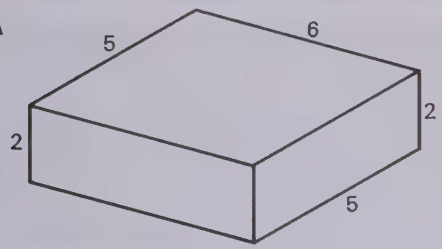


B

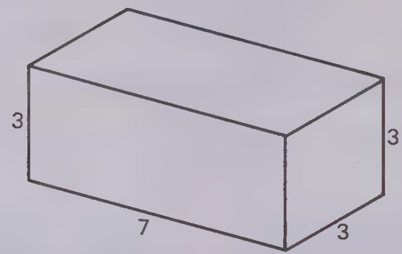


5. Find the volume and surface area of each box.

A

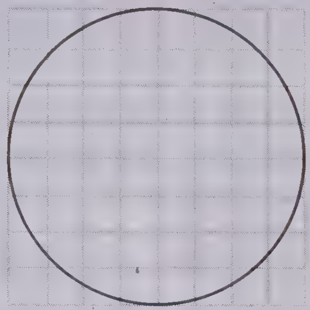


B

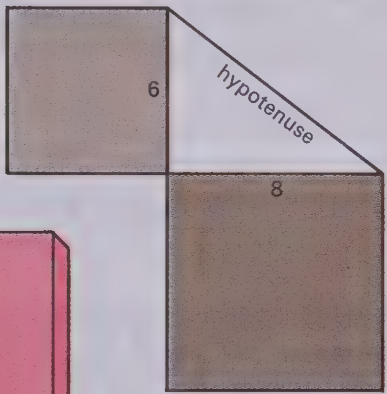


6. A Count squares to **estimate** the area of the circle.

B The diameter of the circle in the figure is 8. Use 3.14 for π to find the area of the circle.



7. If a square were drawn on the hypotenuse of this right triangle, what would its area be?



think

The figures below show two different paths that start at the top and spell LEVEL. Show 7 other paths that start at the top and spell LEVEL.

```
  L Start        L Start
 L E L          L E L
L E V E L      L E V E L
 L E L          L E L
  L              L
```

8. A A cube has __?__ vertices, __?__ faces, and __?__ edges.

B If the area of a face of a cube is 4 square units, what is the surface area of the cube?

1. A List the multiples of 20 (to 100). B List the multiples of 7 (to 35).
   C List five fractions equivalent to $\frac{7}{20}$.

2. Solve these equations.

   A $\frac{7}{20} = \frac{n}{40}$ C $\frac{1}{4} = \frac{n}{16}$ E $\frac{2}{3} = \frac{n}{12}$ G $\frac{1}{6} = \frac{5}{n}$ I $\frac{3}{4} = \frac{n}{20}$

   B $\frac{7}{20} = \frac{n}{100}$ D $\frac{3}{5} = \frac{9}{n}$ F $\frac{4}{9} = \frac{n}{18}$ H $\frac{3}{8} = \frac{n}{24}$ J $\frac{5}{4} = \frac{n}{100}$

3. Give a set of five fractions equivalent to the given fraction.

   A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{4}$ D $\frac{2}{3}$ E $\frac{3}{4}$ F $\frac{1}{5}$ G $\frac{3}{5}$

4. The rings are shown to remind you of a check for equivalent fractions. Give the correct sign ($=$ or $\neq$) for each ◍.

   A $\frac{3}{4}$ ◍ $\frac{7}{8}$ B $\frac{1}{3}$ ◍ $\frac{2}{7}$ C $\frac{3}{4}$ ◍ $\frac{75}{100}$ D $\frac{5}{9}$ ◍ $\frac{5}{8}$

5. Find the totals. Change each answer so that you have the greatest number of the larger unit.

   A 8 h 42 min
   7 h 21 min

   B 17 days 8 h
   9 days 16 h

   C 4 mo 2 days
   5 mo 29 days

   D 4 wk 5 days
   2 wk 6 days

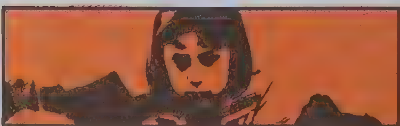
   E 5 min 19 sec
   3 min 18 sec
   2 min 29 sec

   F 4 h 35 min
   1 h 56 min
   8 h 49 min

think

Four sections of chain are shown. It costs 15¢ to cut a link open. It costs 35¢ to have a link welded together.

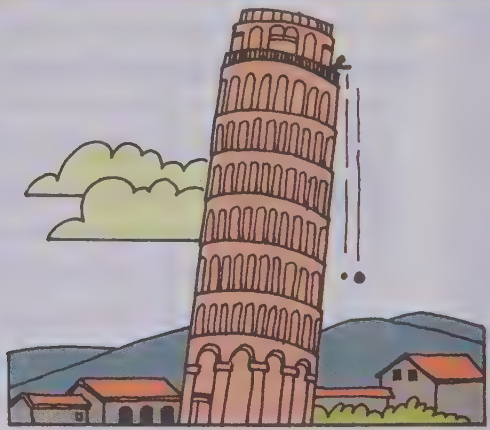
Show how you could form one long chain for $1.50. There is a cheaper way. What would it cost?



You are invited to explore

ACTIVITY CARD 13
Page 361

FALLING BODIES



The famous Leaning Tower of Pisa is located in Pisa, Italy. It has stood for hundreds of years, even though it leans so far to one side it looks as if it will fall. The tower is 54.56 m tall, and it is about 544 centimetres out of line. Galileo, a famous 17th century scientist, thought that two falling bodies would fall at the same speed, even if one were large and the other small. Galileo is said to have dropped objects from the Leaning Tower of Pisa to prove his theory. This theory, although unpopular in its day, is correct (assuming no air resistance).

1. The approximate distance (in metres) that an object falls when dropped to Earth can be found by the formula $d = 4.9 \times t^2$, where t is the time (in seconds). If an object is dropped, how far will it fall in

   A 1 second? B 2 seconds? C 4 seconds? D 7 seconds?

2. If the objects Galileo dropped took 3 seconds to hit the ground, about how high was Galileo when he dropped them?

3. As objects fall, their speed increases. You can give the speed (in metres per second) of a falling object if you multiply 9.8 by the number of seconds the object has been falling.

   A An object dropped from a high place is going how fast after 3 seconds?

   ★ B One kilometre per hour is about 0.28 metres per second. What is the speed in kilometres per hour of the object in part A?

★ 4. If an object takes $5\frac{1}{2}$ seconds to fall to the ground, how far did it fall and how fast was it going when it hit?

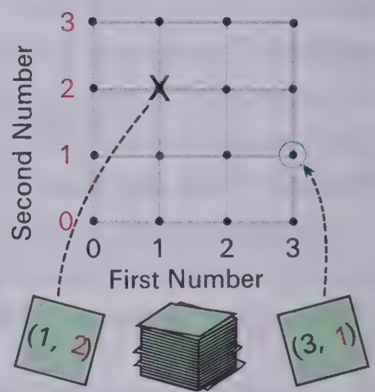
12 Geometry and Graphing

● *How do you graph number pairs?*

Investigating the Ideas

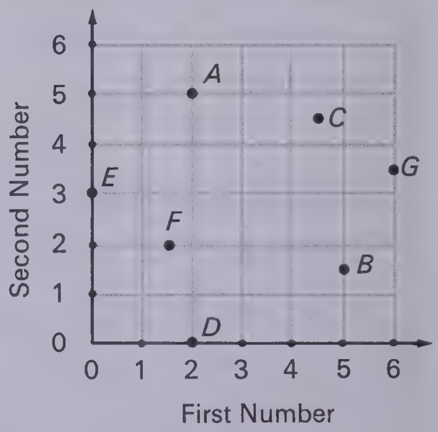
? Can you play the Tic-Tac-Toe game described below with one of your classmates?

On small cards, write the 16 pairs of whole numbers for the points shown on the grid. Shuffle the cards and turn them face down. Each player in turn takes a card from the stack and marks an ○ or an X on the grid at the point for his number pair. The first player to get 4 of his marks in a row, column, or diagonal wins the game.



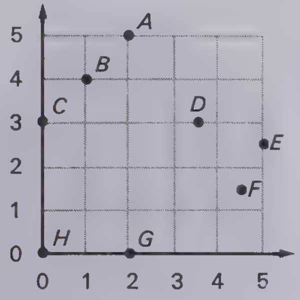
Discussing the Ideas

1. The two numbers which give the location of a point on a grid like this are called the **co-ordinates** of the point. Each of the two number lines is a **co-ordinate axis.** What number pair designates the point where the co-ordinate axes intersect?



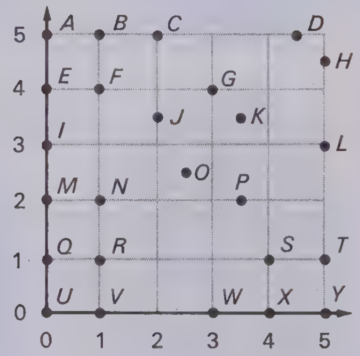
2. To describe the location of some points, you may need to use fractional numbers. For example, in the figure for question 1, the co-ordinates of *B* are $(5,\ 1\frac{1}{2})$.
   - A Which points on the grid have whole-numbered co-ordinates?
   - B Give the co-ordinates of those lettered points which are described by fractional numbers.

1. Give the co-ordinates for each point.



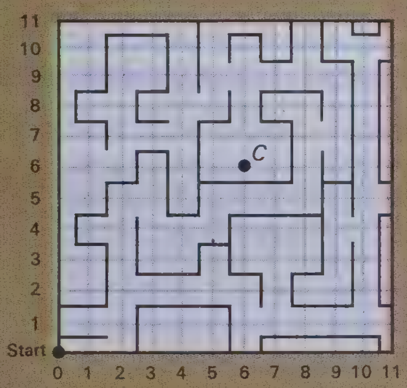
2. From the graph below select the letter of the point for each pair of co-ordinates.

$(2,3\frac{1}{2})$	$(3,4)$	$(1,2)$
$(4,0)$	$(0,5)$	$(3\frac{1}{2},3\frac{1}{2})$
$(4\frac{1}{2},5)$	$(2,5)$	$(5,0)$
$(5,4\frac{1}{2})$	$(0,1)$	$(1,0)$



3. A All the points directly above the point (3,0) have co-ordinates with what first number?
B All the points directly to the right of the point (0,4) have co-ordinates with what second number?

think



Give the shortest path along blue lines to the centre of the maze by writing, in order, the co-ordinates of the points where you turn.

4. What figure is formed when these points are graphed and connected in order?
$(2,2) \rightarrow (4,7) \rightarrow (6,2) \rightarrow (1,5) \rightarrow (7,5) \rightarrow (2,2)$

5. On a 10-by-10 grid, mark at least 10 points in which the second co-ordinate is twice the first. Examples: (1,2), $(2\frac{1}{2},5)$ What do you notice about all of these points?

6. Invent a picture and give the co-ordinates, in order, to a classmate. Ask your classmate to draw the picture.

Investigating the Ideas

Think about these points:

Point A is the midpoint of the segment from (2,2) to (6,6).

Point B is a point on the line through (2,4) and (4,8).

Point C is a point that goes with (2,3) and (8,3) to form the corners of an isosceles triangle.

Point D goes with (2,3), (2,8), and (7,8) to form the corners of a square.

Point E is the centre of a circle that goes through (3,0) and (0,3).

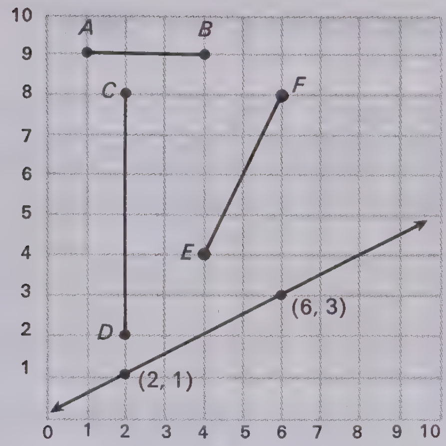
Point F is the vertex of a right angle whose sides go through (8,2) and (4,6).

?	How many of the points described above can you find?	Show your findings on graph paper.

Discussing the Ideas

1. For which of the descriptions above is more than one choice for the point possible?
2. Choose one of the situations you mentioned in exercise 1 and give some other possibilities for the point.
3. Can you make up a description of a point of your choice and see if your classmates can give its co-ordinates?

1. A Give the co-ordinates of the midpoints of each segment.
   B Give the co-ordinates of two more points on the line.



2. What geometric figure is formed when these points are graphed and connected in order?
   (3,4) → (3,7) → (6,9) → (9,7) → (9,4) → (6,2) → (3,4)

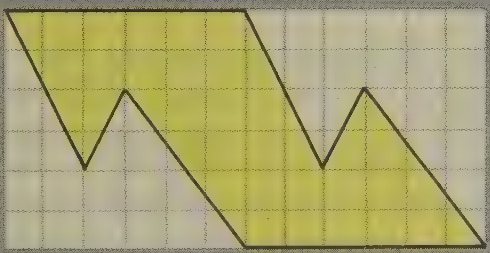
3. Give the co-ordinates of the vertices of:
   A a rectangle with area 8
   B a right triangle with area 10

4. Give the co-ordinates of the fourth vertex of a rectangle with vertices at $(2\frac{1}{2},3\frac{1}{2})$, $(2\frac{1}{2},7\frac{1}{2})$, and $(8\frac{1}{2},3\frac{1}{2})$.

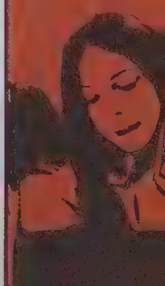
★ 5. (1,3), (6,3), and (3,6) are the co-ordinates of three vertices of a parallelogram. What are possible co-ordinates of the other vertex?

think

Draw a figure like this on your graph paper and cut it out.

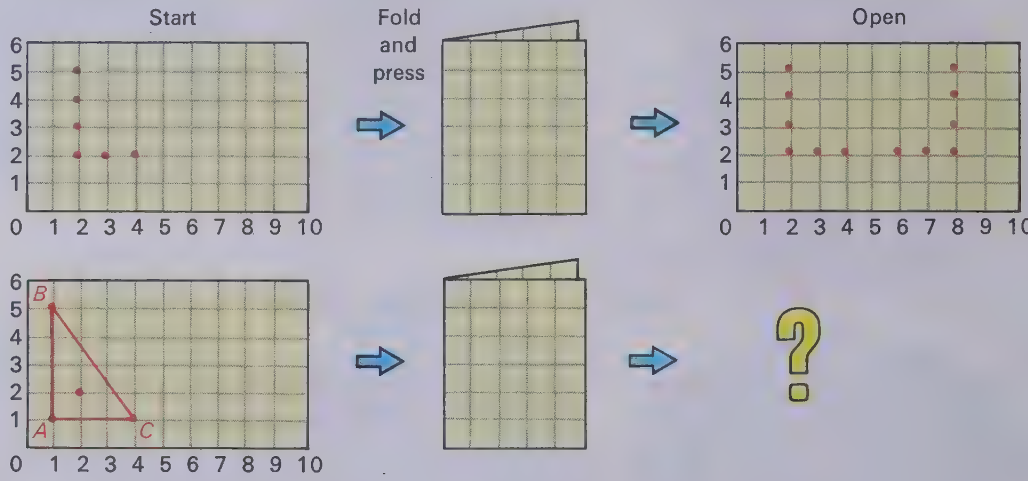


Make one straight cut so that the two parts can be placed together to form a square.



Investigating the Ideas

Imagine "magic" paper that works like this: When the two halves are folded together, anything printed in red on one half is automatically printed on the other half.



Can you show on graph paper what this paper will look like when it is opened?

Discussing the Ideas

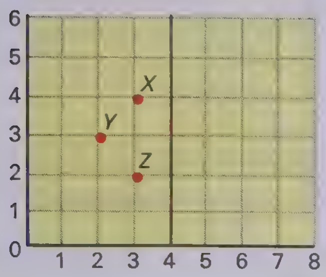
1. The dot pattern made up of two "L's" above is **symmetrical** because it can be folded so that one half exactly matches the other half. The fold line is called **a line of symmetry.** Name a geometrical figure that is symmetrical, and describe a line of symmetry of the figure.

2. In the triangle that will be printed on the right above, the point that matches point *A* is called the **reflection image** of point *A*. Can you give the co-ordinates of the reflection image of

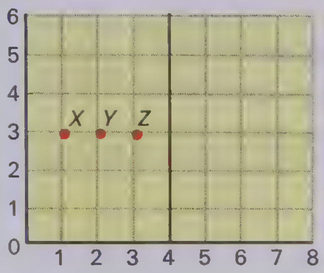
   A point *A*?
   B point *B*?
   C point *C*?
   D the point inside the triangle?

1. Give the co-ordinates of the reflection images of points X, Y, and Z.

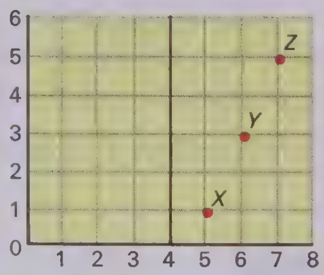
A



B

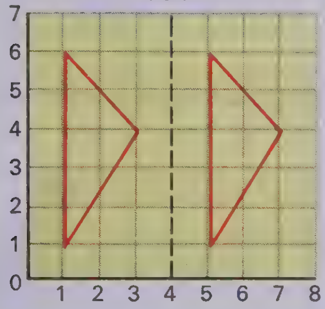


C

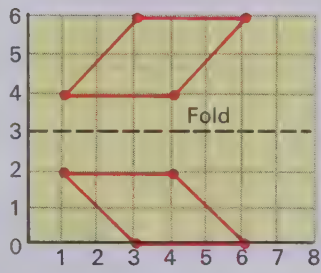


2. Which pairs of figures are reflections of each other?

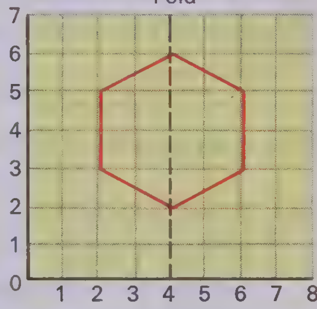
A



B



C

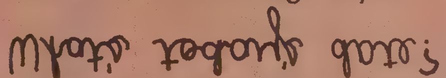


3. Use your graph paper to draw three symmetrical figures. Show a line of symmetry for each one.

think

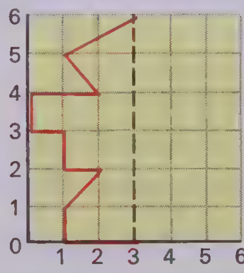
Make a "mirror" by taping plastic or cellophane to a cardboard frame.

Can you use your "mirror" to find out what this message says?



Write a "mirror message" to a friend.

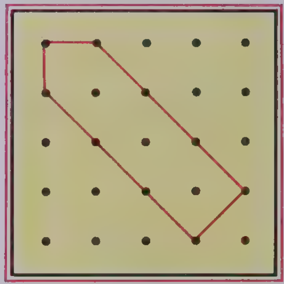
4. Copy this figure on your graph paper. Then draw the other half to make a symmetrical figure.



Let's explore rotations.

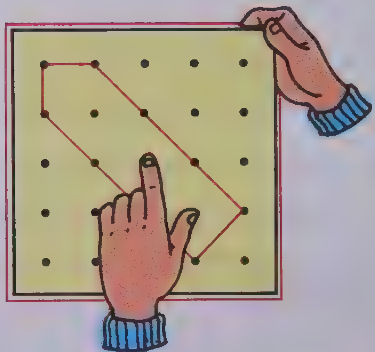
Investigating the Ideas

Draw a red square around a geoboard (or a piece of dot paper) with this figure on it.

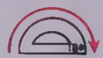


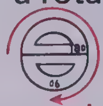
? If you pick up the board and turn it, in how many different ways can you put it back inside the square?	Show each way on dot paper.

Discussing the Ideas



1. A motion that turns the board about a single fixed point is called a **rotation** of the board. The fixed point is called the **centre** of rotation. Rotations are often described in degrees. Which of the positions you drew in the Investigation do you think is a result of

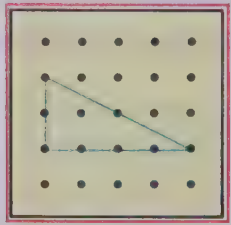
   A a rotation of 90°?  B a rotation of 180°? 

   C a rotation of 270°? 

2. You perform a rotation when you turn a doorknob to open a door. Give three other examples of situations when rotations are performed.

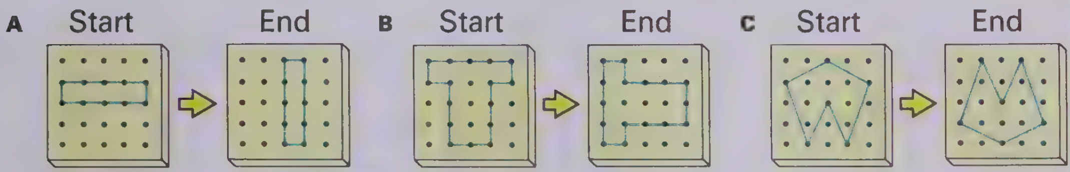
3. Through how many degrees does the minute hand of a clock rotate in

   A 30 minutes? B 15 minutes? C 45 minutes? D 2 hours?



1. Draw a picture on dot or graph paper to show how this geoboard figure will look after:
   A a 90° rotation
   B a 180° rotation
   C a 270° rotation
   D a 360° rotation

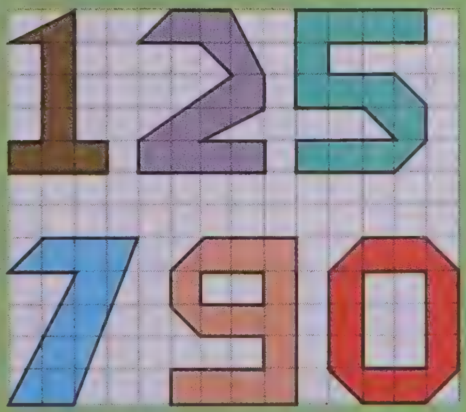
2. Each part shows a geoboard at the start and end of a rotation. What rotation was made?



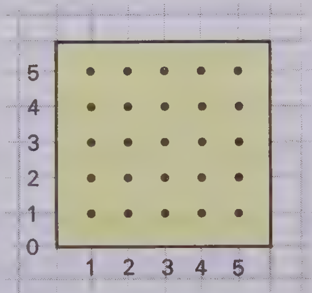
3. Make a figure on your geoboard or on dot paper and show the "start" and "end" of a rotation of your figure. Ask a classmate to tell what rotation was performed.

think

Find the exact area of each digit.

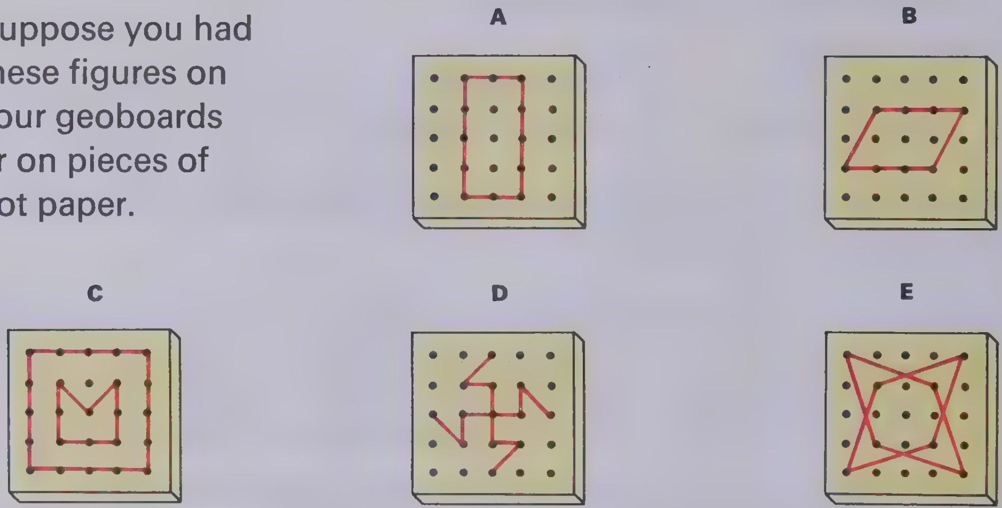


4. If the geoboard were placed on graph paper, each "nail" would have co-ordinates
   A If the board were turned 90° about the centre, what would be the new co-ordinates for the "nail" now at (3,5)?
   B Give the **90° rotation image** of the point (1,5).



Investigating the Ideas

Suppose you had these figures on your geoboards or on pieces of dot paper.



? Can you tell which geoboards would look exactly the same if each one were rotated 180° ?

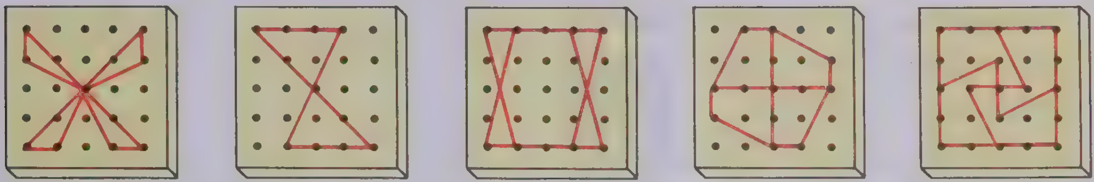
Discussing the Ideas

1. If the board looks exactly the same at the end of a rotation (other than 360°) as it did at the beginning, the figure on the board is said to have **rotational symmetry.**
   A Do you think every rectangle has 180° rotational symmetry ?
   B Name some other quadrilaterals that have 180° rotational symmetry. (The pictures above may help you.)
   C Can you think of a quadrilateral that has 90° rotational symmetry ?

2. A rectangle has both a line of symmetry and 180° rotational symmetry. Which quadrilateral has only rotational symmetry ?

3. A Can you think of any polygons, other than quadrilaterals, that have rotational symmetry ? Explain.
   B Invent a figure that has rotational symmetry.

1. Which of these geoboard designs have 180° rotational symmetry?

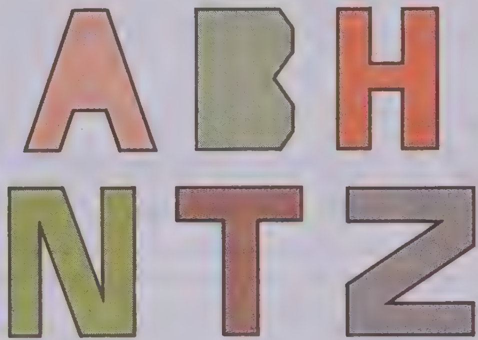


A　　B　　C　　D　　E

2. Which figures in exercise 1 have 90° rotational symmetry?

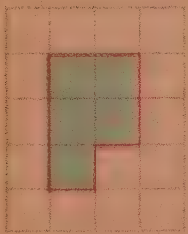
3. Make up a figure on your geoboard or on dot paper that has 90° rotational symmetry; 180° rotational symmetry.

4. Think about polygons that are outlines of the 26 letters of the alphabet. Which of these polygons have 180° rotational symmetry?



think

A pentomino is a figure made from 5 squares. Each square can touch another only on a complete common side. Find as many pentominoes that have 180° rotational symmetry as you can and draw them on graph paper.

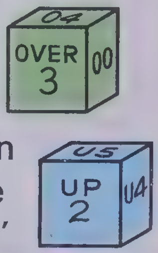


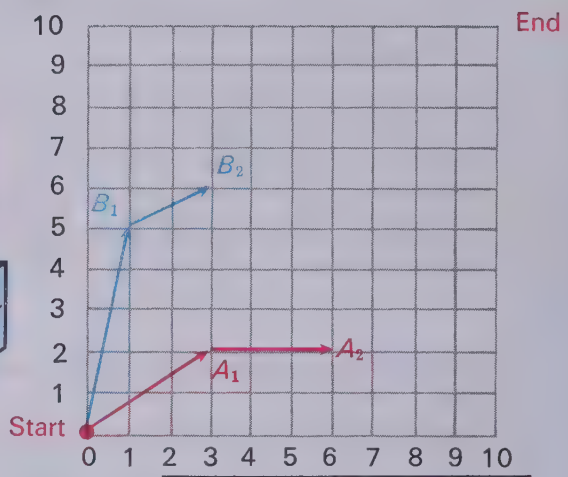
★ 5. Draw a picture of a figure that contains no straight lines and has 180° rotational symmetry.

★ 6. Use only squares and circles to make a design that has 90° rotational symmetry.

Investigating the Ideas

To play the game "Translations," each player in turn rolls two cubes with faces marked **00** through **05** and **U0** through **U5**. Then he draws an arrow on the graph to show the "slide" made by his point. The chart shows the record of the start of a game played by Ann and Bob. The object of the game is to reach the point (10, 10) first.





	Ann	Bob
1st roll	A_1 (03, U2)	B_1 (01, U5)
2nd roll	A_2 (03, U0)	B_2 (02, U1)

? Can you play the game with a classmate?

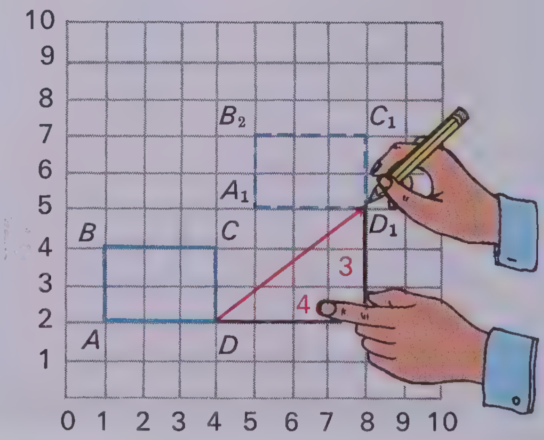
Make up your own rules for the game.

Discussing the Ideas

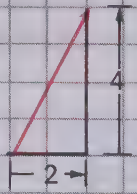
1. What are some rules you would use for the game?

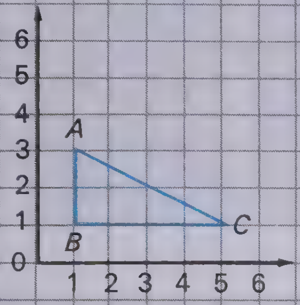
2. How could you win the game in two moves?

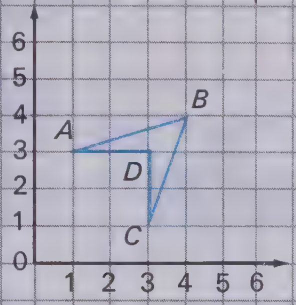
3. A student used an "over 4, up 3" point slider to "slide" each point of a figure to a new position. The red arrow shows the distance and direction of this move, called a **translation** of the figure. How far was the figure moved? Can you give the co-ordinates of the **slide image** of point D?

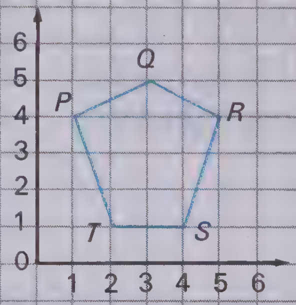


1. Copy each figure on your graph paper. Then use the point slider to translate the figure and draw its slide image.

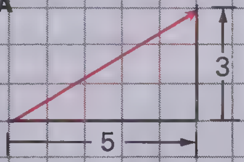


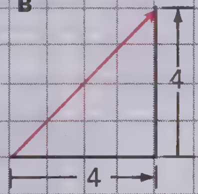
A


B


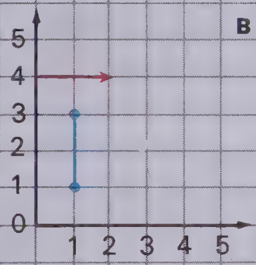
C


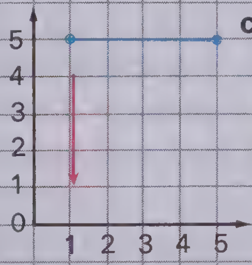
2. Complete exercise 1 for each of these translations.

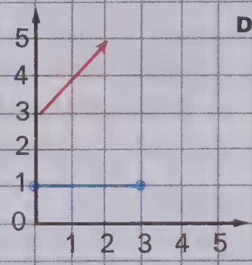
A


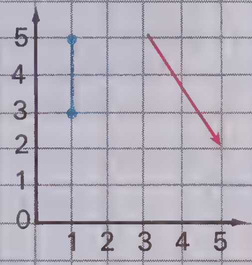
B


3. The red arrow tells the distance and direction of a translation. Copy each segment on your graph paper and draw its image after the translation.

A


B


C


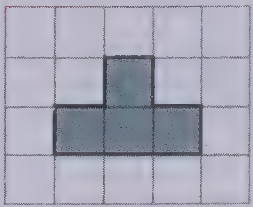
D


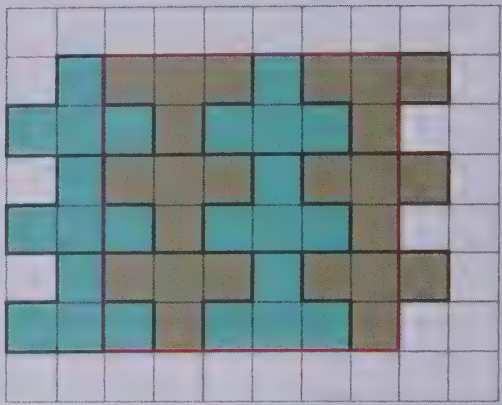
4. Draw a figure on your graph paper and show its new position after these translations.

A right 1, up 6 B right 3, down 2 C left 4, down 3

Investigating the Ideas

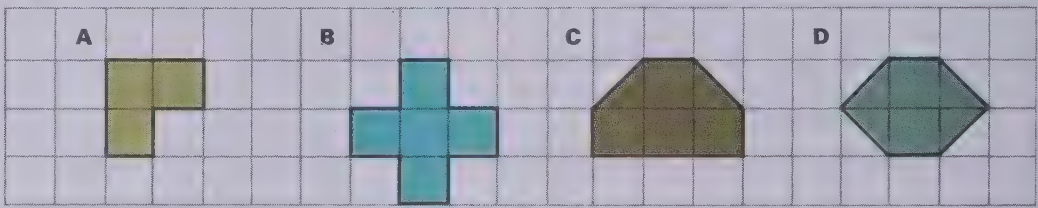
A 6 × 6 red square has been completely covered with copies of this shape. (Some squares fell outside.)





? Which of these shapes can be used to completely cover a 6 × 6 red square?

Draw the covering for those that can.

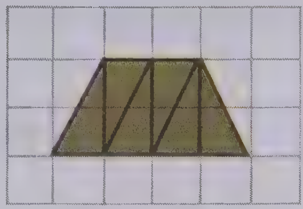


Discussing the Ideas

1. You may remember that a pattern that can "cover a plane" is called a **tessellation** of the plane. If a particular shape is used to form a tessellation, we sometimes say that the shape will **tessellate** the plane.
   - A What patterns do you see in the tessellations in the Investigation?
   - B How can you color the tessellations to make the patterns stand out?

2. Is there more than one way to tessellate a plane with shape A above? with shape D?

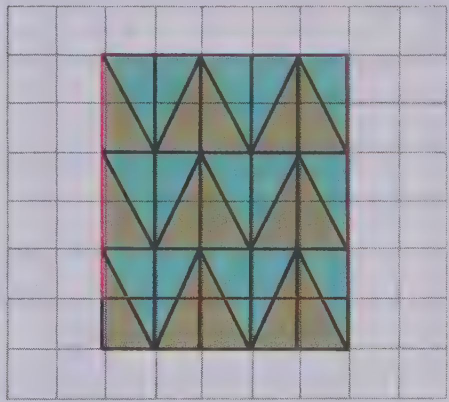
1. Six of these right triangle shapes have been put together to form a trapezoid. On your graph paper show how to put copies of this shape together to form:



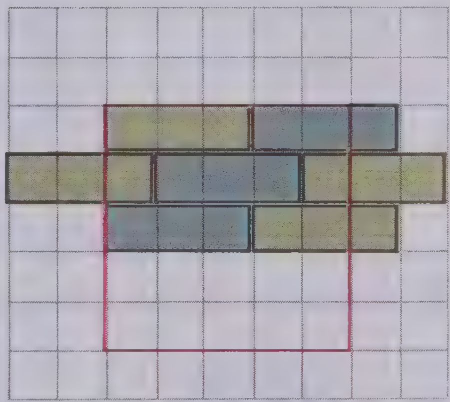
A an isosceles triangle
B a square
C a rectangle
D a parallelogram
E a rhombus
F a hexagon

2. Here is a tessellation with the right triangle as the basic shape.

On your graph paper draw and color two other tessellations with this shape.



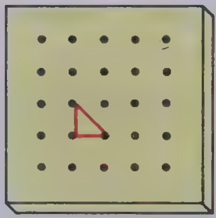
3. A Copy and complete this covering of the red square by using the suggested pattern.
B Use this same basic shape and show a different tessellation.
C Color your tessellation so the pattern stands out.



4. Invent a figure of your own that will tessellate a plane.

Investigating the Ideas

On a geoboard make a triangle like this one.



?	
How many different-sized triangles that have the same shape as this one can you find on a geoboard?	Show the triangles that you find on dot paper.

Discussing the Ideas

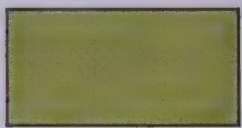
1. Triangles which have the same shape are called **similar triangles.**
   A Did you find a triangle that has sides twice as long as the triangle shown above? three times as long? four times as long?
   B Have you discovered anything about how the lengths of the sides of two similar triangles compare?

2. A Without measuring, give the degree measures of the angles of the triangle in the Investigation. How did you do this?
   B Now find the degree measures of the angles of the similar triangles you drew.
   C What do you think might be true about the measures of the angles of two similar triangles?

3. Which two figures shown below are similar figures?

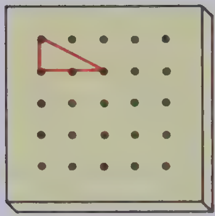
A

B

C


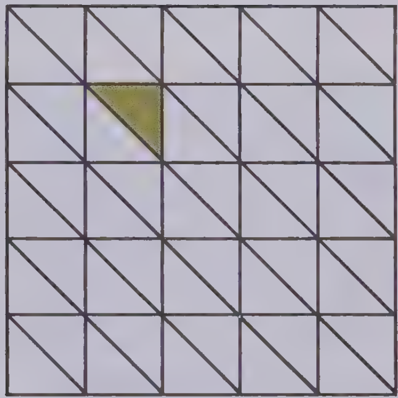
4. What do you think must be true about the angles of two similar figures?

1. On your geoboard find a triangle similar to this triangle. Draw it on dot paper.

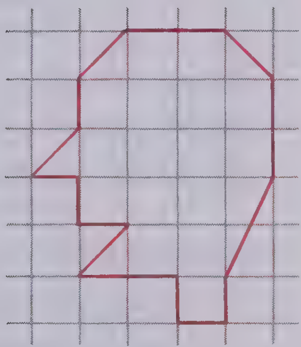


2. A Place tracing paper over this grid and trace as many different-sized triangles that are similar to this triangle as you can. You may include only those triangles that can be traced directly from this grid.

   B Find another familiar figure on the grid and repeat part A.



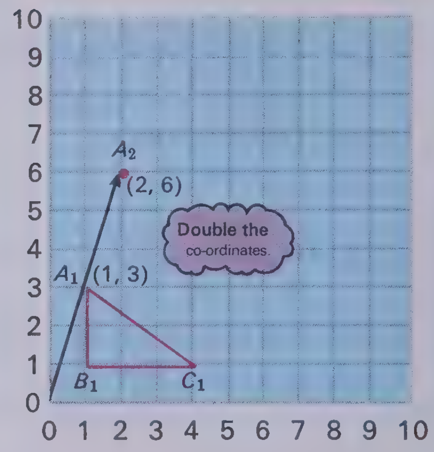
3. Draw a figure like this one on graph paper. Then make a larger figure that is similar to the one shown.



★ 4. Trace a favorite cartoon character or picture on a small grid and reproduce it on a larger grid to make a larger figure similar to it.

Investigating the Ideas

Copy the red triangle on your graph paper.



?

Can you draw the figures that would result if the co-ordinates of A_1, B_1, and C_1 were doubled, graphed, and connected?

Discussing the Ideas

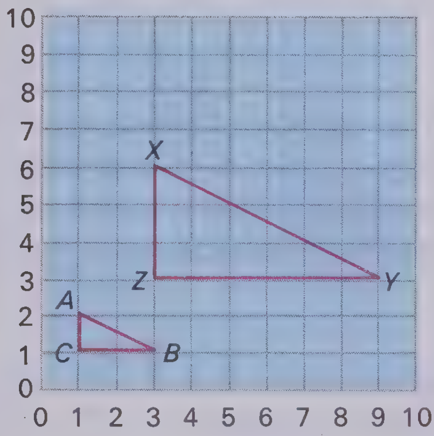
1. A What kind of figure do you get when you double the co-ordinates of A_1, B_1, and C_1 and connect them?
   B How do the sides of the new figure compare with the sides of $\triangle A_1B_1C_1$?
   C If we call the points with the doubled co-ordinates A_2, B_2, and C_2, how does the distance from (0,0) to A_2 compare with the distance from (0,0) to A_1?

2. What are the answers to the questions in exercise 1 when
   A the co-ordinates are tripled?
   B the co-ordinates are quadrupled?

3. We say that $\triangle A_2B_2C_2$ is a **magnification** from (0,0) of $\triangle A_1B_1C_1$, using scale factor 2. Can you describe the magnification from (0,0) of $\triangle A_1B_1C_1$, using scale factor 3?

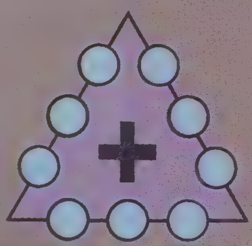
1. On your graph paper, copy these figures and show the magnification from (0,0) of:
   A the square, using scale factor 2
   B the segment, using scale factor $1\frac{1}{2}$
   C the triangle, using scale factor 3

2. $\triangle XYZ$ is a magnification from (0,0) of $\triangle ABC$, using scale factor 3.
   A The length of $\overline{XZ}$ is __?__ times the length of $\overline{AC}$.
   B The length of $\overline{ZY}$ is __?__ times the length of $\overline{CB}$.
   C The length of $\overline{XY}$ is __?__ times the length of $\overline{AB}$.



think

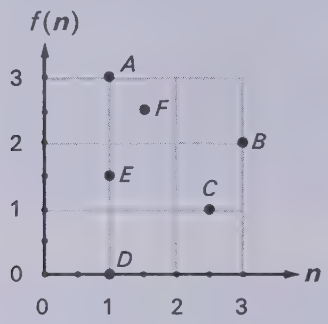
Can you copy the figure and write the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 in the circles so that the sum "on each side of the triangle" is the same?



3. In exercise 2, $\triangle ABC$ is similar to (the same shape as) $\triangle XYZ$. Show on a large sheet of graph paper a triangle similar to $\triangle ABC$ with matching sides 5 times as long.

4. Draw some interesting figures on your graph paper and use magnification to make figures similar to them.

1. Give the number pair for each letter in the graph.



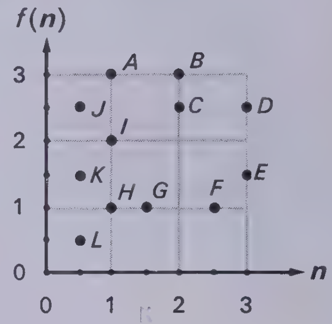
2. Give the letter for each number pair.

(2, 3)

($1\frac{1}{2}$, 1)

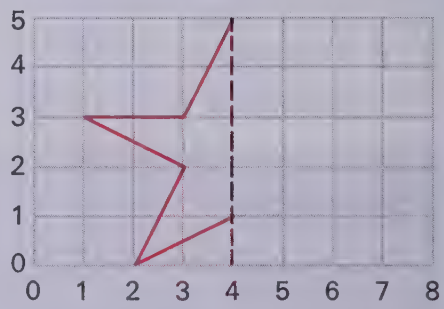
(3, $1\frac{1}{2}$)

($\frac{1}{2}$, $1\frac{1}{2}$)



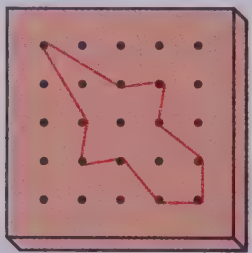
3. A Give the co-ordinates of the midpoint of the segment whose endpoints have co-ordinates (1,1) and (7,7).
   B Three corners of a rectangle are at (2,3), (2,6), and (8,6.) Give the co-ordinates of the fourth corner.

4. Copy this picture and complete the other half so that it will be symmetrical.



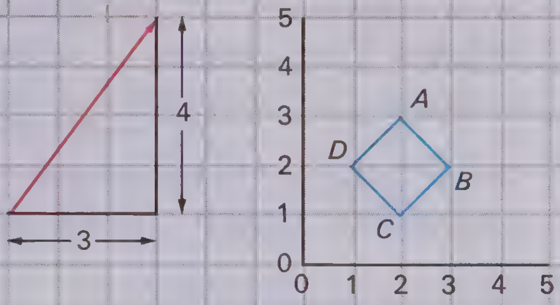
5. How many different lines of symmetry does a rectangle have?

6. Draw a picture on dot paper to show how this geoboard will look after it has been rotated 180°.



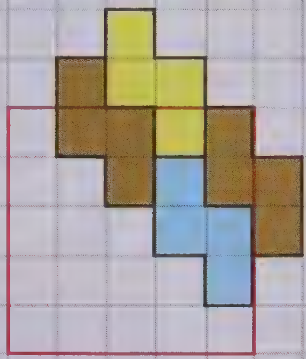
7. Draw a figure that has 180° rotational symmetry on dot paper.

8. Copy figure $ABCD$ on your graph paper and draw the figure in its final position after this translation.



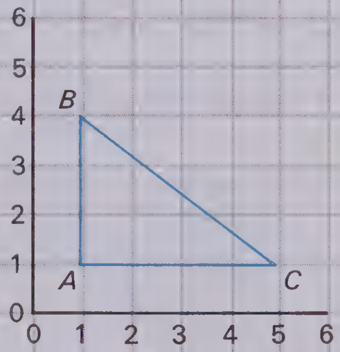
9. A Copy and complete the tessellation of the red square by using the suggested pattern.

B Color your tessellation so that the pattern stands out.

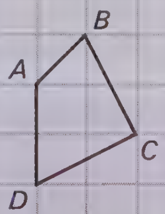


10. A Draw the magnification from (0,0) of $\triangle ABC$, using scale factor 3.

B How long are the sides of $\triangle ABC$? How long are the sides of the "magnified" triangle?



11. Draw a figure like $ABCD$ on graph paper. Then draw a larger, similar figure.



Keeping in Touch with Computing, Place value, Principles, Fractional numbers, Decimals

1. For the decimal 5 407 368.159283, give the number of:
   A tens C hundreds E thousands G hundred thousands
   B tenths D hundredths F thousandths H hundred thousandths

2. Round 3654.563 to the nearest:
   A hundredth B tenth C whole number D ten E hundred F thousand

3. Solve the equations.
   A $19 \times 1 = a$ D $(8 \times 7) \div 7 = t$ G $5 \times 23 = (5 \times n) + (5 \times 3)$
   B $28 + 0 = p$ E $63 - (n \times 7) = 0$ H $976 + 843 = n + 976$
   C $0 \div 5 = n$ F $10^2 \times 2^2 = b$ I $34 + (17 + 19) = (n + 17) + 19$

4. Compute the following.
   A $\begin{array}{r} 3872 \\ +5799 \\ \hline \end{array}$ B $\begin{array}{r} 537 \\ -69 \\ \hline \end{array}$ C $\begin{array}{r} 4007 \\ -2768 \\ \hline \end{array}$ D $\begin{array}{r} 356 \\ \times 47 \\ \hline \end{array}$ E $89\overline{)5976}$ F $16\overline{)35\,472}$

5. A List all the factors of 16. B List the multiples of 4 (to 28).

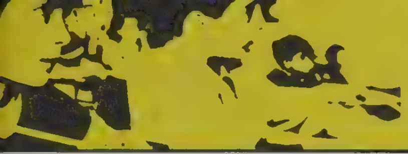
6. Which of these are prime numbers? 5, 7, 9, 21, 29, 33

7. List three fractions equivalent to $\frac{3}{5}$.

8. Give the lowest-terms fraction for each. A $\frac{9}{12}$ B $\frac{24}{100}$ C $\frac{20}{48}$

9. Find the sums and differences.
   A $\frac{3}{8} + \frac{1}{4}$ B $\frac{5}{8} + \frac{1}{6}$ C $\frac{4}{5} + \frac{2}{7}$ D $\frac{5}{6} - \frac{1}{4}$ E $\frac{5}{8} - \frac{3}{10}$
   F $\begin{array}{r} 17\frac{3}{8} \\ +15\frac{1}{2} \\ \hline \end{array}$ G $\begin{array}{r} 34\frac{1}{6} \\ +59\frac{1}{9} \\ \hline \end{array}$ H $\begin{array}{r} 54\frac{1}{4} \\ -21\frac{3}{8} \\ \hline \end{array}$ I $\begin{array}{r} 37\frac{1}{6} \\ -26\frac{2}{3} \\ \hline \end{array}$ J $\begin{array}{r} 51\frac{1}{2} \\ -39\frac{3}{5} \\ \hline \end{array}$

10. Find the products and quotients.
    A $\frac{3}{5} \times \frac{2}{3}$ C $5 \times 4\frac{1}{2}$ E $\frac{20}{24} \times \frac{6}{5}$ G $\frac{2}{3} \div \frac{1}{4}$ I $\frac{8}{5} \div \frac{2}{3}$
    B $\frac{1}{4} \times \frac{2}{5}$ D $3\frac{1}{3} \times 6$ F $1 \div \frac{5}{6}$ H $3 \div 27$ J $2\frac{1}{4} \times \frac{1}{3}$



11. Give the sign ($<$, $>$, $=$) for each ▒.

A 347 ▒ 357   B $4 \div 2$ ▒ $4 \div \frac{1}{2}$ C 5.763 ▒ 5.367

12. Find the average of the numbers in the set {9, 12, 17, 8}.

13. Find the total amounts.

A
$$\begin{array}{r} \$18.95 \\ 7.28 \\ 11.07 \\ \underline{28.36} \end{array}$$

B
$$\begin{array}{r} \$728.46 \\ 819.72 \\ \underline{728.55} \end{array}$$

14. Find the differences in the amounts.

A
$$\begin{array}{r} \$5076.42 \\ \underline{2748.59} \end{array}$$

B
$$\begin{array}{r} \$600.00 \\ \underline{129.95} \end{array}$$

15. Find the products and quotients.

A
$$\begin{array}{r} 3.14 \\ \underline{\times 8} \end{array}$$

B
$$\begin{array}{r} 1.783 \\ \underline{\times 0.24} \end{array}$$

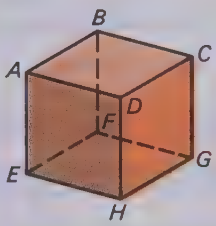
C $5\overline{)154.5}$

D $0.64\overline{)7.68}$

16. A Name a pair of parallel edges.

B Each edge is 2 units. What is the surface area? the volume?

C What is the perimeter of square *ABCD*?



17. Mr. Price can buy an airline ticket to Regina for $36.50
If he uses a family plan, he can take his child for $\frac{1}{2}$ price and his wife for $\frac{1}{4}$ off the regular price.

A How much would it cost for his child?

B How much would it cost for his wife?

C How much would it cost for all three?

think

Can you place a mathematical symbol between the digits 5 and 6 so that you have a name for a number that is between 5 and 6?

You are invited to explore

ACTIVITY CARD 14
Page 362

13 Ratio and Percent

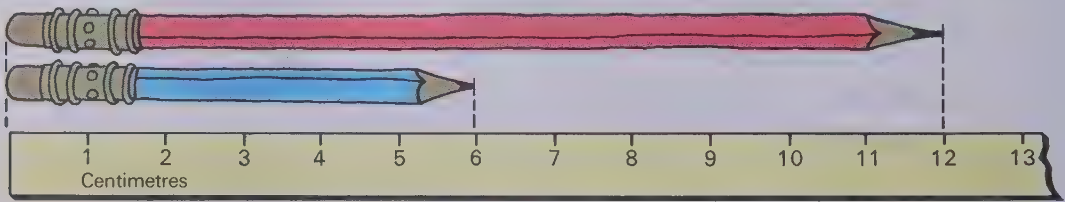
What is the meaning of ratio?

Investigating the Ideas

Since the purple strip is $\frac{1}{2}$ as long as the brown strip, we say the **ratio** of the purple strip to the brown strip is **1** to **2.** Since the purple strip is 4 units long and the brown strip is 8 units, we also say the ratio of the purple strip to the brown strip is **4 to 8.**

?	
How many other pairs of strips that have a ratio of 1 to 2 can you find?	To record your strips, give the ratio suggested by the unit marks on the strips.

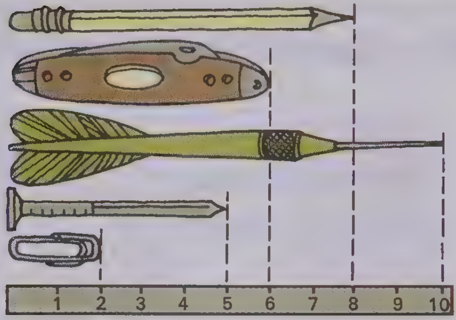
Discussing the Ideas



1. The ratio of the length of the red pencil to that of the blue pencil is 12 to 6, or 2 to 1. What is the ratio of the length of the blue pencil to that of the red pencil?

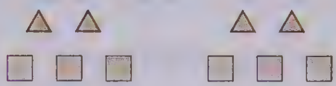
2. When we use ratios to compare the numbers of objects in two sets, we say:

The ratio of the number of **boys** to **girls** is **3** to **6.**
The ratio of the number of **boys** to **girls** is **1** to **2.**
Give two ratios that compare **girls** to **boys.**

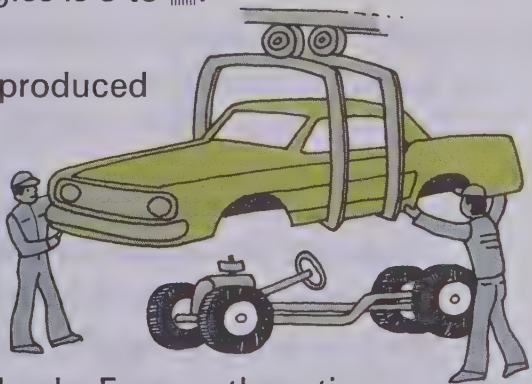


1. Give the ratio of these lengths.
   - A paper clip to nail
   - B nail to pencil
   - C knife to nail
   - D nail to paper clip
   - E pencil to nail

2. For each exercise, give the ratio of the lengths. Use two different pairs of numbers.
   - A dart to nail
     Answer: 10 to 5, 2 to 1
   - B paper clip to knife
   - C knife to pencil
   - D dart to knife
   - E knife to dart
   - F pencil to paper clip
   - G dart to paper clip

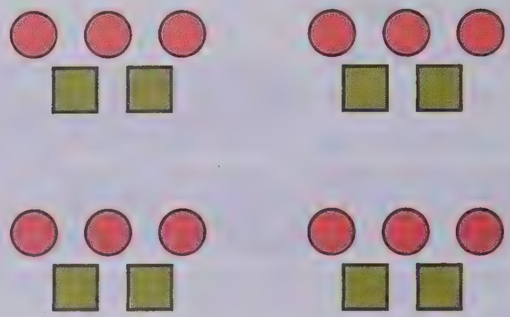


3. Give the missing numbers.
   - A The ratio of triangles to squares is 4 to ▒.
   - B The ratio of triangles to squares is 2 to ▒.
   - C The ratio of squares to triangles is 6 to ▒.
   - D The ratio of squares to triangles is 3 to ▒.



4. A In an auto factory, each car produced requires 5 tires. Give the ratio of cars to tires.
   - B Each car requires 2 headlights. Give the ratio of tires to headlights.
   - C Each 8-cylinder car has 4 wheels. Express the ratio of wheels to cylinders in two different ways.
   - D A machine makes 24 parts for an automobile in 8 hours. Give the ratio that compares parts with hours. How many parts does the machine make per hour?

Investigating the Ideas



There are 3 circles for every 2 squares.

The ratio of circles to squares is 3 to 2.

? Can you show some other groupings of these figures that suggest other ratios ?

Discussing the Ideas

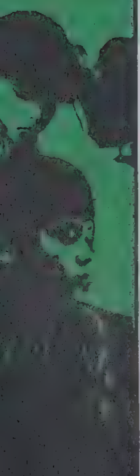
1. The Investigation shows that different pairs of numbers can be used to express the same ratio. Give other ratios that are the same as 3 to 2.

2. To express the ratio of the number of books to **children**, you can write 2 to **4** or you can write 1 to **2**.

   We often use a colon (:) to write a ratio.
   The ratio of books to **children** is 2 : **4**.
   The ratio of books to **children** is 1 : **2**.
   Since 2 : 4 and 1 : 2 are the same ratio, we write: 2 : 4 = 1 : 2
   Give three more ratios that are equal to 1 : 2.

3. Work with equal ratios is much like work with equivalent fractions. Explain how to find the missing numbers in A and B.

   A $\frac{3}{4} = \frac{\square}{12}$ B $3 : 4 = \square : 12$



1. Each table gives several number pairs for the same ratio. Give the missing numbers.

A

2 : 1
4 : 2
6 : $\square$
8 : $\square$
$\square$: 5

B

3 : 2
6 : 4
9 : $\square$
12 : $\square$
$\square$: 10

C

4 : 5
8 : $\square$
12 : $\square$
$\square$: 20
20 : $\square$

D

3 : 4
6 : $\square$
$\square$: 12
12 : $\square$
$\square$: 20

2. For the first part of each exercise, think about fractional numbers. For the second part, think about ratios. Find the missing numbers.

A $\frac{1}{2}=\frac{\square}{8}$
$1:2=\square:8$

B $\frac{3}{10}=\frac{9}{\square}$
$3:10=9:\square$

C $\frac{3}{4}=\frac{6}{\square}$
$3:4=6:\square$

D $\frac{\square}{3}=\frac{8}{6}$
$\square:3=8:6$

E $\frac{\square}{8}=\frac{15}{24}$
$\square:8=15:24$

F $\frac{2}{\square}=\frac{14}{21}$
$2:\square=14:21$

G $\frac{3}{\square}=\frac{12}{4}$
$3:\square=12:4$

H $\frac{\square}{2}=\frac{30}{20}$
$\square:2=30:20$

I $\frac{1}{2}=\frac{\square}{100}$
$1:2=\square:100$

J $\frac{5}{2}=\frac{25}{\square}$
$5:2=25:\square$

K $\frac{2}{\square}=\frac{10}{15}$
$2:\square=10:15$

L $\frac{10}{4}=\frac{\square}{40}$
$10:4=\square:40$

Short Stories RATIO

Write a fractional-number equation and solve it to answer these ratio problems.

1 Ratio of men to women, 4:5. 20 women. How many men? (Solution: $\frac{4}{5}=\frac{n}{20}$, $n=16$)

2 Ratio of batteries to flashlights, 2:1. 9 flashlights. How many batteries?

3 Ratio of scouts to tents, 3:1. 4 tents. How many scouts?

4 Ratio of candy bars to cents, 1:5. 3 candy bars. How many cents?

5 Ratio of length to width, 3:2. Length, 12. What is the width?

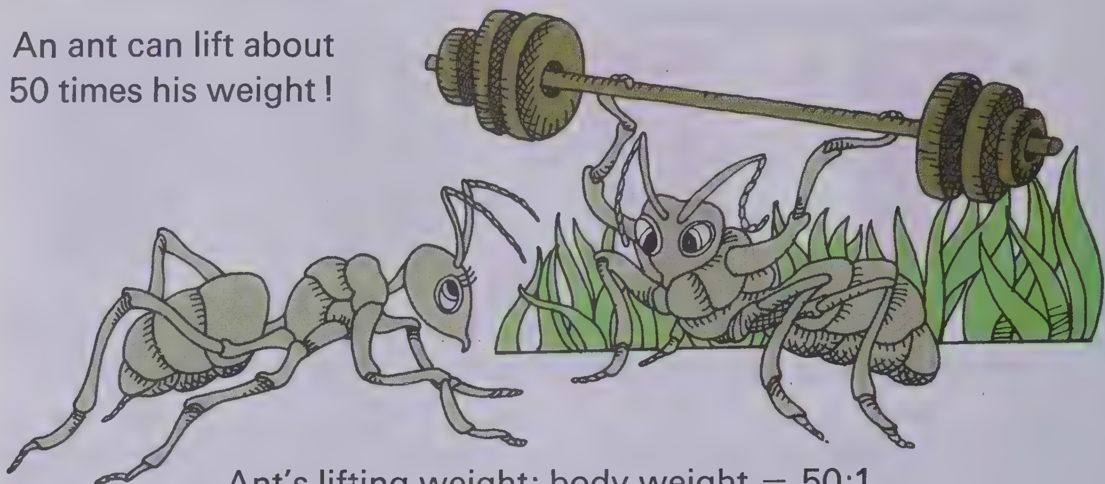
6 Ratio, kilometres to litres, 15:1. 9 litres. How many kilometres?

More practice, page A-23, Set 46

How can you use ratios in problem solving?

Investigating the Ideas

An ant can lift about
50 times his weight!



Ant's lifting weight: body weight = 50:1

? If you could lift as much for your weight as an ant can for his weight, how many kilograms could you lift?

Discussing the Ideas

1. Explain how you found the weight you could lift if the ant ratio were true for you.

2. Is the weight you found more than one tonne?

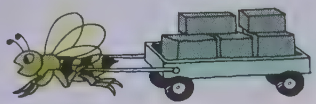
3. Suppose 20 000 ants weigh a total of one kilogram. If they could all lift together, how much weight could they lift?

4. Suppose that a man can safely lift twice his weight.
   - **A** What is the ratio of his "lifting weight" to his weight?
   - **B** How many kilograms should such a person weigh to lift 150 kilograms? 200 kilograms? 90 kilograms?

5. Suppose you could lift only 18 kilograms. What is the ratio of this weight to your body weight?

Ratio Fantasy

1. Assuming that for a man, as for an ant, the ratio of the weight lifted to body weight is 50 to 1, how much could a 90-kilogram man lift ? How many tonnes is this ?



2. The ratio of the weight a bee can pull (on wheels) to the bee's body weight is 300:1.
   - A If the ratio were the same for a man, how many kilograms could a 90-kg man pull ?
   - B How many tonnes is this ?
   - C How many kilograms could you pull at this ratio ?

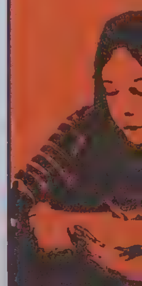
3. The ratio of a grasshopper's length to the distance he can jump is about 1:20. Use this ratio to find how many metres a 2-metre man could jump.

4. The ratio of a flea's body length to the distance he can jump is about 1:200. How far could you jump if you could jump in the 1:200 ratio ?

5. The ratio of the number of metres travelled to the number of minutes to travel that distance is 5:1 for the very slow giant tortoise of the Mauritius.
   - A How long would it take the tortoise to "run" the 100-metre dash ?
   - B How long would it take the tortoise to "run" the 400-metre dash ?

★6. The ratio of a champion athlete's height to the distance he can broad jump is 2:9.
   - A Given the same ratio, how far could a 2-cm cricket jump ?
   - B A cricket can actually jump about 60 cm. How many of the "short jumps" must he make to pass the 60-cm mark ?

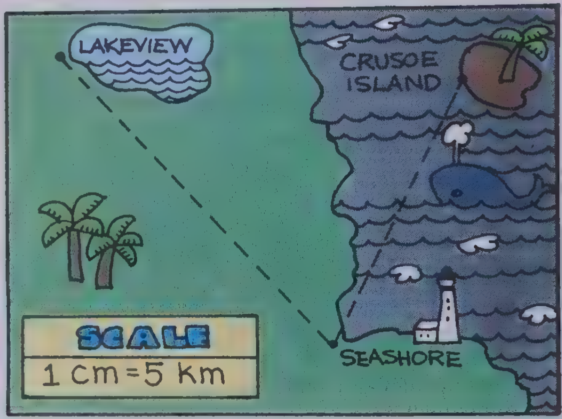
★7. In Diane's math class the ratio of girls to boys was 5 to 6. 3 new girls came into the class. Now there are as many girls as boys. How many students are now in the class ?



Investigating the Ideas

Maps that are drawn to scale give information about distances. 1 cm on this map represents an actual distance of 5 kilometres. We say the scale is:

1 centimetre = 5 kilometres

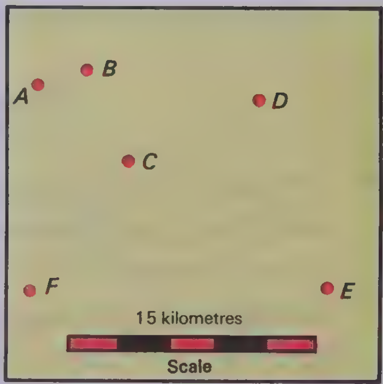


?

Can you use your ruler and the map to find the distance from Lakeview to Seashore and the distance from Seashore to Crusoe Island?

Discussing the Ideas

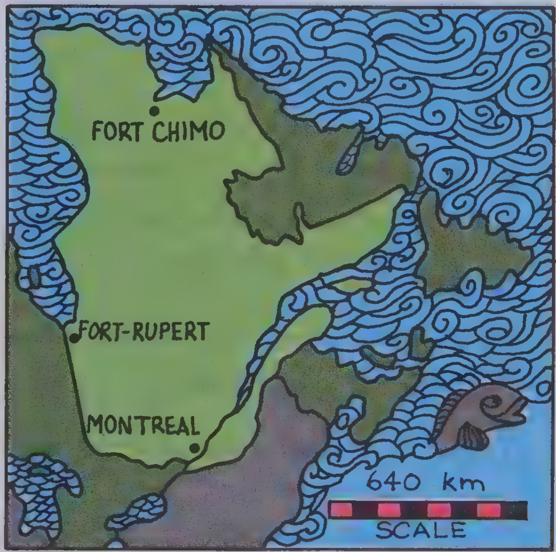
1. Explain how you used your ruler on the map to find distances.
2. How can you think of the scale on the map as a ratio?
3. Give the two distances for the map above if the scale is:
   - A $\frac{1}{4}$ cm = 8 km
   - B 1 mm = 4 km
   - C $\frac{2}{5}$ cm = 10 km
   - D $\frac{1}{2}$ cm = 20 km
4. Sometimes the scale for a map is shown as a segment that represents a given number of kilometres. On this map the segment is 30 mm and represents 15 kilometres. Explain how to find the following distances:
   - A City *A* to City *B*
   - B City *B* to City *C*
   - C City *C* to City *D*
   - D City *D* to City *E*
   - E City *E* to City *F*
   - F City *F* to City *D*



1. Use this map and scale to give the following distances:
   - A Newville to Clearlake
   - B Vista Villa to Clearlake
   - C El Rancho to Newville
   - D Vista Villa to El Rancho
   - E Clearlake to El Rancho

2. Where on the map would you show a city that is 18 km from Newville and between Newville and El Rancho?

3. Use the map and the scale to give the following distances:
   - A Fort Chimo—Fort Rupert
   - B Fort Rupert—Montreal
   - C Montreal—Fort Chimo



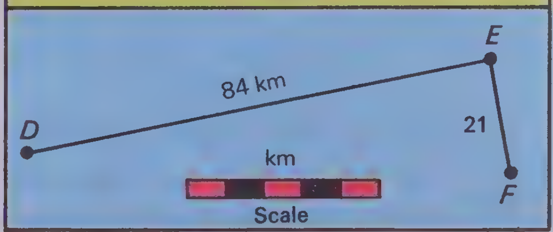
4. Toronto is about 560 km from Montreal. On a map where 1 cm = 60 km, how many centimetres would Toronto be from Montreal?

5. Study the maps carefully.
   - A Use your ruler to find the scale for map A.
   - B Use your ruler to find the scale for map B.

A

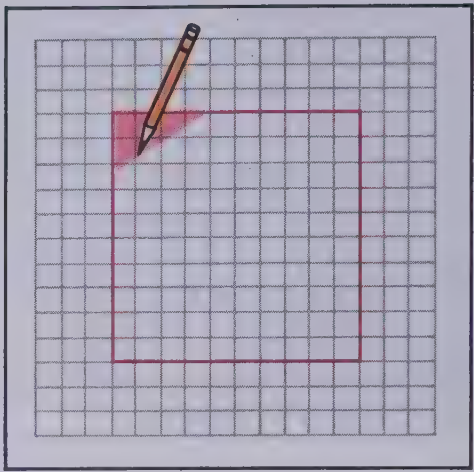
6. Draw your own map, using the scale 1 cm = 10 km. Show 3 cities, *A*, *B*, and *C*, and tell the distance between each pair.

B



Investigating the Ideas

Outline a 10-by-10 square region on graph paper.



? Can you color $\frac{1}{2}$ of the square red, $\frac{1}{4}$ blue, $\frac{1}{10}$ yellow, and the rest green?

Discussing the Ideas

1. A How many small squares make up the region in the Investigation?
   B How many did you color red? blue? yellow? green?
   C What is the ratio of red squares to all squares?

2. A special ratio which compares a number to 100 is called a **percent**. The symbol for percent is %.
   Examples: 50% means 50:100, $\frac{50}{100}$, or .50.
   13% means 13:100, $\frac{13}{100}$, or .13.
   A In the Investigation, what percent of the square region was colored blue?
   B One-tenth of the square region was colored yellow. What percent was colored yellow?
   C What percent of the square region did you color green?

1. Study the first row of the chart to see how the percent symbol (%) is used. Give the missing symbols for the rest of the table.

Region	Fraction: Lowest terms	Fraction: Hundredths	Decimal	Percent
	$\frac{3}{4}$	$\frac{75}{100}$	0.75	75%
	$\frac{1}{4}$	$\frac{25}{100}$	0.20	25%
	$\frac{1}{5}$	$\frac{20}{100}$	.20	B
	$\frac{3}{5}$	$\frac{60}{100}$	C	D
	$\frac{7}{10}$	E	F	G

2. Give the missing numerators.

A $25\% = \frac{▯}{100}$ B $20\% = \frac{▯}{100}$ C $80\% = \frac{▯}{100}$ D $10\% = \frac{▯}{100}$

3. Give the correct percent.

A $\frac{25}{100} =$ ▯ C $\frac{15}{100} =$ ▯

B $\frac{75}{100} =$ ▯ D $\frac{18}{100} =$ ▯

4. Give the percent for each exercise.

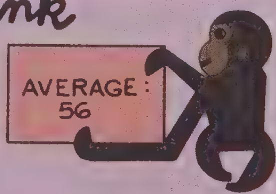
A Since $\frac{3}{5} = \frac{60}{100}$, we know $\frac{3}{5} =$ ▯.

B Since $\frac{9}{10} = \frac{90}{100}$, we know $\frac{9}{10} =$ ▯.

C Since $\frac{1}{4} = \frac{25}{100}$, we know $\frac{1}{4} =$ ▯.

D Since $\frac{1}{20} = \frac{5}{100}$, we know $\frac{1}{20} =$ ▯.

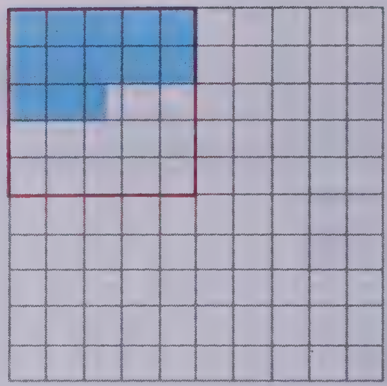
think



The average of 8 numbers is 56. Three of the numbers are 72, 57, and 49. What is the average of the other 5 numbers?

Investigating the Ideas

One eighth of the large square region is shaded. How many small squares make up $\frac{1}{8}$ of the large square?



? Can you show some other ways to shade $\frac{1}{8}$ of a 10-by-10 square?

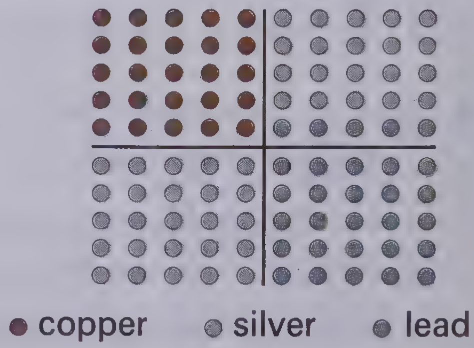
Discussing the Ideas

1. The shaded region above is $12\frac{1}{2}$ small squares. What percent of the large square is shaded?

2. A Since $\frac{1}{3} = 0.33\frac{1}{3}$, how many small squares would you have to shade if you wanted to shade $\frac{1}{3}$ of a 10-by-10 square?
   B What percent of the large square would be shaded?

3. A Each small square is what percent of the 10-by-10 square?
   B How would you shade $\frac{1}{2}\%$ of the large square?

4. A What percent of the coins are copper?
   B What percent are lead?
   C What percent are silver?
   D What percent are **not** lead?



1. Find the numbers for a, b, and c.

A Since $\frac{2}{8} = \frac{1}{4} = 25\%$, we know that $\frac{1}{8} = a\%$.

B Since $\frac{1}{8} = a\%$ and $\frac{2}{8} = b\%$, we know that $\frac{3}{8} = c\%$.

C Since $\frac{4}{8} = a\%$ and $\frac{1}{8} = b\%$, we know that $\frac{5}{8} = c\%$.

D Since $\frac{8}{8} = a\%$ and $\frac{1}{8} = b\%$, we know that $\frac{7}{8} = c\%$.

E Since $\frac{3}{3} = a\%$, we know that $\frac{1}{3} = b\%$.

F Since $\frac{1}{3} = a\%$, we know that $\frac{2}{3} = b\%$.

★ G Since $1.25 = \frac{125}{100}$, we know that $1.25 = a\%$.

★ H Since $2 = \frac{200}{100}$, we know that $2 = a\%$.

★ I Since $\frac{1}{6} = 0.16\frac{2}{3}$, we know that $\frac{1}{6} = a$

★ J Since $\frac{1}{8} = 12\frac{1}{2}\%$, we know that $\frac{1}{16} = a\%$.

2. Copy each table and give the missing percents.

Fraction	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{0}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{5}$
Percent	A	B	C	D	E	F	G	H	I

Fraction	$\frac{0}{8}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$	$\frac{8}{8}$
Percent	J	K	L	M	N	O	P	Q	R

Fraction	$\frac{1}{3}$	$\frac{2}{3}$
Percent	S	T

$\frac{1}{6}$	$\frac{5}{6}$
★ U	★ V

think

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. What percent of the whole numbers from 1 through 100

A have 2 as a factor?

B have 5 as a factor?

C are prime numbers?

2. What percent of these numbers are prime?

A 1 through 50 B 51 through 100

Let's solve some percent problems.

Investigating the Ideas

Your body is about $66\frac{2}{3}\%$ water.

? Can you change this percent to a decimal or a fraction and then find out how many kilograms of your body are water?

Discussing the Ideas

When you work with percent problems, you use either fractions or decimals to do the computing. You will find that sometimes it is easier to use a decimal, while other times it is easier to use a fraction. In each problem below a decimal and a fraction are given for the percent. Tell which one you would use to solve the problem and explain why.

1. A man saved 25% of \$848. How much did he save?
   $25\% = 0.25 = \frac{1}{4}$
2. 44% of the people in a small town are adults. There are 725 people. How many adults? $44\% = 0.44 = \frac{11}{25}$
3. In a basketball game, Tom made exactly $37\frac{1}{2}\%$ of the shots he attempted. He attempted 24 shots. How many shots did he make? $37\frac{1}{2}\% = 0.375 = \frac{3}{8}$
4. About 68% of 97 square kilometres is covered by water. How much of the area is covered with water? $68\% = 0.68 = \frac{17}{25}$
5. A man paid 24% of his yearly income in taxes. His income was \$8357. How much tax did he pay? $24\% = 0.24 = \frac{6}{25}$

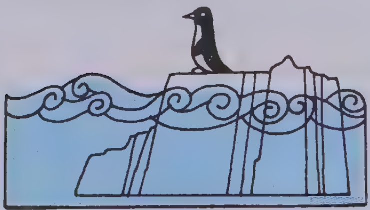
Short Stories Water

1. Earth's surface: 75% water.
   A What fraction is this?
   B What percent of Earth's surface is land?

2. Ocean: water and dissolved minerals. Minerals: $3\frac{1}{2}$%. What percent water?

3. A certain type of rock is 3% water. How much water in a 45-kg rock of this type?

4. Blood: 90% water.
   A Adult: 4.75 litres of blood. How much water?
   B Child: 2.38 litres of blood. How much water?



5. Floating ice: $12\frac{1}{2}$% above the surface.
   A What percent is below the surface?
   B What fraction is above the surface?

6. When water freezes, its volume increases by about 9%. Freeze 500 cubic centimetres of water.
   A How much does the volume increase?
   B What is the volume of the ice?

7. A Milk: $87\frac{1}{2}$% water. What fraction is this?
   B Boiled potato: 80% water. What fraction is this?
   C Fried chicken: 52% water. Give the lowest-terms fraction for this percent.
   D Egg: about $\frac{3}{4}$ water. What percent is this?
   E Bread: 36% water. How many grams of water in a 480-gram loaf?
   F Apple: 85% water. How many kilograms of water in 10 kilograms of apples?

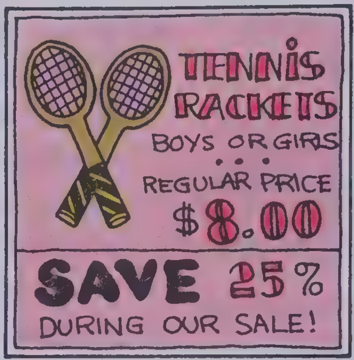
★ 8. About $\frac{1}{30}$% of all water on Earth evaporates each year. Give a fraction for this percent.

★ 9. When water changes to steam, its volume increases by 167 000%. What is the volume of the steam from 1 cubic centimetre of water?

More practice, page A-24, Set 47

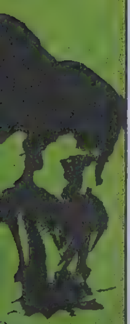
● *How can you find the amount of discount and the sale price?*

Investigating the Ideas



? Can you find how much you would save by buying one of the items on sale?

Discussing the Ideas



1. The amount you save by buying on sale is called the **amount of discount**. The percent off the regular price is the **discount rate.**
   - A Which of the **discount rates** above is largest?
   - B Which item has the largest amount of discount, the bicycle or the record player?

2. What fraction is the discount rate for the record player?

3. Explain how you would find the **sale price** of the bicycle.

Reg. Price	$75	Sale Price
Discount Rate	×0.20	?
Discount	$15.00	

4. How would you find the sale price of a tennis racket?

1. Study the first two rows of the table. Then give the missing entries.

Original price	Percent discount	Amount of discount	Sale price
\$10.00	10%	\$1.00	\$9.00
\$10.00	25%	\$2.50	\$7.50
\$ 5.00	10%	\$0.50	A
\$ 5.00	25%	\$1.25	B
\$12.00	$33\frac{1}{3}$%	C	D
\$ 8.00	20%	E	F
\$16.00	$12\frac{1}{2}$%	G	H
\$30.00	15%	I	J

2. Each of the following is an original price. Give the sale price if the discount is 10%.

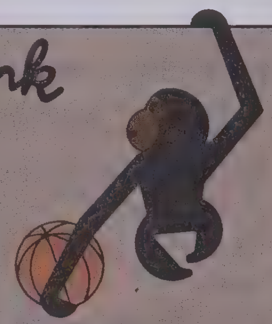
A \$10.00 C \$50.00 E \$6.40
B \$4.00 D \$25.00 F \$1.80

3. Give the sale price for each part of exercise 2 if the discount is 25%.

4. Use the given price and percent discount to find the sale price.

A \$10.00; 5% D \$25.00; 50%
B \$5.00; 20% E \$50.00; 40%
C \$9.00; 20% F \$36.00; $33\frac{1}{3}$%

think



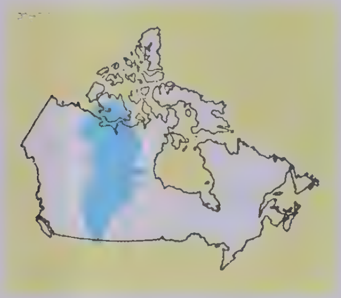
A basketball team won 4 more games than it lost. If it won $\frac{3}{5}$ of its games, how many games did it play?

★ 5. Which store will sell the dress at the lowest price? What is the sale price?

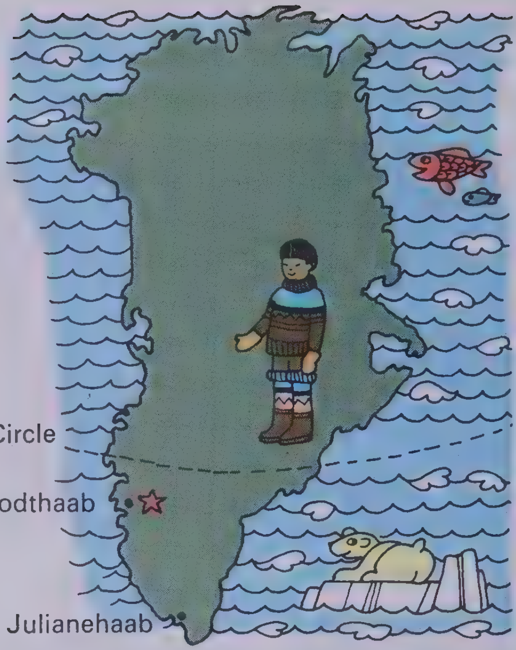
Solving Story Problems

GREENLAND

Greenland is the world's largest island. It is a county of Denmark, even though it is separated from Denmark by 2080 km of ocean.



Greenland is about 20% of the size of Canada.

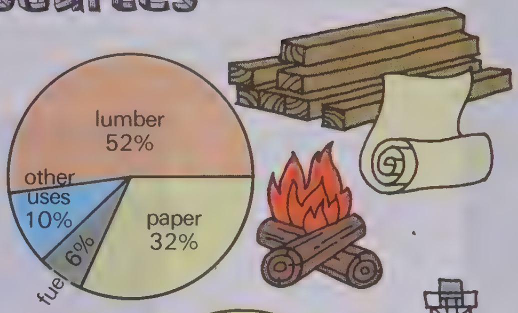


1. Greenland is about 20% of the size of Canada.
   The area of Canada is 9 975 800 square kilometres.
   About what is the area of Greenland ?
2. The area of Denmark is about 2% of the area of Greenland.
   About what is the area of Denmark ?
3. The ratio of the population of Greenland to the population of Denmark is 1:150. If the population of Denmark is 4 581 000, about what is the population of Greenland ?
4. About 85% of Greenland is covered with ice.
   About how many square kilometres is that ?
5. Greenland is 2670 kilometres from north to south. The northern tip of Greenland is just 716 kilometres from the North Pole. It is the world's northernmost land. About how far is the southern tip of Greenland from the North Pole ?
6. Most of Greenland lies above the Arctic Circle. Look at the map above and estimate the percent of Greenland that lies above the Arctic Circle.

Solving Story Problems

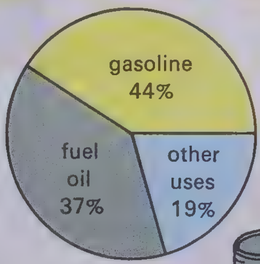
Using Our Natural Resources

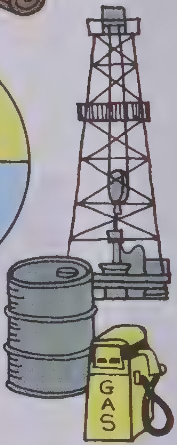
1. The circle graph shows approximate percents of three major uses of wood.



   A The amount of wood used for lumber is about how many times the amount used for paper ?

   B If wood were measured by the kilogram, how many kilograms of each tonne of wood are used for fuel ?





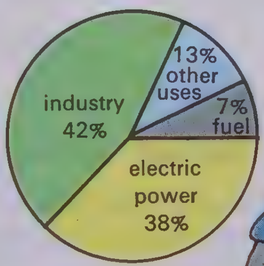
2. This graph shows the approximate percent of petroleum used for two products.

   A An average oil well produces about 2160 litres of oil a day. According to the graph, how many of those litres are used for gasoline ?

   B Oil is measured by the barrel. A barrel contains 159 litres. According to the graph, about how much petroleum from each barrel is used for fuel oil ?

   C About 4 litres of each barrel of petroleum is used for jet fuel. Is this less than 1%, about 2%, or about 20% ?

3. The graph shows approximate percents for some common uses of coal.



   A Industry and production of electric power together use what percent of the total amount of coal produced ?

   B Of each tonne of coal, how many kilograms are needed for fuel ?

   C Railroads use about 10 kilograms of each tonne of coal mined. What percent is that ?

What is the meaning of interest?

Investigating the Ideas

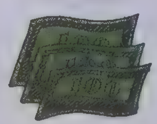
? Can you find how much the bank would earn all together in these two transactions?

Discussing the Ideas

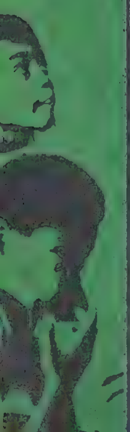
1. When you put your money in a savings account, the money the bank pays you for the use of your money is called **interest**. How much interest would $100 earn at a 5% yearly **interest rate**?

2. When you borrow you must pay interest for using the bank's money. How much interest would you pay for $100 if the yearly interest rate for loans were 10%?

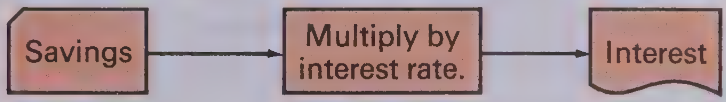
3. How would you find the interest on these savings?

Save	Yearly interest rate	Amount of interest
 for one year.	4%	?

4. How much money would you have at the end of one year?



1. Follow the interest flow chart to find the amount of interest for each amount of savings and rate of interest.



A Savings: \$200 Rate: 4%

B Savings: \$500 Rate: 5%

C Savings: \$1000 Rate: 6%

D Savings: \$700 Rate: 3%

2. Study the first row of the table. Then give the missing entries for the rest of the table.

Savings	Interest rate (per year)	Amount of interest in one year	Savings + interest
\$100.00	2%	\$2.00	\$102.00
\$ 10.00	4%	A	B
\$100.00	3%	C	D
\$ 50.00	2%	E	F
\$ 25.00	5%	G	H
\$ 76.00	4%	I	J
\$124.00	$3\frac{1}{2}$%	★ K	L

★ **3.** Suppose a man invests \$1000 at 5 percent per year. At the end of each year, he leaves his interest in the account to draw interest the next year. How much would he have in his account at the end of 3 years?

More practice, page A-24, Set 48

Discussing the Ideas

1. Three types of simple equations are given in the examples. To solve example **A**, you must find the product of the two factors. In examples **B** and **C**, the products are given and you must find the missing factors.

A	B	C
$5 \times 7 = n$	$5 \times n = 35$	$n \times 7 = 35$

Complete the sentences.

A You can solve **A** if you multiply 5 by ▒▒.

B You can solve **B** if you divide 35 by ▒▒.

C You can solve **C** if you __?__ 35 by 7.

2. Problems using percents are solved much the same as the examples in 1 above. Give the missing numbers.

A To solve $8 \times 25\% = n$, you multiply 8 by ▒▒.

B To solve $n \times 25\% = 2$, you divide 2 by ▒▒.

C To solve $8 \times n = 2$, you divide ▒▒ by 8.

3. How can you express the number for n in **2c** as a percent?

4. It is often helpful to write an equation before attempting to solve a percent problem. Study example **A**, then give the number for n.

5. Study example **B**. Then give the missing percent. Note that you must change the fraction to a percent.

A 10% of what number is 7?

$$10\% \times n = 7$$
$$n = 7 \div 10\%$$
$$n = 7 \div \tfrac{1}{10}$$
$$n = ▒▒$$

B What percent of 12 is 9?

$$n \times 12 = 9$$
$$n = \tfrac{9}{12} = \tfrac{3}{4}$$
$$n = ▒▒\%$$

1. Give the missing words and numbers.
   - A To solve $6 \times 7 = n$, you multiply 6 by ▒.
   - B To solve $9 \times n = 54$, you divide 54 by ▒.
   - C To solve $n \times 4 = 48$, you __?__ 48 by 4.
   - D To solve $30 \times 10\% = n$, you __?__ 30 by 10%.
   - E To solve $n \times 10\% = 3$, you __?__ 3 by 10%.
   - F To solve $n \times 50\% = 4$, you divide 4 by ▒.

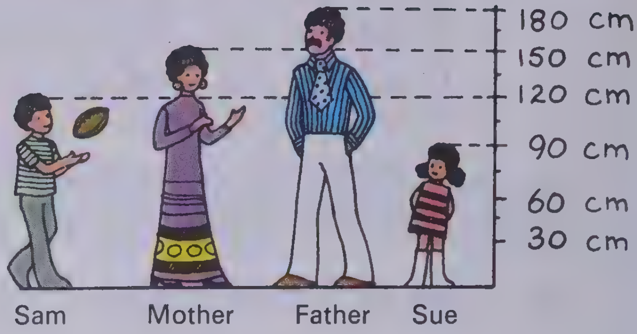
2. Solve each of the equations in exercise 1.

3. Solve the equations. When your solution is a decimal or a fraction, give a percent for the solution.
   - A $10 \times n = 2$
     Answer: $n = 20\%$
   - B $12 \times n = 3$
   - C $n \times 10\% = 12$
   - D $60 \times n = 15$
   - E $20 \times n = 10$
   - F $n \times 60\% = 12$
   - G $n \times 25\% = 7$
   - H $16 \times n = 2$

For each exercise below, write and solve an equation. Be sure to give a percent when you are asked to.

4. 25% of what number is 8?
5. What percent of 25 is 5?
6. 20% of what number equals 9?
7. What percent of 50 is 40?
8. 10% of what number is 15?
9. What percent of 60 is 45?
10. $12\frac{1}{2}\%$ of what number is 6?
11. What percent of 8 is 1?
12. 50% of what number is $7\frac{1}{2}$?
13. What percent of 4 is 3?
14. The sign in the store window said 20% off on all items. Tom figured that this sale would save him \$2 on the football he wanted. What was the original cost of the football?
15. Jane bought a \$12 mini skirt. It was on sale for \$9. What was the percent of discount?
16. Tim had \$20. If he spent \$2, what percent of his money did he spend?

1. Give the following ratios.
   - A Sam's height to Sue's
   - B Father's height to Mother's
   - C Sue's height to Father's
   - D Sam's height to Father's



2. Copy the table and give the missing numbers. Each pair should give the same ratio.

4:3
8:____
____:9
16:____
____:30

3. Give the missing numbers.
   - A 2:1 = 4:____
   - B 2:____ = 8:12
   - C 4:1 = ____:3
   - D 5:2 = 10:____
   - E 3:____ = 30:40
   - F ____:8 = 77:88

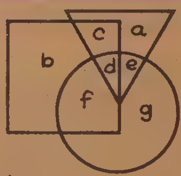
4. Give the correct percent for each decimal.
   - A 0.75 B 0.70 C 0.07 D 0.23 E 1.00

5. Give the lowest-terms fraction for each percent.
   - A 90% B 80% C $87\frac{1}{2}$% D 36% E 45%

6. Compute.
   - A 10% of 70
   - B 25% of 20
   - C 30% of 40
   - D 25% of 100
   - E $33\frac{1}{3}$% of 36
   - F 75% of 4
   - G 20% of 50
   - H 50% of 20
   - I 40% of 10
   - J 10% of 40

★ 7. Solve the equations.
   - A $n \times 10\% = 10$
   - B $n \times 10\% = 1$
   - C $n \times 10\% = 2$
   - D $n \times 25\% = 8$
   - E $n \times 33\frac{1}{3}\% = 12$
   - F $n \times 50\% = 18$
   - G $n \times 20\% = 20$
   - H $n \times 40\% = 80$

think



Give the letter or letters that are:
- A in □ or ○ but not ▽.
- B in both □ and ○ but not ▽.
- C in □ and ○ and ▽.
- D in both □ and ▽ but not ○.

★ **8.** Give the percent for ***n***.

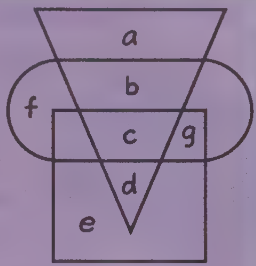
A $8 \times n = 2$
B $8 \times n = 4$
C $8 \times n = 1$
D $80 \times n = 8$
E $80 \times n = 4$
F $80 \times n = 10$

9. If the discount is 20%, give the sale price for each of the following.

A $10.00
B $5.00
C $1.00
D $4.00
E $15.00
F $100.00
G $25.00
H $17.00

think

Example:
d is in ▽ and □ but not ⬭.

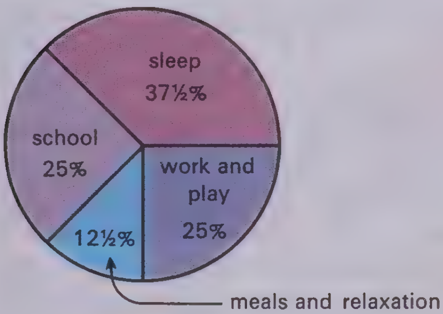


A Describe the location of ***b***.
B Describe the location of ***c***.
C Describe the location of ***e***.
D Describe the location of ***g***.

10. For the following amounts, give the amount of interest earned in one year at 4% per year.

A $100　B $10　C $50　D $25　E $15

11.



The circle graph shows how Bill might spend an average 24-hour period. Give the numbers of hours for each of the four activities.

12. In Alice's class, $33\frac{1}{3}\%$ of the children ride the bus. If there are 36 children in the class, how many ride the bus?

13. Robert said, "$\frac{1}{8}$ of the children in this school are in my class." What percent of the children are in Robert's class?

14. Pam said, "I spend 9 out of 12 months in school." What percent of the months does Pam spend in school?

★ **15.** Tim brought home 6 boxes of strawberries. He said, "This is 10% of what I picked." How many boxes of strawberries did Tim pick?

Keeping in Touch with Computing, Measurement, Geometry, Fractional numbers

1. Give the correct sign (<, >, =) for each ▒.

A $\frac{1}{3}$ ▒ 0.334 C 56 754 ▒ 57 654 E $3\frac{1}{8}$ ▒ 3.25

B $\frac{1}{2}$ ▒ $\frac{1}{4}$ D 3.07 ▒ 3.007 F $2\frac{5}{8}$ ▒ 2.50

2. Solve the equations.

A $587+395=n$ C $324\times 32=n$ E $617+n=1076$ G $21\times n=1827$

B $803-695=n$ D $1421\div 53=n$ F $n-374=297$ H $n\div 45=19$

3. Solve the equations.

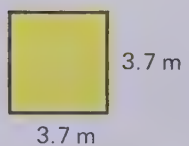
A $\frac{3}{4}+\frac{1}{6}=n$ D $3\frac{1}{2}-1\frac{3}{4}=n$ G $5\times 3\frac{1}{2}=n$ J $2\div\frac{3}{5}=n$

B $2\frac{1}{3}+4\frac{1}{2}=n$ E $\frac{1}{3}\times\frac{3}{5}=n$ H $2\frac{1}{3}\times 1\frac{2}{5}=n$ K $\frac{1}{2}\div\frac{3}{5}=n$

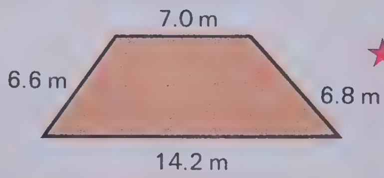
C $\frac{7}{8}-\frac{2}{5}=n$ F $\frac{3}{8}\times\frac{2}{3}=n$ I $1\div\frac{3}{5}=n$ L $\frac{3}{8}\div\frac{5}{8}=n$

4. Find the perimeter of each figure.

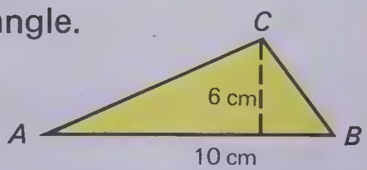
A



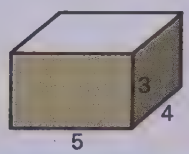
B



5. Find the area of this triangle.



6. Find the volume and surface area of this figure.



★ 7. This table is about squares. Find the missing numbers.

Perimeter	Area
8	A
4	B
C	16
D	49
10	E

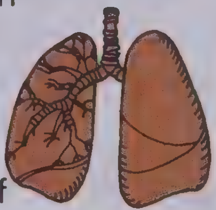
You are invited to explore

ACTIVITY CARD 15
Page 362

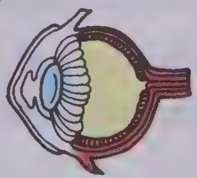
THE HUMAN BODY

1. Your body contains 206 bones. About $\frac{1}{7}$ of these bones are in your head. About how many bones are there in your head ?

2. Muscles make up about 0.4 of a man's body weight. What do the muscles of an 80-kg man weigh ?

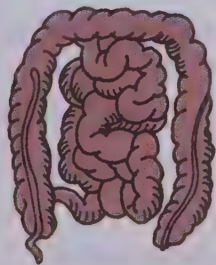


3. In one day, a man breathes in about 11.4 cubic metres of air. About 0.05 of this is oxygen that is absorbed into the blood stream. How many cubic metres of oxygen is this ?

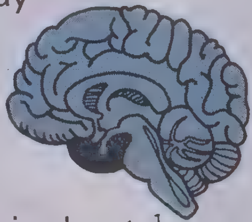


4. An eye blinks about 25 times each minute. About how many times does it blink in a day (24 hours) ?

5. Your intestines (large and small) are about 7.6 m long. Your small intestine is about $\frac{4}{5}$ of this length. How long is your small intestine ?

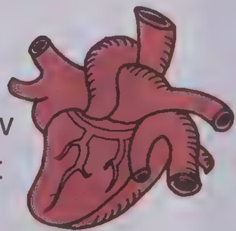


6. A man's brain is between 0.02 and 0.03 of his body weight. Between what two numbers is the brain weight of a 70-kg man ?



7. The brain of a baby is about $\frac{1}{10}$ its body weight. About how many grams does the brain of a 3.7-kg baby weigh ?

8. Your heart beats about 80 times a minute. About how many times does it beat in a day ?



9. The body of an adult contains about 4.7 litres of blood. A blood donor usually gives 470 ml of blood. What fraction of his blood does the donor give ?

10. Your body contains about 160 900 km of blood vessels. Write this number as a power of ten.

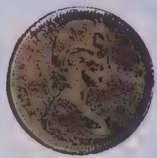
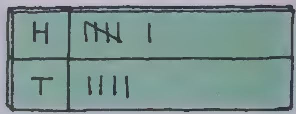
11. The liver is the largest gland in your body. In a 66.5-kg person, the liver is about $\frac{1}{35}$ his body weight. How many grams would that be ?

14 Probability

Let's explore some probability experiments.

Investigating the Ideas

Here is a probability experiment for you to try.

Experiment	Possible Outcomes		Tally
 Toss a penny.	 Tails	 Heads	 Keep a tally of the outcomes.

? How many times do you think you will have to flip a penny before you get 50 heads?

Try it and record your results.

Discussing the Ideas

1. A What fractional part of the tosses of a coin should come up "heads"?
   B What part should come up "tails"?
   C Do you think you will always get the same number of heads as tails when you toss a coin several times?

2. Predict, as closely as you can, the number of heads and tails that you think will appear after
   A 10 tosses. B 20 tosses. C 50 tosses. D 1000 tosses.
   E your classmates each toss a penny once, and the results are combined.

   Give reasons, if any, for your predictions.

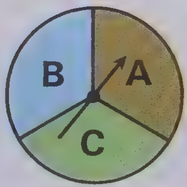
Using the Ideas

1. Complete a table like the one below for the coin-tossing experiment. Use the combined results of your class.

Total number of heads	Total number of tails	Total number of tosses

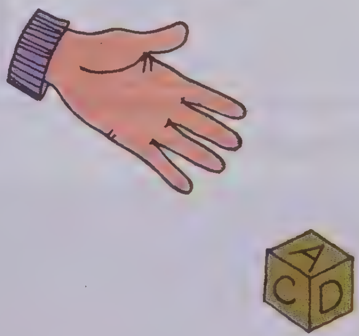
2. Use the table in exercise 1.
   A Write the ratio: $\frac{\text{total number of heads}}{\text{total number of tosses}}$
   B Is this ratio more than $\frac{1}{2}$ or less than $\frac{1}{2}$?

3. The spinner is divided into 3 parts of equal size. If you spin the arrow several times, for what fractional part of the spins will the arrow probably stop in region **B**?



4. How many times do you think you would have to spin the arrow to get 10 "**B**" outcomes?

5. Suppose a cube has the letters **A, B, C, D, E,** and **F** on its faces. If you toss the cube several times, for what fractional part of the tosses should the cube land with the face lettered **C** on top?

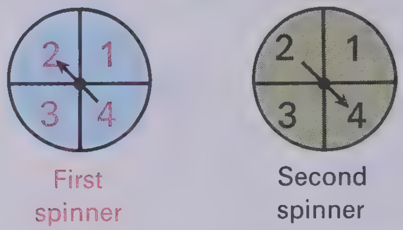


6. Predict, as closely as you can, the number of times each letter on the cube in exercise 5 should appear after
   A 24 tosses. B 48 tosses. C 72 tosses. D 600 tosses.

Investigating the Ideas

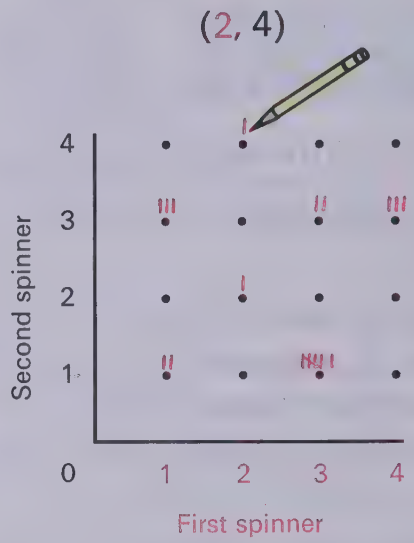
A

Spin two spinners like these 100 times.



B

For each spin, give the outcome as an ordered pair.



C

Tally the outcomes on a co-ordinate grid.

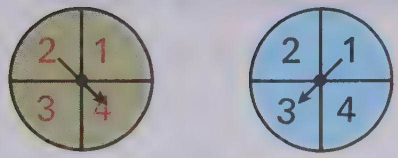
?	Can you use spinners like these to carry out this experiment?	For 100 spins, tally the number of times each number pair occurs.

Discussing the Ideas

1. A How many different outcomes can this experiment have?
   B Do you think each outcome has the same chance of happening?
   C For what fraction of the spins should each outcome occur?

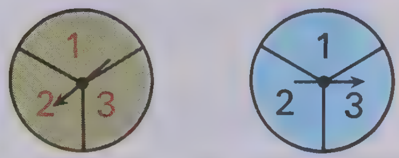
2. A Tabulate the combined results of your class for this experiment on a co-ordinate grid like the one above.
   B Did each outcome occur about the same number of times?

1. A Write the ordered pair that represents the outcome shown here.
   B How many times did this outcome appear in your class experiment?
   C Did this outcome appear for more than, less than, or exactly $\frac{1}{16}$ of the number of spins?



First spinner   Second spinner

2. An experiment consists of spinning the pointers on both spinners and recording the outcomes. The outcome shown is (2, 3).



First spinner   Second spinner

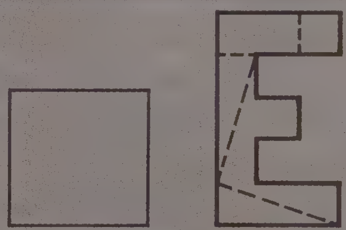
   A List all of the possible outcomes for this experiment.
   B For what fraction of the spins should outcome (2, 3) occur?

3. Suppose a penny and a nickel are tossed at the same time.
   A Complete the table of outcomes.
   B How many outcomes are possible?
   C If both coins are tossed several times, for what part of the time would you expect to get heads on both coins?
   D For what part of the time would you expect the coins not to "match"?

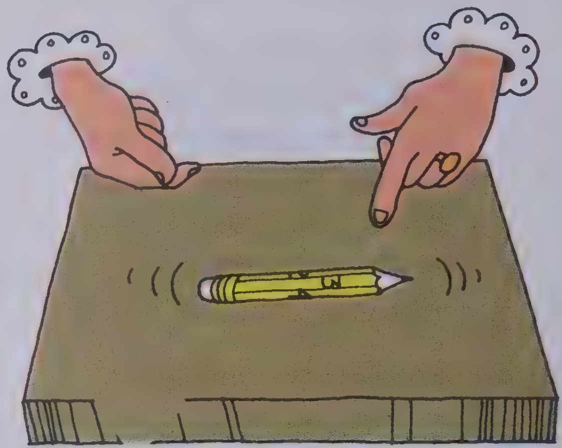
Outcomes	
Nickel	Penny
Heads	Heads

think

Trace the letter E and cut along the dotted lines. Then paste these pieces so that they exactly cover a tracing of the square.



Investigating the Ideas



Label each side of a six-sided pencil with one of the digits 1, 2, 3, 4, 5, 6. Roll the pencil along a table and record the top number when it stops.

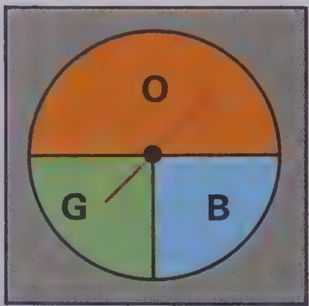
> **?** How close can you come to predicting the number of rolls you must make before recording ten 2's?

Discussing the Ideas

1. If all possible outcomes of an experiment have the same chance of occurring, we say these outcomes are **equally likely.** Do you think the outcomes in the Investigation are equally likely?

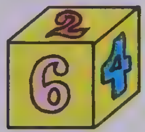
2. Explain how you made your prediction in the Investigation.

3. A What are the possible outcomes for the spinner if "liners" are not counted?
   B Do you think the outcomes are equally likely?
   C Which outcome do you think will occur most often?



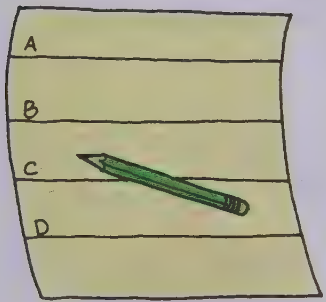
For each probability experiment below, tell whether or not the outcomes are equally likely. If the outcomes are not equally likely, tell which outcome is most likely to occur.

1. Toss a cube with faces numbered 1 to 6. Record the number on the top face.

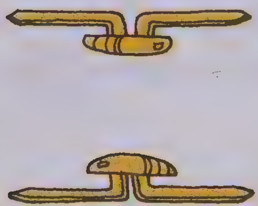


2. Open a dictionary at random and choose the last digit of the page numeral.

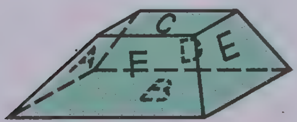
3. Drop a 20-cm pencil on a paper with lines drawn 7.5 cm apart. Record whether or not the pencil touches a line.



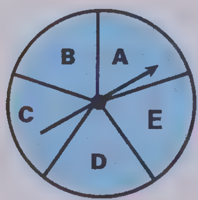
4. Open a paper fastener as shown here, and toss it onto your desk. Record which way it lands.



5. Toss an object like this and record the letter on the top face.

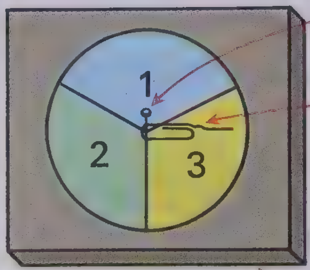


6. Spin the arrow and record the letter of the region in which it stops.



Investigating the Ideas

Make a spinner like this one. When you spin the paper clip, do you think you are more likely to get an even number or an odd number?

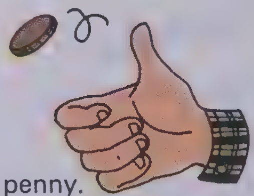


How closely can you predict the number of odd numbers you will get in 30 spins?

Discussing the Ideas

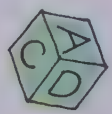
1. A Are the three outcomes for the spinner experiment equally likely outcomes?
   B The **chances** of getting a **2** are 1 in 3. The **probability** of getting a **2** is $\frac{1}{3}$. What are the chances of getting a **3**?
   C What is the probability of getting a **3**?

2. The chances of getting a head are 1 in 2. What is the probability of getting a head?

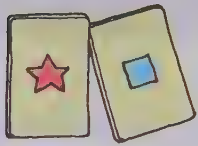


Toss a penny.

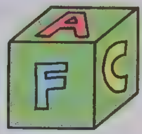
3. A What are the chances of getting a vowel?
   B What is the probability of getting a vowel?



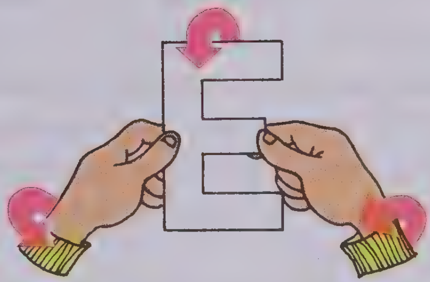
Toss a cube that has one of the letters **A, B, C, D, E, F** on each face.



1. These two cards are turned over and shuffled. If one card is then turned up, two equally likely outcomes are possible.
   A What is the probability that a star will be turned up?
   B What is the probability that a square will be turned up?



2. A cube with letters **A, B, C, D, E, F** is tossed.
   A What is the probability that the letter **A** will appear on top?
   B Think of the 6 equally likely outcomes. What are the chances that a consonant will appear on top?
   C What is the probability that a consonant will appear on top?
   D Certain letters, like **E,** look about the same after they have been turned upside down, as in this picture. Which of the letters **A, B, C, D, E, F** have this property? Think of the 6 equally likely outcomes. What are the chances that a letter with this property will appear on the top face?



   E What is the probability of getting a letter with this property?

3. Two spinners like these are spun.
   A What is the probability that the pair (2,2) will appear?
   B In the 9 possible outcomes, how many chances are there of a pair appearing with both numerals the same?
   C What is the probability of getting a pair with both numerals the same?
   ★ D What is the probability of getting a pair that has a sum less than 5?

1. A If you do not count spins that end on a line, how many outcomes are there for this spinner?

B Do you think these outcomes are equally likely?

C What are the chances of spinning a 2 in one spin?

D What is the probability of spinning a 2?

E Of the 4 equally likely outcomes for the experiment, how many are even numbers?

F What are the chances of spinning an even number in one spin?

G What is the probability of spinning an even number?

2. The outcome shown in the picture represents the pair (3, 4).

A List the 16 outcomes. Are they equally likely?

B (2, 1) is one of the 16 outcomes. What are the chances of spinning a (2, 1) in one trial?

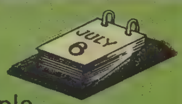
C What is the probability of spinning a (2, 1)?

D Of the 16 equally likely outcomes for the experiment, how many have both numbers the same, like (1, 1)?

E What are the chances of spinning the same two numbers?

F What is the probability of spinning the same two numbers?

think



Mathematical calculations have shown that if you choose a group of 25 people, the chances are about 1 in 2 that two of the people will have the same birthday. If there are 35 people, the chances are about 3 in 4 that this will happen.

A Find a list of the birthdays of famous Canadians. Did any of the famous Canadians have the same birthday?

B Did any members of the group you chose die on the same day?

C Do any members of your family have the same birthday?

D Make a survey of the classes in your school to see if any two people in a class have a birthday on the same day, and report your results.

Keeping in Touch with

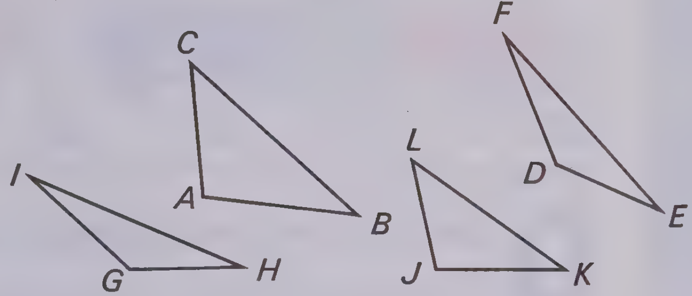
Computing
Measurement
Principles
Fractional numbers
Number theory

1. Remember that a prime number has exactly 2 factors.
   - A Name an odd number that is not prime.
   - B Name a prime number that is not odd.
   - C List the prime numbers between 20 and 40.

2. Compute the following.

   A $\begin{array}{r} 7428 \\ 6917 \\ +3844 \\ \hline \end{array}$ B $\begin{array}{r} 7090 \\ -3774 \\ \hline \end{array}$ C $\begin{array}{r} 924 \\ \times 38 \\ \hline \end{array}$ D $2052 \div 38$

3. A Which two triangles are congruent?
   B Which two are similar triangles?



4. Find the sum or difference.

   A $\frac{3}{4} + \frac{1}{2}$ C $\frac{1}{3} + \frac{1}{4}$ E $\frac{5}{6} - \frac{1}{3}$ G $7\frac{1}{3} + \frac{2}{3}$

   B $\frac{3}{8} + \frac{3}{4}$ D $\frac{7}{8} - \frac{1}{2}$ F $1\frac{1}{2} - \frac{1}{4}$ H $6\frac{3}{4} - 5\frac{1}{8}$

5. Change each fraction to lowest terms.

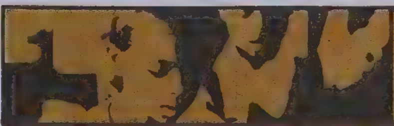
   A $\frac{12}{16}$ B $\frac{14}{20}$ C $\frac{9}{15}$ D $\frac{40}{30}$ E $\frac{27}{36}$

6. Express as an improper fraction.

   A $3\frac{1}{2}$ B $1\frac{2}{3}$ C $4\frac{3}{4}$ D $7\frac{1}{2}$ E $3\frac{1}{10}$

7. Express as a mixed numeral.

   A $\frac{15}{8}$ B $\frac{23}{10}$ C $\frac{45}{7}$ D $\frac{33}{9}$ E $\frac{27}{4}$



You are invited to explore

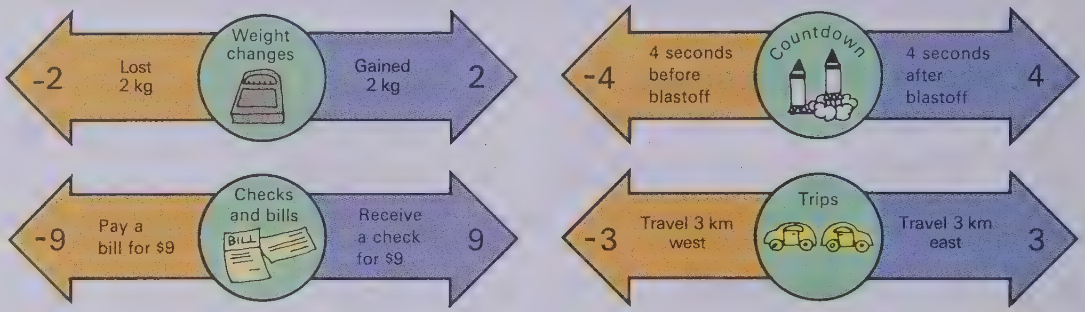
ACTIVITY CARD 16
Page 363

15 Positive and Negative Numbers

Let's explore positive and negative numbers.

Investigating the Ideas

Here are some situations that can be described by using positive and negative numbers.



? Can you show some other situations like these ?

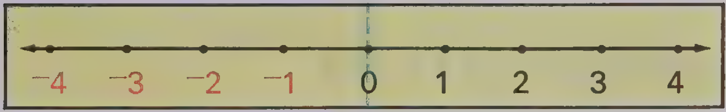
Discussing the Ideas

Situations such as those above suggest the invention of numbers for the points **opposite** the whole-number points on the number line. The symbols for these numbers are given in red on the number line below. These numbers are called **negative numbers.** The non-zero whole numbers are often called **positive numbers.**

$^{-}6 \quad ^{-}5 \quad ^{-}4 \quad ^{-}3 \quad ^{-}2 \quad ^{-}1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

1. Why do you think pairs of positive and negative numbers, such as 4 and −4 are sometimes called **opposites ?**
2. Choose some positive and negative numbers and give the opposite of each one.
3. How could you locate the point for $^{-}27$ on the number line?

1. Imagine cutting out a number-line strip and folding it along the blue line.

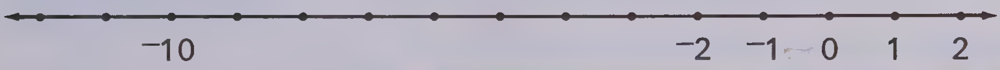


   A The point for 3 would fall on the point for what number?
   B The point for ⁻4 would match with the point for what number?
   C If the strip were long enough, ⁻62 would match with what number?
   D What number would match 103?

2. Give the opposite of each:

A 2	C ⁻2	E 9	G 56	I ⁻18	K ⁻268	M ⁻786
B 6	D ⁻6	F ⁻20	H 100	J ⁻59	L 597	N ⁻260

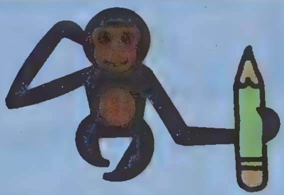
3. Copy and complete this number-line picture.



4. The planet Pluto has an estimated surface temperature of 194°C below zero.
   A How could you represent this temperature?
   B Give the opposite of this temperature.

★ 5. What is the opposite of the opposite of the opposite of 9?

think



Each of the first four equations gives two symbols that name the same number. Study the first four equations to understand the new symbol. Then give the missing numbers in the other equations.

$\sqrt{4} = 2$ $\quad$ $\sqrt{16} = 4$ $\quad$ $\sqrt{36} = \square$

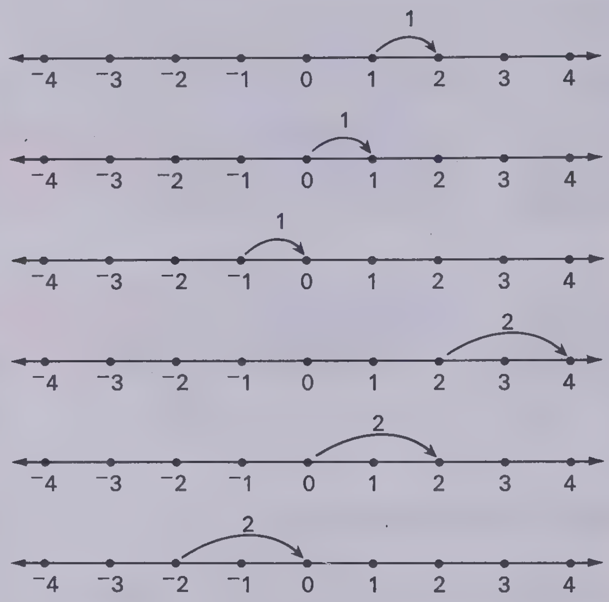
$\sqrt{64} = \square$

$\sqrt{9} = 3$ $\quad$ $\sqrt{25} = 5$ $\quad$ $\sqrt{100} = \square$

Let's explore basic principles for addition of positive and negative numbers.

Investigating the Ideas

Study the patterns.



A $1 + 1 = \square$

B $0 + 1 = \square$

C $^{-}1 + 1 = \square$

D $2 + 2 = \square$

E $0 + 2 = \square$

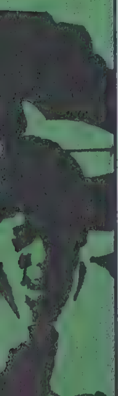
F $^{-}2 + 2 = \square$

? Can you find each of these sums and then write a rule for adding any negative number and its opposite?

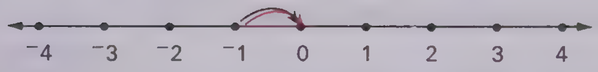
Discussing the Ideas

Use positive and negative numbers to give examples.

Opposites principle	The sum of any negative number and its opposite is zero.
0 principle (+)	If you choose a number and add 0, the sum is the number you chose.
Commutative principle (+)	When finding the sum of two numbers, you can change the order of the addends and the sum is the same.
Associative principle (+)	When finding the sum of three numbers, you can change the grouping of the addends and the sum is the same.



1. The number line suggests that we can solve the equation $^-1 + 1 = n$ by thinking about a jump 1 unit to the right from $^-1$. What is this sum? Solve each equation. It may help to think about the number line.



A $^-2+2=n$ C $^-4+4=n$ E $^-10+10=n$ G $^-97+97=n$
B $^-3+3=n$ D $^-5+5=n$ F $^-53+53=n$ H $^-508+508=n$

2. Find the sums. Because of the **commutative** and **associative principles,** you can first find the sum of the addends given in red.

A $1 + 2 + {}^-2$ C $4 + 1 + {}^-1$ E $6 + 5 + {}^-5$ G $24 + 6 + {}^-6$
B $2 + 3 + {}^-3$ D $^-4 + 4 + 6$ F $^-7 + 7 + 3$ H $^-8 + 8 + 4$

3. The **basic principles** will help you complete each exercise. Give the number for ***a***. Then give the number for ***b***.

A Since $^-25 + 25 = a$, we know that $25 + {}^-25 = b$.
B Since $^-3 + 0 = a$, we know that $0 + {}^-3 = b$.
C Since $10 + {}^-7 = a$, we know that $^-7 + 10 = b$.
D Since $^-5 + ({}^-4 + 4) = a$, we know that $({}^-5 + {}^-4) + 4 = b$.

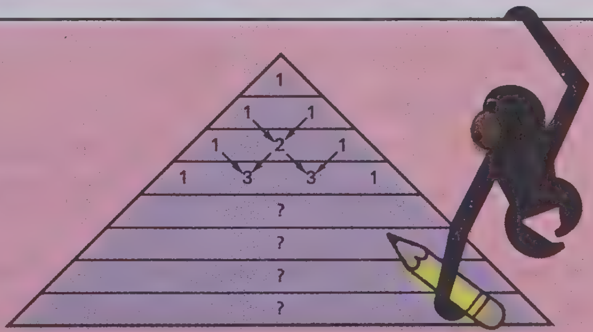
4. These temperatures were recorded on board a trawler in the North Atlantic. Tell whether the temperature rose or fell and give the number of degrees of change for each morning.

	9 a.m.	noon
1st day	-5 °C	8 °C
2nd day	-6 °C	3 °C

	9 a.m.	noon
3rd day	-2 °C	-4 °C
4th day	-3 °C	11 °C

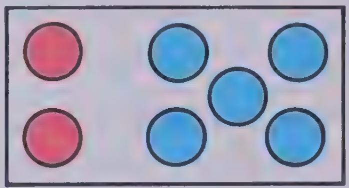
think

Here are 4 rows of a "triangle." Can you discover the pattern and make 4 more rows?



Investigating the Ideas

Use counters or checkers.
Think of the two colors as opposites.



$^{-}2 + 5 = 3$

? Can you use this idea and your counters to find these sums?

A $^{-}3 + 7 = ?$ C $^{-}2 + 2 = ?$

B $8 + {}^{-}5 = ?$ D $^{-}4 + {}^{-}3 = ?$

Discussing the Ideas

1. What key ideas did you need to find the sums by using the counters?

2. A One student found the sum of $^{-}2$ and 5 by finding the sum in the box.

$$\boxed{^{-}2 + 2 + 3}$$
$$^{-}2 + 5 = n$$

How is this method like using the counters?

B What basic principles were used to find this sum?

3. A How would you find the sum $5 + {}^{-}8$ by using counters? By using the method in exercise 2?

B Can you think of another way to find this sum?

1. Find the sums. It may help you to think about the counters.

A 

$3 + {}^{-}2 = n$

B 

${}^{-}3 + 5 = n$

C 

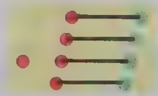
${}^{-}4 + 2 = n$

D 

$5 + {}^{-}1 = n$

E 

${}^{-}2 + {}^{-}3 = n$

F 

${}^{-}5 + 4 = n$

2. Find the sums. The red and blue numerals may help you think of the counters.

A $6 + 5 + {}^{-}5$
$11 + {}^{-}5 = n$

B ${}^{-}9 + 9 + 11$
${}^{-}9 + 20 = n$

C ${}^{-}4 + {}^{-}3 + 3$
${}^{-}7 + 3 = n$

D $5 + {}^{-}5 + {}^{-}13$
$5 + {}^{-}18 = n$

E $24 + 6 + {}^{-}6$
$30 + {}^{-}6 = n$

F ${}^{-}8 + {}^{-}4 + 4$
${}^{-}12 + 4 = n$

3. Find these sums.

A $4 + {}^{-}2$ C ${}^{-}2 + {}^{-}3$ E ${}^{-}9 + 5$ G $8 + {}^{-}17$

B ${}^{-}8 + 5$ D $10 + {}^{-}6$ F $12 + {}^{-}2$ H $56 + {}^{-}5$

4. Think about jumps on the number line, **starting at 0.**

A

Jump 5 units right ( ).	Jump 3 units left ( ).	Left or right of 0? How far?

$5 + {}^{-}3 = n$

B

Jump 4 units left.	Jump 2 units left.	How far left or right of 0?

${}^{-}4 + {}^{-}2 = n$

C

Jump 7 units left.	Jump 5 units right.	How far left or right of 0?

${}^{-}7 + 5 = n$

D

Jump 4 units right.	Jump 9 units left.	How far left or right of 0?

$4 + {}^{-}9 = n$

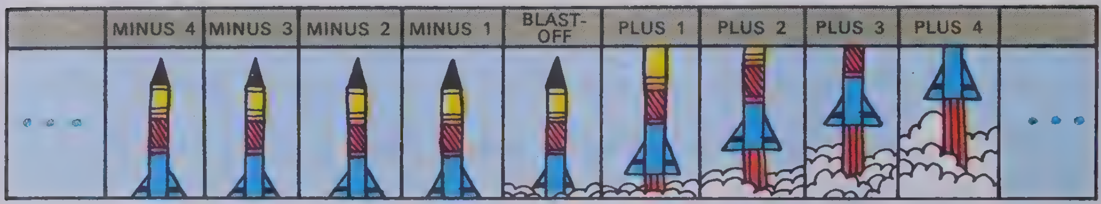
5. Draw number lines and show the jumps for each sum. Write an equation for each number-line picture.

A $4 + 3$ B ${}^{-}4 + {}^{-}3$ C $3 + {}^{-}3$ D $6 + {}^{-}3$ E ${}^{-}6 + 3$

Solving Story Problems

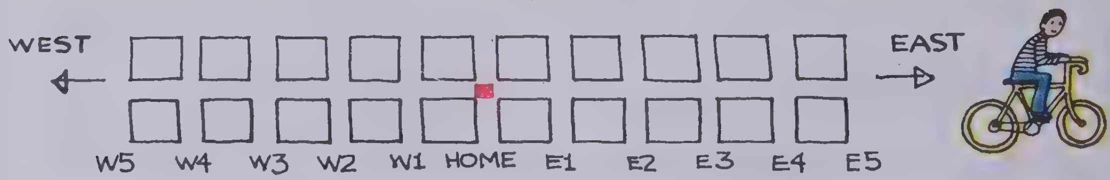
Write and solve an equation to answer the question in each problem.

1. These are pictures shown on television during the countdown and launching of a rocket.



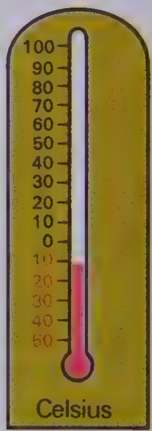
If you started watching at "minus 4," which screen would you be watching after 7 seconds?

2. Jerry keeps track of some of his bike trips on a map like this:



If Jerry starts at home, how far from home is he after a trip 4 blocks west, followed by a trip 9 blocks east?

3. Water freezes at the temperature of zero degrees Celsius. We often speak of temperatures **above** and **below** zero. If the temperature was 12° below zero and rose 24°, what was the new temperature?



4. A If you spend \$5 and earn \$8, how much richer or poorer are you?

B If you earn \$6 and spend \$9, how much richer or poorer are you?

Solving Inequality Problems

This number line may help you complete some of the exercises below.

1. Complete each sentence. Then give the correct sign (<, >) for each ●.

A A temperature **above** zero is __?__ (higher than, lower than) a temperature **below** zero. 10 ● $^-24$

B When the temperature is 5 degrees **below** zero, it is __?__ (warmer, colder) than when the temperature is 6 degrees **below** zero. $^-5$ ● $^-6$

2. Jed and Sandy are playing a game in which they can get scores less than zero. They put rings around these below-zero scores. Who has the higher score for this game?

Score	
Jed	Sandy
6	8

3. $^-7$ is less than the number you get when you add 1 to $^-7$. What number is this? Write an inequality statement about this.

4. Use "greater than" or "less than" to complete each sentence.

A Any positive number is __?__ a negative number.

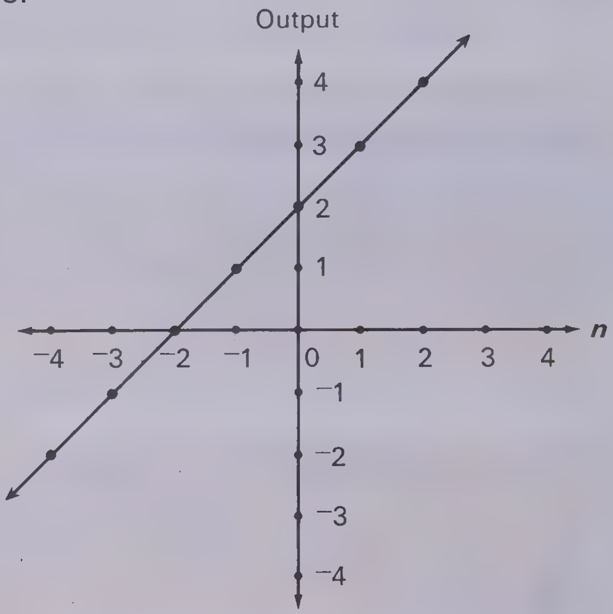
B A number is __?__ a number that is to the left of it on the number line.

Investigating the Ideas

The input-output pairs of numbers produced by a function machine can be used as co-ordinates.

Function Rule

$n + 2$		
n	Output	
$^-4$	$^-2$	→ $(^-4, ^-2)$
$^-3$	$^-1$	→ $(^-3, ^-1)$
$^-2$	0	→ $(^-2, 0)$
$^-1$	1	→ $(^-1, 1)$
0	2	→ $(0, 2)$
1	3	→ $(1, 3)$
2	4	→ $(2, 4)$



When you draw the graph for the set of co-ordinates produced by this function rule, the points all lie on a straight line.

? Can you choose another function rule and draw a graph for the co-ordinates of points produced by your rule ?

Discussing the Ideas

1. Explain how to graph the points for $(3, ^-4)$, $(^-2, ^-4)$, and $(^-2, 4)$. Are any of these input-output pairs from the function rule used above ?

2. If you graphed the number pairs produced by the function rule
$$\text{output} = \begin{cases} 0 \text{ if input is even} \\ 1 \text{ if input is odd} \end{cases}$$
would the points all fall on one straight line ?

Find the missing numbers. Graph the number pairs.

1. Function Rule: $n + 3$

n	Output
$^-4$	$^-1$
$^-3$	A
$^-2$	B
$^-1$	C
0	D
1	E

2. Function Rule: $n + {}^-2$

n	Output
$^-2$	$^-4$
$^-1$	A
0	B
1	C
2	D
3	E

3. Function Rule: $0 - n$

n	Output
$^-3$	3
$^-2$	A
$^-1$	B
0	C
1	D
2	E

4. Function Rule: $(2 \times n) + 1$

n	Output
$\frac{1}{2}$	A
$1\frac{1}{2}$	B
$2\frac{1}{2}$	C
$3\frac{1}{2}$	D
$4\frac{1}{2}$	E

5. Function Rule: $(12 \div n) - 1$

n	Output
1	A
2	B
3	C
6	D
12	E

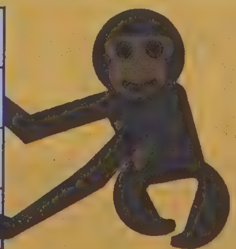
6. Function Rule: $n \times n$

n	Output
0	A
1	B
2	C
3	D
4	E

think

Give the missing numbers so that this will be a magic square.

7	$^-7$	$^-6$	4
$^-4$	2	1	
	$^-2$		3
		6	



1. List the positive and negative whole numbers from -15 to 15.

2. Give the opposite of each:
   A 3 B $^{-}3$ C 9 D $^{-}7$ E 75 F $^{-}75$ G $^{-}183$

3. Give the sign ($<$, $>$, $=$) for each $\bigcirc$.
   A $6 \bigcirc {}^{-}6$ B $0 \bigcirc {}^{-}6$ C $^{-}30 \bigcirc 7$ D $^{-}9 \bigcirc {}^{-}7$

4. List the whole numbers greater than -10 and less than -5.

5. Find the sums.
   A $9 + {}^{-}5$ C $^{-}9 + {}^{-}8$ E $11 + {}^{-}11$ G $17 + {}^{-}4$ I $^{-}4 + 9$
   B $^{-}8 + 3$ D $^{-}8 + 12$ F $^{-}7 + 12$ H $^{-}12 + {}^{-}5$ J $8 + {}^{-}15$

6. Find the missing addends.
   A $4 - {}^{-}3 = n$ B $^{-}4 - 6 = n$ C $^{-}16 - {}^{-}7 = n$ D $0 - {}^{-}8 = n$

7. Use integers to write an addition equation for this problem. You lose \$9 and then find \$12. How much do you have?

8. Complete each sentence.
   A A negative number is __?__ than a positive number.
   B __?__ is the only number that is never positive or negative.
   C A positive number is __?__ than another positive number that is to the right of it on the number line. Would this also be true for two negative numbers?

★ 9. Solve the equations.
   A $2 - {}^{-}3 = n + 3$ D $4 - {}^{-}2 = 4 + n$
   B $^{-}3 - 5 = n + {}^{-}5$ E $^{-}2 - 7 = {}^{-}2 + n$
   C $^{-}1 - {}^{-}3 = n + 3$ F $^{-}4 - {}^{-}6 = {}^{-}4 + n$

★ 10. Find this sum.
   $5 + {}^{-}3 + 7 + {}^{-}9 + {}^{-}5 + {}^{-}6 + {}^{-}1 + 9 = n$

Keeping in Touch with Computing Fractional numbers

1. Find the sums and differences.

A $\begin{array}{r} 34.3 \\ +47.7 \\ \hline \end{array}$ B $\begin{array}{r} 29.6 \\ +54.8 \\ \hline \end{array}$ C $\begin{array}{r} 73.7 \\ -26.5 \\ \hline \end{array}$ D $\begin{array}{r} 56.2 \\ -37.5 \\ \hline \end{array}$ E $\begin{array}{r} 9.643 \\ +8.27 \\ \hline \end{array}$ F $\begin{array}{r} 58.3 \\ -9.764 \\ \hline \end{array}$

2. Find the products and quotients.

A $\begin{array}{r} 9.3 \\ \times 0.7 \\ \hline \end{array}$ B $\begin{array}{r} 5.65 \\ \times 3 \\ \hline \end{array}$ C $\begin{array}{r} 4.26 \\ \times 5.3 \\ \hline \end{array}$ D $0.6\overline{)54.6}$ E $0.04\overline{)3.076}$ F $2.4\overline{)1.368}$

3. Find the missing percent or fraction.

Fraction	$\frac{1}{2}$	B	$\frac{3}{4}$	D	$\frac{3}{5}$	F	$\frac{1}{3}$	H	$\frac{3}{8}$	J
Percent	A	25%	C	20%	E	70%	G	$12\frac{1}{2}$%	I	$66\frac{2}{3}$%

4. Copy and complete the function table.

Function Rule

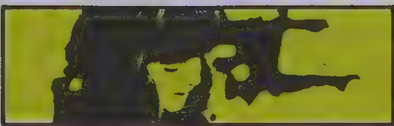
$n + {}^{-}3$

n	Output
4	1
3	A
2	B
1	C
0	D
$^{-}1$	E

5. Old Faithful, a geyser in Yellowstone National Park, erupts on the average of once every 64.5 minutes.

A About how many times does it erupt in a 24-hour day ?

B Starting right after an eruption, how many minutes pass while Old Faithful erupts 56 times ?



You are invited to explore

ACTIVITY CARD 17
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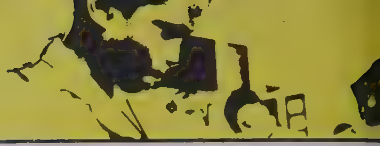
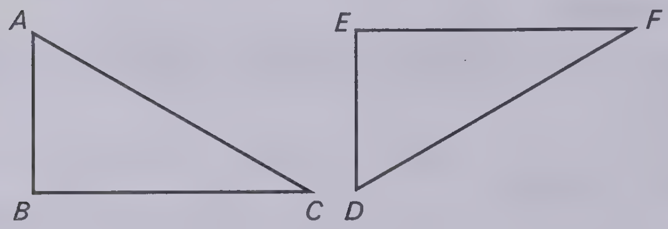
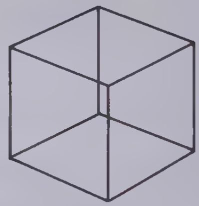


1. Assume that triangle ABC is congruent to triangle DEF. Complete the table.



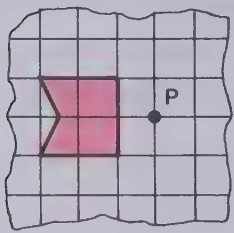
	$\triangle ABC$		$\triangle DEF$
A	$\overline{AC}$	$\cong$	
B		$\cong$	$\overline{ED}$
C	$\overline{BC}$	$\cong$	
D		$\cong$	$\angle DEF$
E	$\angle BAC$	$\cong$	
F	$\angle BCA$	$\cong$	

2. Look at a cube.
   A How many vertices has it ?
   B How many edges has it ?
   C What can you say about the lengths of the edges ?

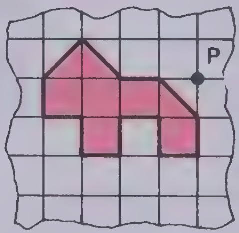


3. Using centimetre squared paper, copy each figure and mark the point P. Then rotate each figure through 180° about P and draw its image. Use tracing paper if you wish.

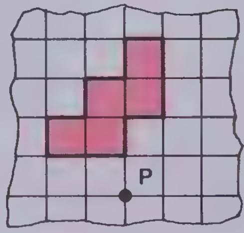
A



B

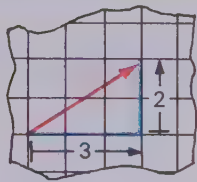


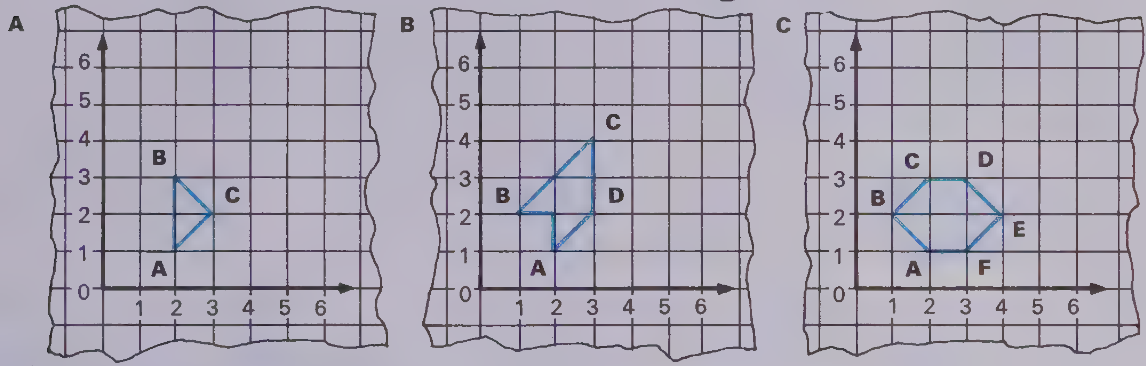
C



4. Draw a figure of your own on centimetre squared paper. Rotate the figure through 180° and draw its image.

5. Copy each figure on your graph paper. Then use the point slider to translate the figure and draw its slide image.

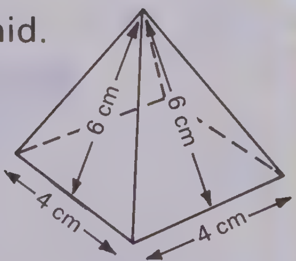




6. How many different lines of symmetry does a square have?

7. A Give the co-ordinates of the midpoint of the segment whose endpoints have co-ordinates (1,1) and (5,5).
   B Three corners of a rectangle are at (2,2), (2,7), and (5,7). Give the co-ordinates of the fourth corner.

8. Look at this pyramid.



   A On centimetre squared paper, draw a pattern for the pyramid. Be as accurate as you can.
   B Cut out the pattern, fold it to form the pyramid. Use tape to fasten the edges together.

think

Sue thought of a number.

Multiplied the number by 4.

Added 6.

Divided by 2.

Subtracted 4.

If the result is 39, what number was Sue thinking of?

Antique Arithmetic

Arithmetic students were busy scribbling these exercises into their "copybooks" over 100 years ago, in 1856. The exercises first appeared in *Ray's Higher Arithmetic,* by Joseph Ray, M.D. At the time, most North Americans lived on farms and bought their supplies at general stores. Can you do some of them ?

1. Which is the nearer number to 920736; 1816045 or 25427 ? *Ans.* Neither. Why ?

2. A grocer gave 153 barrels of flour, worth \$6 a barrel, for 54 barrels of sugar: what did the sugar cost per barrel ? *Ans.* \$17.

3. A grocer bought 135 barrels of pork for \$2295; he sold 83 barrels at the purchase price and the remainder at an advance of \$2 per barrel: how much did he gain ? *Ans.* \$104.

4. A drover bought 5 horses at \$75 each, and 12 at \$68 each; he sold them all at \$73 each: what did he gain ? *Ans.* \$50.

5. If a pulse beats 28 times in 16 seconds, how many times a minute is that ? *Ans.* 105.

6. I can pasture 10 horses or 15 cows on my ground; if I have 9 cows, how many horses can I keep ? *Ans.* 4.

7. If 13 men can build a wall in 15 days, in how many days can it be done if 8 men leave ? *Ans.* 39.

8. If 14 men can perform a job of work in 24 days, in how many days can they perform it with the assistance of 7 more men ? *Ans.* 16 days.

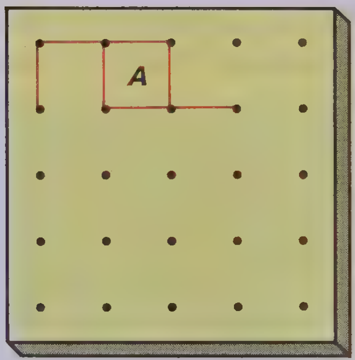
9. A company of 45 men have provisions for 30 days: how many men must depart, that the provisions may last the remainder 50 days ? *Ans.* 18 men.

How to Use the Activity Cards

Do you like to explore things for yourself? These Activity Cards will give you some exciting experiences with mathematics. Each card presents a different idea for you to explore. Often you will find that a card will give you ideas for additional activities on your own.

ACTIVITY CARD 1

Use dot paper and try this game with a classmate.



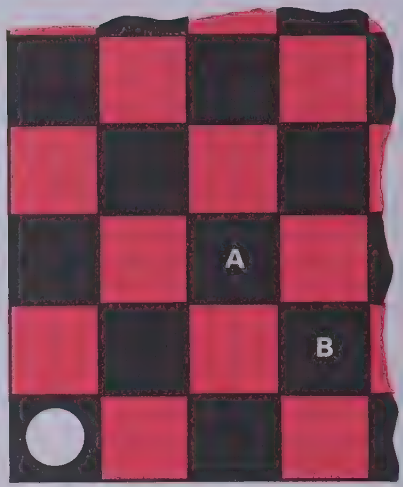
At your turn, you must connect two adjacent dots in the same row or same column. If the connection results in a square, place your initial inside, score one point, and play again. The first player to score 9 points wins.

ACTIVITY CARD 2

Put a checker in one corner of a checkerboard. To move, flip a penny and

(1) move one space up for a head.

(2) move one space right for a tail.



Use graph paper to show 4 possible paths to **B**.

Can you show all 6 ways to get to **A**? Which square are you more likely to land on, **A** or **B**?

ACTIVITY CARD 3

Can you find all the numbers less than 50 that can be represented by the triangular patterns below?

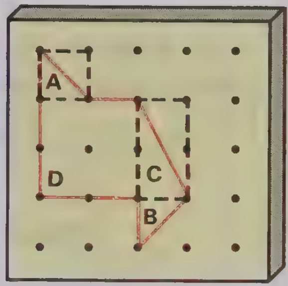
First	•	1	Third	• • • • • •	6
Second	• • •	3	Fourth	• • • • • • • • • •	10

Can you find a pattern?

Use your pattern to find the thirtieth triangular number.

ACTIVITY CARD 4

The area of the dotted square is 1,
so the area of triangle **A** is __ ? __.
The area of triangle **B** is __ ? __.
The area of the dotted rectangle is 2,
so the area of triangle **C** is __ ? __.
The area of square **D** is __ ? __.
The area of the figure outlined in red is __ ? __.



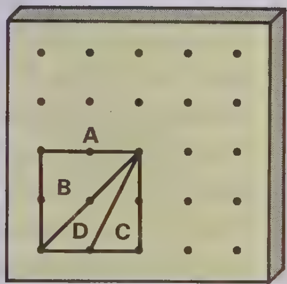
Can you find on the geoboard and draw on dot paper as many as 10 different figures with area 6? (Find one such figure that is unusual and compare it with others in your class.)

ACTIVITY CARD 5

What is the area of square **A**?
What is the area of triangle **B**?
What is the area of triangle **C**?

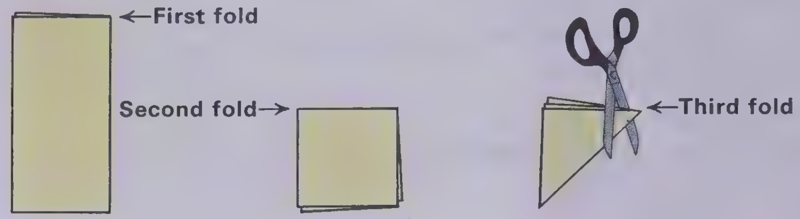
Using the information about the areas of **A**, **B**, and **C**, what is the area of triangle **D**?

There are 8 triangles of different shapes with area 1 that can be formed on the geoboard. How many of them can you find and draw on dot paper?



ACTIVITY CARD 6

Fold a square piece of paper 3 times, as shown, the last time along the diagonal. Then cut off a corner.



Can you make a cut so that the piece you cut off will unfold to be a square? an octagon? a 4-pointed star? another interesting figure?

ACTIVITY CARD 7

There are at least a dozen three- and four-letter mathematical words hidden in the large square.
You can spell out each word by going from square to square, making sure that each new square touches the one before it.
An example is **AREA**, marked by the red path in the large square.

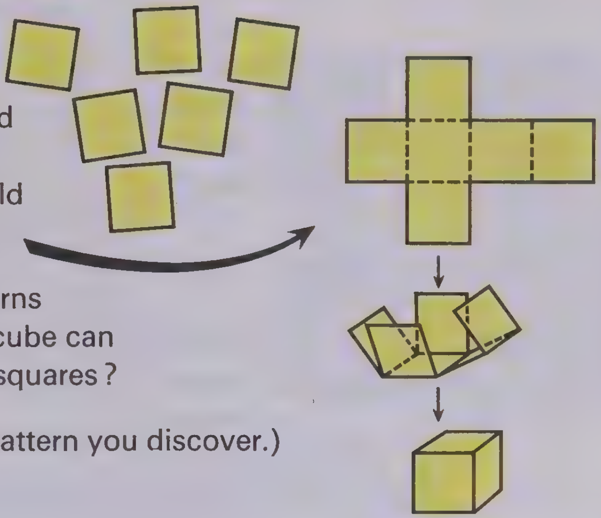
L	T	H	R
S	I	U	C
R	E	N	O
A	T	W	F

How many mathematical words can you find in the square?

ACTIVITY CARD 8

Cut out 6 squares.

Here is one way you could tape the squares together to form a pattern that could be folded to make a cube.



How many different patterns that can be folded into a cube can you make by using the 6 squares?

(Draw a picture of each pattern you discover.)

ACTIVITY CARD 9

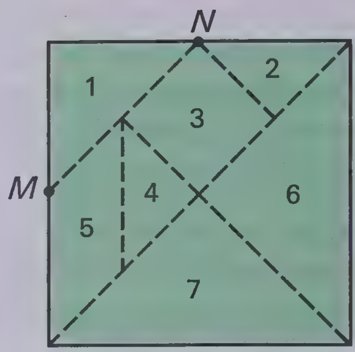
One of the numbers below is approximately the number of times an ordinary bicycle wheel goes around in one kilometre. Which do you think it is?

48 480 4800

Can you find out how many times your own bicycle wheel goes around in one kilometre?

ACTIVITY CARD 10

Draw a 10-cm square and cut it into 7 tangram pieces as shown by the dotted lines. (*M* and *N* are the middle points of the sides.)



Can you place pieces 1, 2, 3, 4, and 5 together to form

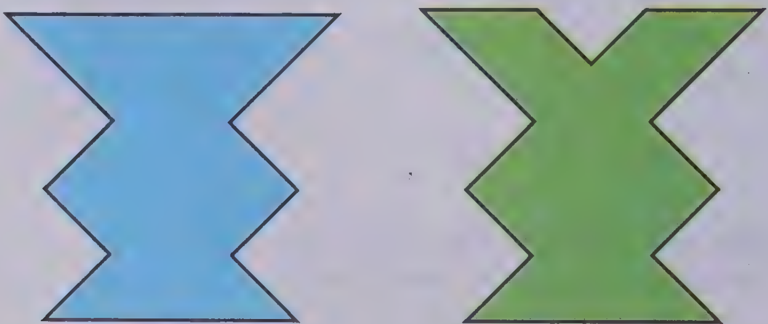
a square?	a parallelogram?
a long rectangle?	a triangle?
a trapezoid?	a hexagon?

another interesting figure?

(Draw a picture to show how you made each figure.)

ACTIVITY CARD 11

These two "vases" are almost alike, except that the "rim" of one is "chipped."

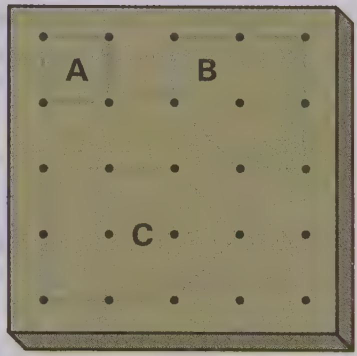


Can you form each vase by using the 7 tangram pieces?

ACTIVITY CARD 12

Copy and fill in the table below.

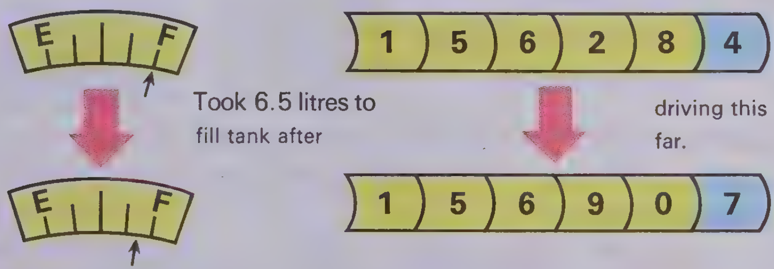
Polygon	A	B	C
Number of nails on the edges			
Number of interior nails			
Area of polygon			



Construct several more polygons on dot paper. Make a table like the one above. Can you discover a way to find the areas of polygons on the geoboard if you know the number of edge nails and interior nails?

ACTIVITY CARD 13

Figuring gasoline consumption

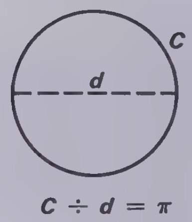


Can you find how many kilometres this car can travel per litre of gasoline?

Try this with your family's or someone else's car.

ACTIVITY CARD 14

A decimal approximation for the number π (the ratio of the circumference of a circle to its diameter) is 3.141592655389. . . .



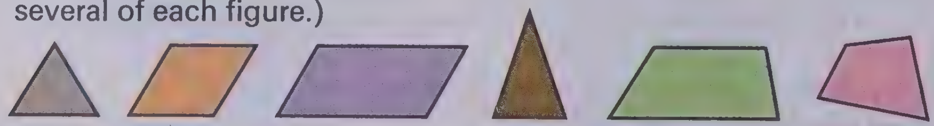
Which of these fractions is the best approximation for π ?

A $\frac{22}{7}$ **B** $\frac{377}{120}$ **C** $\frac{355}{113}$ **D** $\frac{223}{71}$

ACTIVITY CARD 15

You can put squares together to form a larger square.
You can put rectangles together to form a larger rectangle.

Which of the figures below can you put together to make a larger figure of the same kind? (You may want to draw and cut out several of each figure.)



(Draw pictures to show how you could do it.)

ACTIVITY CARD 16

Can you solve these fractional-number problems?

1. Can you find the 3 different missing denominators in the equation $\frac{1}{\square} + \frac{1}{\square} + \frac{1}{\square} = 1$?

2. What are the next four fractional numbers in the sequence
   $\frac{3}{2}, \frac{7}{5}, \frac{17}{12}, \frac{41}{29}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \ldots$?

3. What fractional number is $\frac{1}{3}$ of the way between $\frac{3}{7}$ and $\frac{9}{14}$?

ACTIVITY CARD 17

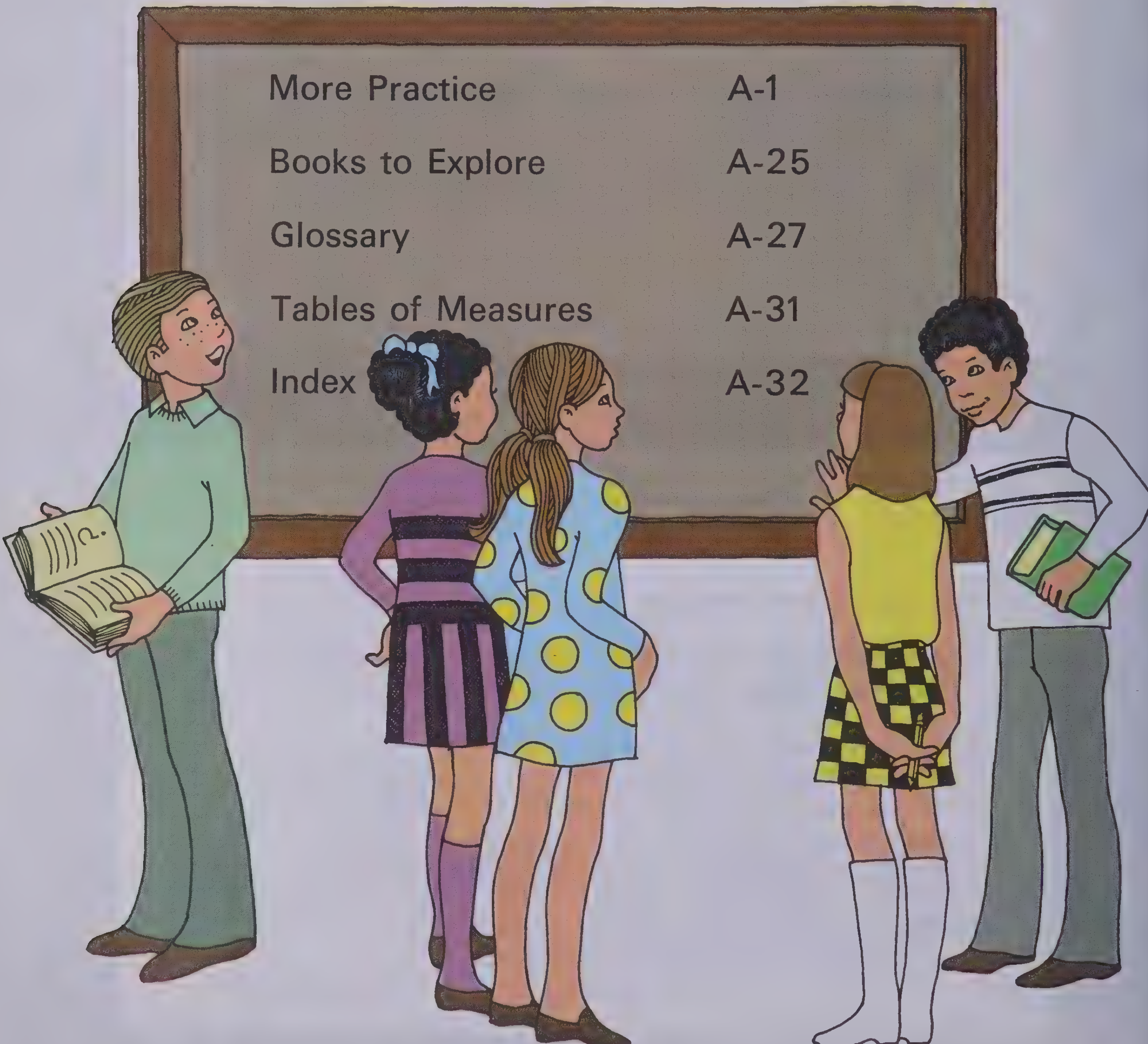
How many of these problems can you solve?

1. Find two whole numbers whose product is 1 000 000 but neither factor has a zero in its representation.

2. Write a numeral of ten digits so that the first digit on the left tells the number of zeros in the numeral, the second digit from the left tells the number of ones, the third digit tells the number of twos and so on until the tenth digit tells the number of nines in the numeral.

3. Place a mathematical symbol between 6 and 7 so that a number between 6 and 7 is represented.

Appendix


More Practice A-1
Books to Explore A-25
Glossary A-27
Tables of Measures A-31
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More Practice

Set 1 *For use with page 21*

Use expanded notation to represent each number.

1. 37	**4.** 4611	**7.** 7041	**10.** 20 631
2. 549	**5.** 7309	**8.** 36 482	**11.** 761 265
3. 603	**6.** 9620	**9.** 83 722	**12.** 900 340

Give the correct sign ($<$ or $>$) for each ▒.

13. 261 ▒ 281	**16.** 6721 ▒ 6689	**19.** 23 621 ▒ 22 899
14. 639 ▒ 631	**17.** 12 407 ▒ 13 006	**20.** 32 507 ▒ 31 607
15. 4368 ▒ 4568	**18.** 84 732 ▒ 84 901	**21.** 53 688 ▒ 53 788

Reflected answers, Set 1: **1.** $(3 \times 10) + 7$, **4.** $(4 \times 1000) + (6 \times 100) + (1 \times 10) + 1$, **7.** $(7 \times 1000) + (4 \times 10) + 1$, **10.** $(2 \times 10\,000) + (6 \times 100) + (3 \times 10) + 1$, **13.** $<$ **16.** $>$ **19.** $>$

Set 2 *For use with page 27*

Give the number for ***a***. Then give the product for ***b***.

1. $6 \times 10^3 = 6 \times a = b$	**4.** $21 \times 10^5 = 21 \times a = b$
2. $13 \times 10^2 = 13 \times a = b$	**5.** $42 \times 10^3 = 42 \times a = b$
3. $7 \times 10^4 = 7 \times a = b$	**6.** $35 \times 10^4 = 35 \times a = b$

Give the number for ***a***. Then give the power of ten for ***b***.

7. $700 = 7 \times a = 7 \times b$	**10.** $15\,000 = 15 \times a = 15 \times b$
8. $4000 = 4 \times a = 4 \times b$	**11.** $60\,000 = 6 \times a = 6 \times b$
9. $38\,000 = 38 \times a = 38 \times b$	**12.** $40\,000 = 4 \times a = 4 \times b$

Write each of the following as a product of a number between 1 and 10 and a power of 10.

13. 700	**15.** 7000	**17.** 400 000
14. 3000	**16.** 80 000	**18.** 900 000

Reflected answers, Set 2: **1.** 1000; 6000, **4.** 100 000; 2 100 000, **7.** 100; 10^2, **10.** 1000; 10^3, **13.** 7×10^2, **16.** 8×10^4, **17.** 4×10^5

Set 3 *For use with page 33*

Find the sums and differences. Use base-six numerals.

1. $2_{(6)}$ $+4_{(6)}$	2. $4_{(6)}$ $+3_{(6)}$	3. $3_{(6)}$ $+5_{(6)}$	4. $10_{(6)}$ $-5_{(6)}$
5. $10_{(6)}$ $+4_{(6)}$	6. $15_{(6)}$ $-5_{(6)}$	7. $13_{(6)}$ $+5_{(6)}$	8. $11_{(6)}$ $+4_{(6)}$

Find the products. Use base-six numerals.

9. $3_{(6)}$ $\times 4_{(6)}$	10. $5_{(6)}$ $\times 2_{(6)}$	11. $5_{(6)}$ $\times 5_{(6)}$	12. $3_{(6)}$ $\times 4_{(6)}$
13. $2_{(6)}$ $\times 4_{(6)}$	14. $10_{(6)}$ $\times 2_{(6)}$	15. $13_{(6)}$ $\times 3_{(6)}$	16. $14_{(6)}$ $\times 4_{(6)}$

Reflected answers, Set 3: 1. $10_{(6)}$, 2. $11_{(6)}$, 9. $20_{(6)}$, 10. $14_{(6)}$

Set 4 *For use with page 41*

Find the missing number in the addition and multiplication equations.
Then solve the subtraction and division equations.

1. $a + 4 = 13$ $13 - 4 = a$	5. $c + 5 = 11$ $11 - 5 = c$	9. $c \times 7 = 35$ $35 \div 7 = c$	13. $b \times 6 = 0$ $0 \div 6 = b$
2. $b + 8 = 15$ $15 - 8 = b$	6. $s + 6 = 14$ $14 - 6 = s$	10. $r \times 3 = 18$ $18 \div 3 = r$	14. $w \times 5 = 45$ $45 \div 5 = w$
3. $x + 4 = 12$ $12 - 4 = x$	7. $r + 9 = 16$ $16 - 9 = r$	11. $s \times 7 = 21$ $21 \div 7 = s$	15. $t \times 7 = 56$ $56 \div 7 = t$
4. $w + 8 = 17$ $17 - 8 = w$	8. $w + 6 = 13$ $13 - 6 = w$	12. $a \times 8 = 56$ $56 \div 8 = a$	16. $d \times 6 = 48$ $48 \div 6 = d$

Solve the equations.

17. $9 + 6 = z$
18. $r - 9 = 4$
19. $3 \times s = 24$
20. $(64 \div 8) \times 8 = a$
21. $72 \div w = 8$
22. $6 = 14 - b$
23. $36 = t \times 6$
24. $(68 - 38) + 38 = r$

Reflected answers, Set 4: 1. 9, 5. 6, 9. 5, 13. 0

Set 5 *For use with page 49*

Give the numbers for ***a*** and ***b***.

1. $10 \times 1000 = a$
$10^1 \times 10^3 = 10^b$

2. $100 \times 1000 = a$
$10^2 \times 10^3 = 10^b$

3. $10 \times 10\,000 = a$
$10^1 \times 10^4 = 10^b$

4. $40 \times 700 = a$
$40 \times 700 = 28 \times 10^b$

5. $500 \times 80 = a$
$500 \times 80 = b \times 10^3$

6. $900 \times 800 = a$
$900 \times 800 = 72 \times 10^b$

Find the products and quotients.

7. 30×60	**12.** $4800 \div 60$	**17.** 700×9
8. $4000 \div 80$	**13.** 90×80	**18.** 80×60
9. 9×400	**14.** $1200 \div 60$	**19.** $4900 \div 70$
10. $1800 \div 20$	**15.** 60×70	**20.** 800×3
11. 4×700	**16.** 50×50	**21.** $3000 \div 50$

Reflected answers, Set 5: 1. 10 000; 4, 4. 28 000; 3, 7. 1800, 8. 50, 12. 80, 13. 7200, 17. 6300, 18. 4800

Set 6 *For use with page 53*

Write and solve an equation for each exercise.

1. The sum of 6 and 3 is multiplied by 7. What is the product?

2. If you divide 42 by 6, then add 2, and multiply by 4, what is the result?

3. Find the product of 5 times the difference of 15 and 8.

4. What is the quotient when you divide the sum of 35 and 5 by 5?

5. If you multiply 4 and 6, then add 16 to this product, what is the final number?

6. If 6 is multiplied by 9 and 12 is subtracted from the product, what is the result?

7. The sum of 6 and 3 is multiplied by 4. What is the result?

8. Beginning with 36, what is the result if you divide by 9 and then multiply by 8?

Reflected answers, Set 6: 1. $(6 + 3) \times 7 = 63$, 5. $(4 \times 6) + 16 = 40$

Set 7 *For use with page 69*

Measure each of these segments to the nearest centimetre.

1.

2.

3.

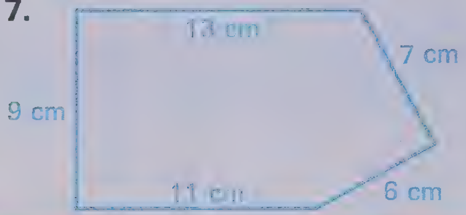
4.

5.

6.

Find the perimeter of each figure below.

7.



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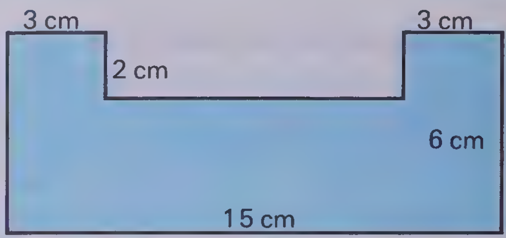
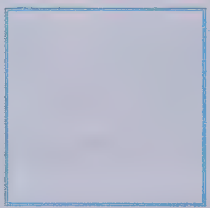


Reflected answers, Set 7: 1. 11 cm, 4. 12 cm

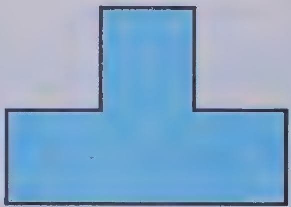
Set 8 *For use with page 75*

Give the number of lines of symmetry for each figure.

1.



2.



3.

Which figures have exactly 1 line of symmetry?
Which figures have no line of symmetry?

4.

5.

6.

Reflected answers, Set 8: 1. 4, 4. 1

Set 9 *For use with page 85*

Find these sums and differences.

1. 42 +38	**2.** 56 +48	**3.** 37 −19	**4.** 66 +78	**5.** 321 +468
6. 640 −558	**7.** 461 −163	**8.** 533 +729	**9.** 520 −483	**10.** 769 +845
11. 825 +466	**12.** 925 −776	**13.** 1376 +4842	**14.** 1761 −1247	**15.** 6538 +5473
16. 5049 −3369	**17.** 4231 −2146	**18.** 2642 −1888	**19.** 7487 +6591	**20.** 8076 +8935
21. 561 48 +361	**22.** 31 257 +3405	**23.** 6483 21 +378	**24.** 3651 4888 +31	**25.** 4863 295 +5906

Reflected answers, Set 9: 1. 80, 2. 104, 3. 18, 4. 144, 5. 789

Set 10 *For use with page 93*

Find the products.

1. 42 ×56	**2.** 83 ×36	**3.** 71 ×25	**4.** 17 ×93	**5.** 62 ×34
6. 27 ×95	**7.** 54 ×63	**8.** 74 ×56	**9.** 243 ×37	**10.** 576 ×61
11. 738 ×42	**12.** 409 ×73	**13.** 283 ×96	**14.** 465 ×64	**15.** 827 ×36
16. 543 ×72	**17.** 615 ×324	**18.** 753 ×466	**19.** 358 ×722	**20.** 408 ×807
21. 945 ×438	**22.** 2648 ×321	**23.** 7407 ×648	**24.** 6532 ×483	**25.** 4309 ×243

Reflected answers, Set 10: 1. 2352, 2. 2988, 3. 1775, 4. 1581, 5. 2108

Set 11 — *For use with page 103*

Find the quotients and remainders.

1. 27)166	**2.** 51)258	**3.** 34)243	**4.** 69)209	**5.** 56)226
6. 42)255	**7.** 21)107	**8.** 64)389	**9.** 73)295	**10.** 32)262
11. 75)455	**12.** 48)243	**13.** 81)409	**14.** 58)353	**15.** 25)180
16. 67)610	**17.** 19)117	**18.** 92)371	**19.** 31)157	**20.** 88)355
21. 95)478	**22.** 46)371	**23.** 70)354	**24.** 83)501	**25.** 28)115

Find the divisor for each exercise.

26. ▒)240 = 48	**27.** ▒)576 = 72	**28.** ▒)448 = 64	**29.** ▒)312 = 52	**30.** ▒)549 = 61

Reflected answers, Set 11. 1. 6 R4, 2. 5 R3, 3. 7 R5, 4. 3 R2, 5. 4 R2

Set 12 — *For use with page 105*

Find the quotients and remainders.

1. 34)2494	**2.** 61)2547	**3.** 27)1527	**4.** 87)2054	**5.** 46)4303
6. 93)2336	**7.** 41)3083	**8.** 66)4828	**9.** 51)3387	**10.** 22)1662
11. 38)1610	**12.** 95)5812	**13.** 39)2465	**14.** 63)3930	**15.** 83)4663
16. 24)798	**17.** 48)2567	**18.** 88)3006	**19.** 55)4023	**20.** 69)1663
21. 96)6828	**22.** 29)1400	**23.** 32)1799	**24.** 60)5071	**25.** 45)1921
26. 84)6137	**27.** 67)2022	**28.** 73)5189	**29.** 57)5310	**30.** 92)3336

Reflected answers, Set 12: 1. 73 R12, 2. 41 R46, 3. 56 R15, 4. 23 R53, 5. 93 R25

Set 13 — *For use with page 111*

Find the quotients.

1. 36)11 808	**2.** 42)21 252	**3.** 53)33 973	**4.** 81)35 154
5. 64)32 960	**6.** 72)24 840	**7.** 26)10 608	**8.** 33)20 592
9. 19)13 927	**10.** 84)34 944	**11.** 93)19 995	**12.** 46)29 164
13. 51)16 626	**14.** 65)29 445	**15.** 38)33 136	**16.** 15)8040
17. 94)38 728	**18.** 74)23 310	**19.** 86)44 204	**20.** 54)39 042
21. 28)17 976	**22.** 35)16 170	**23.** 62)23 870	**24.** 15)53 475
25. 34)27 710	**26.** 52)26 468	**27.** 81)35 316	**28.** 46)43 286

Solve each story problem.

29. There are 12 eggs in one dozen. How many dozen in 60 816 eggs ?

30. Mr. Perez gets 16 km to 1 litre of gas. How many litres of gas is needed to travel 2302 kilometres ?

Reflected answers, Set 13: 1. 328, 2. 506, 3. 641, 4. 434, 17. 412, 18. 315, 19. 514, 20. 723

Set 14 — *For use with page 113*

Find the quotients and remainders.

1. 36)18 576	**2.** 48)29 808	**3.** 71)30 814	**4.** 66)22 770
5. 82)34 932	**6.** 53)27 295	**7.** 27)17 666	**8.** 38)28 234
9. 62)83 454	**10.** 15)93 180	**11.** 73)315 240	**12.** 21)113 646
13. 91)219 583	**14.** 56)190 523	**15.** 45)182 880	**16.** 17)109 370
17. 28)129 864	**18.** 84)222 699	**19.** 63)116 130	**20.** 32)162 061
21. 400)2800	**22.** 300)2700	**23.** 544)2176	**24.** 432)3024

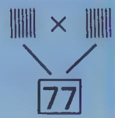
Reflected answers, Set 14: 1. 516, 2. 621, 3. 434, 4. 345, 17. 4638, 18. 2651 R15, 19. 1843 R21, 20. 5064 R13

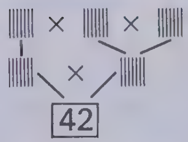
Set 15 *For use with page 127*

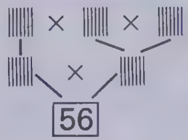
Copy each factor tree and give the missing factors.

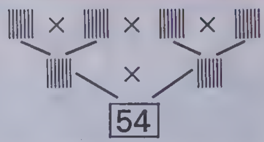
1. 

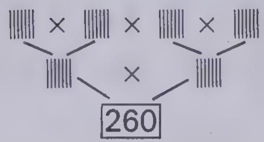
2. 

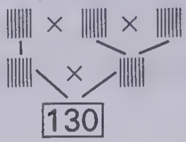
3. 

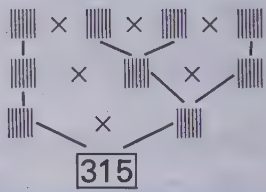
4. 

5. 

6. 

7. 

8. 

9. 

Reflected answers, Set 15: **1.** 2, 7 **2.** 3, 13 **3.** 7, 11

Set 16 *For use with page 133*

Complete the following.

1. List the factors of 14.
2. List the factors of 20.
3. List the common factors of 14 and 20.
4. What is the greatest common factor of 14 and 20?

Use the method above to give the GCF for each pair of numbers.

5. 15, 50 **6.** 12, 36 **7.** 24, 60 **8.** 36, 66

Use the prime factorization method to find the GCF for the two numbers.

9. 38, 57	**11.** 48, 56	**13.** 49, 63	**15.** 45, 72
10. 26, 52	**12.** 39, 65	**14.** 42, 70	**16.** 36, 160

Reflected answers, Set 16: **1.** 1, 2, 7, 14 **2.** 1, 2, 4, 5, 10, 20 **3.** 1, 2 **4.** 2

Set 17 *For use with page 135*

Complete the following.

1. List all the multiples (to 54) of 6.
2. List all the multiples (to 52) of 4.
3. List the common multiples (to 48) of 6 and 4.
4. Give the least common multiple of 6 and 4.

Use the method above to find the LCM for each pair of numbers.

5. 3, 7
6. 14, 18
7. 12, 14
8. 21, 30
9. 4, 24
10. 21, 9
11. 28, 40
12. 24, 32

Use prime factorization to find the LCM for each pair of numbers.

13. 30, 4
14. 18, 28
15. 15, 36
16. 21, 42

Find each of the following.

17. The LCM of 3, 9, and 21.
18. The LCM of 4, 7, and 12.
19. The LCM of 9, 30, and 42.
20. The LCM of 7, 15, and 20.
21. The LCM of 14, 24, and 33.
22. The LCM of 18, 36, and 42.

Reflected answers, Set 17: 1. 6, 12, 18, 24, 30, 36, 42, 48, 54, 2. 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 3. 12, 24, 36, 48, 4. 12

Set 18 *For use with page 147*

Tell whether or not these fractions are in lowest terms.

1. $\frac{4}{3}$
2. $\frac{21}{24}$
3. $\frac{10}{31}$
4. $\frac{11}{31}$
5. $\frac{13}{26}$
6. $\frac{4}{21}$
7. $\frac{8}{32}$
8. $\frac{15}{40}$
9. $\frac{16}{32}$
10. $\frac{90}{100}$

Give these fractions in lowest terms.

11. $\frac{9}{15}$
12. $\frac{27}{30}$
13. $\frac{16}{24}$
14. $\frac{12}{18}$
15. $\frac{20}{25}$
16. $\frac{14}{36}$
17. $\frac{8}{18}$
18. $\frac{6}{32}$
19. $\frac{24}{42}$
20. $\frac{30}{50}$
21. $\frac{20}{30}$
22. $\frac{32}{54}$
23. $\frac{28}{36}$
24. $\frac{15}{75}$
25. $\frac{18}{21}$
26. $\frac{22}{66}$
27. $\frac{42}{54}$
28. $\frac{32}{48}$
29. $\frac{72}{81}$
30. $\frac{75}{100}$

Reflected answers, Set 18: 1. Yes, 3. Yes, 5. No, 7. No, 9. No, 11. $\frac{3}{5}$, 15. $\frac{4}{5}$, 19. $\frac{4}{7}$, 23. $\frac{7}{9}$, 27. $\frac{7}{9}$

Set 19 *For use with page 151*

Give the correct sign (= or ≠) for each $\square$.
Use the cross-product to help you find your answer.

1. $\frac{5}{8} \square \frac{15}{24}$	4. $\frac{6}{9} \square \frac{12}{20}$	7. $\frac{14}{20} \square \frac{28}{40}$	10. $\frac{18}{20} \square \frac{27}{30}$
2. $\frac{7}{10} \square \frac{28}{48}$	5. $\frac{12}{21} \square \frac{20}{35}$	8. $\frac{6}{9} \square \frac{8}{10}$	11. $\frac{25}{30} \square \frac{30}{36}$
3. $\frac{3}{4} \square \frac{21}{28}$	6. $\frac{15}{18} \square \frac{25}{38}$	9. $\frac{9}{12} \square \frac{3}{18}$	12. $\frac{12}{18} \square \frac{20}{24}$

Give the missing numerator or denominator.

13. $\frac{4}{5} = \frac{12}{\square}$	16. $\frac{12}{\square} = \frac{6}{10}$	19. $\frac{12}{5} = \frac{\square}{20}$	22. $\frac{\square}{150} = \frac{10}{50}$
14. $\frac{7}{10} = \frac{\square}{30}$	17. $\frac{\square}{4} = \frac{21}{3}$	20. $\frac{17}{18} = \frac{34}{\square}$	23. $\frac{24}{\square} = \frac{12}{30}$
15. $\frac{13}{15} = \frac{\square}{30}$	18. $\frac{8}{\square} = \frac{32}{40}$	21. $\frac{12}{20} = \frac{\square}{10}$	24. $\frac{81}{9} = \frac{\square}{5}$

Give the whole numbers named by these fractions.

25. $\frac{36}{4}$	26. $\frac{56}{8}$	27. $\frac{72}{9}$	28. $\frac{42}{7}$

Reflected answers, Set 19: 1. =, 4. ≠, 7. =, 10. =, 13. 15, 16. 20, 19. 48, 22. 30

Set 20 *For use with page 153*

Give the correct sign (< or >) for each $\square$.

1. $\frac{3}{4} \square \frac{6}{1}$	3. $\frac{5}{8} \square \frac{3}{16}$	5. $\frac{15}{4} \square \frac{7}{2}$	7. $\frac{6}{7} \square \frac{2}{3}$
2. $\frac{4}{5} \square \frac{2}{3}$	4. $\frac{7}{5} \square \frac{33}{20}$	6. $\frac{31}{6} \square 6$	8. $\frac{7}{9} \square \frac{6}{8}$

Solve the following short story problems.

9. 57 centimetres. $\frac{2}{3}$ metre.
Which is longer?

10. John works $\frac{3}{5}$ of an hour.
Ted works $\frac{5}{6}$ of an hour.
Who works longer?

11. Mary answered $\frac{3}{4}$ of the test questions correctly. There were 80 problems. How many did she get wrong?

12. 748 metres. $\frac{3}{4}$ kilometre.
Which is longer?

13. Sam ate $\frac{5}{6}$ of a pie.
Mark ate $\frac{7}{8}$ of a pie.
Who ate more?

14. Joan bought $\frac{7}{8}$ of a metre of ribbon. Sue bought 88 cm of the same ribbon. Who bought more?

Reflected answers, Set 20: 1. <, 3. >, 5. >, 7. >, 9. $\frac{2}{3}$, 12. $\frac{3}{4}$

Set 21 *For use with page 159*

Make your own lists of equivalent fractions until you find two fractions with the same denominator. Then find the sum or difference.

1. $\frac{3}{8} + \frac{3}{4}$	**2.** $\frac{2}{3} - \frac{1}{6}$	**3.** $\frac{1}{3} + \frac{1}{12}$	**4.** $\frac{2}{5} - \frac{3}{10}$
5. $\frac{7}{10} + \frac{3}{8}$	**6.** $\frac{5}{6} - \frac{3}{5}$	**7.** $\frac{3}{4} + \frac{1}{10}$	**8.** $\frac{3}{5} + \frac{1}{20}$
9. $\frac{9}{10} - \frac{3}{4}$	**10.** $\frac{53}{100} - \frac{9}{20}$	**11.** $\frac{2}{3} + \frac{7}{10}$	**12.** $\frac{2}{5} + \frac{3}{100}$
13. $\frac{1}{6} + \frac{3}{10}$	**14.** $\frac{3}{4} - \frac{3}{8}$	**15.** $\frac{4}{5} - \frac{3}{7}$	**16.** $\frac{1}{6} - \frac{1}{10}$
17. $\frac{9}{20} + \frac{11}{25}$	**18.** $\frac{3}{20} + \frac{3}{5}$	**19.** $\frac{2}{3} + \frac{5}{8}$	**20.** $\frac{3}{5} - \frac{2}{7}$

Reflected answers, Set 21: 1. $\frac{9}{8}$, 2. $\frac{3}{6}$, 3. $\frac{5}{12}$, 4. $\frac{1}{10}$

Set 22 *For use with page 163*

Find the sums and differences. Give your answers in lowest terms.

1. $\frac{2}{3} - \frac{1}{5}$	**2.** $\frac{4}{5} + \frac{3}{4}$	**3.** $\frac{5}{8} - \frac{4}{7}$	**4.** $\frac{3}{100} + \frac{1}{10}$
5. $\frac{5}{7} + \frac{1}{10}$	**6.** $\frac{3}{4} - \frac{3}{8}$	**7.** $\frac{7}{10} + \frac{5}{6}$	**8.** $\frac{1}{3} - \frac{1}{8}$
9. $\frac{5}{8} + \frac{7}{10}$	**10.** $\frac{5}{8} + \frac{3}{4}$	**11.** $\frac{5}{8} - \frac{1}{6}$	**12.** $\frac{5}{7} + \frac{2}{3}$
13. $\frac{4}{5} - \frac{1}{3}$	**14.** $\frac{2}{5} + \frac{3}{8}$	**15.** $\frac{2}{5} - \frac{1}{4}$	**16.** $\frac{7}{10} - \frac{4}{15}$

17. $\frac{1}{6} + \frac{2}{3} + \frac{1}{4}$	**18.** $\frac{3}{5} + \frac{1}{4} + \frac{3}{10}$	**19.** $\frac{5}{8} + \frac{1}{4} + \frac{1}{2}$
20. $\frac{1}{6} + \frac{3}{8} + \frac{1}{4}$	**21.** $\frac{3}{20} + \frac{2}{5} + \frac{7}{10}$	**22.** $\frac{3}{4} + \frac{1}{5} + \frac{1}{2}$
23. $\frac{2}{3} + \frac{3}{4} + \frac{5}{8}$	**24.** $\frac{3}{4} + \frac{5}{6} + \frac{1}{2}$	**25.** $\frac{3}{5} + \frac{1}{2} + \frac{1}{6}$

Reflected answers, Set 22: 1. $\frac{7}{15}$, 2. $1\frac{11}{20}$, 3. $\frac{3}{56}$, 4. $\frac{13}{100}$, 17. $1\frac{1}{12}$, 18. $1\frac{3}{20}$, 19. $1\frac{3}{8}$

Set 23 *For use with page 167*

Write an improper fraction for each mixed numeral.

1. $3\frac{5}{8}$	2. $6\frac{1}{3}$	3. $7\frac{3}{5}$	4. $4\frac{3}{4}$	5. $5\frac{2}{3}$
6. $2\frac{3}{10}$	7. $5\frac{5}{6}$	8. $2\frac{1}{10}$	9. $3\frac{1}{5}$	10. $10\frac{3}{5}$

Write a mixed numeral for each improper fraction.

11. $\frac{19}{5}$	12. $\frac{17}{3}$	13. $\frac{23}{8}$	14. $\frac{44}{5}$	15. $\frac{27}{4}$
16. $\frac{65}{10}$	17. $\frac{27}{4}$	18. $\frac{150}{100}$	19. $\frac{16}{5}$	20. $\frac{48}{20}$

Solve the equations. Give all fractions in lowest terms.

21. $7\frac{4}{3} = 8 + n$	22. $21\frac{12}{10} = 22 + n$	23. $32\frac{13}{9} = n + \frac{4}{9}$
24. $36\frac{13}{4} = 39 + n$	25. $17\frac{10}{8} = n + \frac{1}{4}$	26. $9\frac{7}{5} = 10 + n$
27. $43\frac{10}{7} = 44 + n$	28. $72\frac{13}{6} = 74 + n$	29. $26\frac{11}{4} = n + \frac{3}{4}$

Reflected answers, Set 23: 1. $\frac{29}{8}$, 2. [illegible], 3. [illegible], 4. [illegible], 5. $\frac{17}{3}$, 21. [illegible], 22. $\frac{1}{5}$, 23. 33

Set 24 *For use with page 169*

Find the sums and differences.

1. $\frac{3}{4} + \frac{7}{12}$	2. $\frac{7}{10} + \frac{2}{5}$	3. $\frac{5}{8} + \frac{5}{6}$	4. $\frac{8}{9} + \frac{1}{6}$	5. $\frac{5}{12} + \frac{7}{8}$	6. $\frac{8}{15} + \frac{9}{10}$
7. $5\frac{3}{4} + 3\frac{7}{8}$	8. $7\frac{3}{10} + 8\frac{4}{5}$	9. $6\frac{1}{2} + 4\frac{1}{6}$	10. $2\frac{2}{3} + 9\frac{3}{5}$	11. $5\frac{1}{4} + 6\frac{1}{12}$	12. $10\frac{2}{5} + 3\frac{1}{2}$
13. $22\frac{5}{8} + 31\frac{1}{6}$	14. $46\frac{1}{3} + 10\frac{1}{5}$	15. $13\frac{2}{7} + 28\frac{5}{6}$	16. $43\frac{3}{4} + 21\frac{9}{10}$	17. $66\frac{1}{3} + 18\frac{3}{10}$	18. $37\frac{3}{4} + 48\frac{7}{10}$
19. $7\frac{1}{6} + 3\frac{3}{10}$	20. $12\frac{1}{2} + 8\frac{3}{4}$	21. $9\frac{1}{3} - 4\frac{2}{5}$	22. $22\frac{3}{50} + 11\frac{1}{20}$	23. $6\frac{1}{4} - 2\frac{1}{3}$	24. $8\frac{3}{7} - 3\frac{1}{6}$

Reflected answers, Set 24: 1. [illegible], 2. [illegible], 3. $1\frac{11}{24}$, 4. $1\frac{1}{18}$, 5. $1\frac{7}{24}$, 6. [illegible]

Set 25 — *For use with page 175*

Stock Exchange Prices (dollars per share)

	1967	1968	1969	1970
Industrial	52	58	$57\frac{1}{2}$	48
Transportation	$53\frac{1}{2}$	$50\frac{3}{5}$	47	$32\frac{1}{10}$
Utilities	$45\frac{2}{3}$	$44\frac{2}{5}$	$42\frac{4}{5}$	$37\frac{1}{5}$
Finance	$49\frac{4}{5}$	$65\frac{9}{10}$	$70\frac{1}{2}$	$54\frac{3}{5}$

1. In 1968 the average share of transportation stock cost \$$50\frac{3}{5}$. In 1970, the price had fallen to \$$32\frac{1}{10}$ per share. How much more did an average share cost in 1968 than in 1970?
2. An average share of finance cost \$$49\frac{4}{5}$ in 1967. How much more did an average share cost in 1969?
3. How much would Mr. Lawson have paid for an average share of transportation and an average share of finance in 1968?
4. The price of an average share of utilities steadily decreased from 1967 to 1970. How much more did it cost to buy an average share in 1967 than in 1969?

Reflected answers, Set 25: 1. \$$18\frac{1}{2}$

Set 26 — *For use with page 177*

Give the numerator for ***a*** and the whole number for ***b***.

1. $\frac{42}{4} = \frac{a}{4} + \frac{2}{4} = b + \frac{2}{4}$
2. $\frac{68}{5} = \frac{a}{5} + \frac{3}{5} = b + \frac{3}{5}$
3. $\frac{124}{7} = \frac{a}{7} + \frac{5}{7} = b + \frac{5}{7}$
4. $\frac{37}{3} = \frac{a}{3} + \frac{1}{3} = b + \frac{1}{3}$
5. $\frac{51}{2} = \frac{a}{2} + \frac{1}{2} = b + \frac{1}{2}$
6. $\frac{93}{8} = \frac{a}{8} + \frac{5}{8} = b + \frac{5}{8}$

Give the number for ***q*** and the number for ***r***.
Then give a mixed numeral for ***m***.

7. $\frac{39}{6} = \frac{q \times 6}{6} + \frac{r}{6} = m$
8. $\frac{47}{2} = \frac{q \times 2}{2} + \frac{r}{2} = m$
9. $\frac{57}{4} = \frac{q \times 4}{4} = \frac{r}{4} = m$
10. $\frac{49}{3} = \frac{q \times 3}{3} + \frac{r}{3} = m$
11. $\frac{63}{4} = \frac{q \times 4}{4} + \frac{r}{4} = m$
12. $\frac{72}{5} = \frac{q \times 5}{5} + \frac{r}{5} = m$

Give the mixed numeral for each fraction.

13. $\frac{58}{3}$
14. $\frac{134}{5}$
15. $\frac{75}{4}$
16. $\frac{93}{6}$
17. $\frac{98}{3}$
18. $\frac{144}{5}$
19. $\frac{326}{7}$
20. $\frac{175}{3}$
21. $\frac{61}{2}$
22. $\frac{632}{5}$

Reflected answers, Set 26: 1. 40; 10, 3. 119; 17, 5. 50; 25

Set 27 — *For use with page 185*

Solve the equations.

1. $7 \times \frac{1}{3} = n$
2. $\frac{1}{7} \times \frac{1}{4} = n$
3. $5 \times \frac{1}{10} = n$
4. $\frac{1}{10} \times \frac{1}{10} = n$
5. $4 \times \frac{1}{5} = n$
6. $6 \times \frac{1}{4} = n$
7. $\frac{1}{3} \times \frac{1}{8} = n$
8. $\frac{1}{10} \times \frac{1}{2} = n$
9. $\frac{1}{6} \times \frac{1}{4} = n$
10. $15 \times \frac{1}{10} = n$
11. $12 \times \frac{1}{5} = n$
12. $\frac{1}{8} \times \frac{1}{4} = n$
13. $\frac{1}{3} \times \frac{1}{9} = n$
14. $\frac{1}{5} \times \frac{1}{4} = n$
15. $\frac{1}{2} \times \frac{1}{6} = n$
16. $11 \times \frac{1}{7} = n$

Write and solve a multiplication equation for each problem.

17. Mary has read 15 books this year. Joe has read only $\frac{1}{5}$ as many books as Mary. How many books has Joe read?
18. John and Steve decided to run from John's house to school. Steve ran $\frac{1}{3}$ of the way to school. John ran only $\frac{1}{2}$ as far as Steve. What part of the way to school did John run?

Reflected answers, Set 27: 1. $\frac{7}{3}$, 5. $\frac{4}{5}$, 9. $\frac{1}{24}$, 13. $\frac{1}{27}$, 17. $15 \times \frac{1}{5} = 3$

Set 28 — *For use with page 189*

Give the number for ***n***.

1. $\frac{1}{2} \times \frac{3}{5} = \frac{1}{2} \times n \times \frac{1}{5}$
2. $\frac{5}{6} \times \frac{1}{8} = n \times \frac{1}{6} \times \frac{1}{8}$
3. $13 \times \frac{2}{3} = 13 \times n \times \frac{1}{3}$
4. $\frac{3}{4} \times \frac{1}{6} = n \times \frac{1}{4} \times \frac{1}{6}$
5. $\frac{7}{9} \times 15 = n \times \frac{1}{9} \times 15$
6. $\frac{1}{4} \times \frac{3}{8} = \frac{1}{4} \times n \times \frac{1}{8}$

Give the whole number for ***a***. Give the fraction for ***b***.
Then give the missing product for ***p***.

7. $\frac{3}{4} \times \frac{3}{5} = (3 \times \frac{1}{4}) \times (3 \times \frac{1}{5}) = a \times b = p$
8. $\frac{5}{6} \times \frac{2}{3} = (5 \times \frac{1}{6}) \times (2 \times \frac{1}{3}) = a \times b = p$

Copy each equation. Give the product for ***n***.

9. $\frac{3}{7} \times \frac{2}{3} = n$
10. $\frac{2}{5} \times \frac{4}{6} = n$
11. $\frac{3}{4} \times \frac{6}{5} = n$
12. $\frac{3}{8} \times \frac{4}{3} = n$
13. $\frac{7}{10} \times \frac{3}{2} = n$
14. $\frac{2}{5} \times \frac{3}{20} = n$
15. $\frac{7}{8} \times \frac{3}{10} = n$
16. $\frac{3}{10} \times \frac{5}{2} = n$
17. $\frac{2}{7} \times \frac{3}{7} = n$
18. $\frac{7}{9} \times \frac{5}{6} = n$
19. $\frac{3}{5} \times \frac{2}{5} = n$
20. $\frac{5}{8} \times \frac{5}{6} = n$

Reflected answers, Set 28: 1. 3, 3. 2, 5. 7, 9. $\frac{6}{21}$ or $\frac{2}{7}$, 12. $\frac{12}{24}$ or $\frac{1}{2}$, 15. $\frac{21}{80}$, 18. $\frac{35}{54}$

Set 29 *For use with page 193*

Use the distributive principle to find the products.

1. $\frac{3}{4} \times 7\frac{2}{3}$ 2. $8 \times 9\frac{3}{5}$ 3. $6 \times 4\frac{1}{4}$ 4. $\frac{2}{3} \times 6\frac{1}{5}$

5. $5\frac{3}{5} \times 3$ 6. $4\frac{5}{6} \times 7\frac{2}{3}$ 7. $2\frac{7}{8} \times 8$ 8. $3\frac{7}{10} \times 6\frac{3}{5}$

Replace each mixed numeral with an improper fraction to find these products.

9. $3\frac{3}{4} \times 4\frac{1}{3}$
10. $\frac{2}{3} \times 5\frac{1}{6}$
11. $1\frac{1}{8} \times 6\frac{1}{3}$
12. $2\frac{1}{10} \times 1\frac{2}{5}$
13. $\frac{5}{2} \times 5\frac{1}{3}$
14. $7\frac{1}{3} \times 2\frac{1}{5}$
15. $\frac{7}{8} \times 3\frac{7}{10}$
16. $\frac{5}{3} \times 1\frac{3}{8}$
17. $6\frac{3}{5} \times \frac{5}{7}$
18. $8\frac{1}{4} \times 9\frac{1}{3}$
19. $5\frac{5}{6} \times 1\frac{3}{4}$
20. $3\frac{5}{9} \times 4\frac{5}{8}$
21. $\frac{5}{7} \times 8\frac{3}{5}$
22. $\frac{7}{6} \times 4\frac{3}{8}$
23. $2\frac{7}{10} \times 4\frac{1}{3}$
24. $6\frac{1}{6} \times 3\frac{1}{10}$

Reflected answers, Set 29: 1. $5\frac{3}{4}$, 2. $76\frac{4}{5}$, 3. $25\frac{1}{2}$, 4. $4\frac{2}{15}$, 5. $16\frac{4}{5}$, 13. $13\frac{1}{3}$, 17. $4\frac{5}{7}$, 21. $6\frac{1}{7}$

Set 30 *For use with page 197*

Use the shortcut to find these products.

1. $\frac{3}{5} \times \frac{7}{9}$
2. $\frac{4}{7} \times \frac{9}{8}$
3. $\frac{5}{6} \times \frac{9}{20}$
4. $\frac{2}{3} \times \frac{9}{10}$
5. $\frac{5}{12} \times \frac{16}{30}$
6. $\frac{3}{4} \times \frac{12}{18}$
7. $\frac{3}{8} \times \frac{12}{21}$
8. $\frac{2}{9} \times \frac{15}{16}$
9. $\frac{7}{16} \times \frac{32}{42}$
10. $\frac{27}{30} \times \frac{24}{36}$
11. $\frac{14}{22} \times \frac{33}{49}$
12. $\frac{6}{16} \times \frac{24}{33}$
13. $\frac{5}{8} \times \frac{16}{25}$
14. $\frac{16}{7} \times \frac{35}{48}$
15. $\frac{9}{14} \times \frac{32}{45}$
16. $\frac{7}{11} \times \frac{55}{28}$

Complete the function tables.

Function Rule: $\frac{2}{3} \times n$

	n	$f(n)$
17.	$\frac{9}{16}$	
18.	$\frac{21}{32}$	
19.	$2\frac{3}{4}$	

Function Rule: $n \times \frac{3}{5}$

	n	$f(n)$
20.	$\frac{20}{21}$	
21.	$5\frac{1}{3}$	
22.	$\frac{35}{39}$	

Function Rule: $2\frac{1}{4} \times n$

	n	$f(n)$
23.	$\frac{32}{27}$	
24.	$3\frac{1}{3}$	
25.	$4\frac{2}{5}$	

Reflected answers, Set 30: 1. $\frac{7}{15}$, 5. $\frac{2}{9}$, 9. $\frac{1}{3}$, 13. $\frac{2}{5}$, 17. $\frac{3}{8}$, 18. $\frac{7}{16}$, 19. $1\frac{5}{6}$

Set 31 *For use with page 205*

Solve the equations.

1. $7 \div \frac{3}{4} = n \times \frac{4}{3}$
2. $11 \div \frac{7}{8} = n \times \frac{8}{7}$
3. $13 \div \frac{11}{13} = n \times \frac{13}{11}$
4. $\frac{2}{3} \div \frac{7}{8} = n \times \frac{8}{7}$
5. $\frac{11}{12} \div \frac{4}{7} = n \times \frac{7}{4}$
6. $\frac{5}{9} \div \frac{4}{5} = \frac{5}{9} \times n$
7. $\frac{6}{13} \div \frac{3}{8} = \frac{6}{13} \times n$
8. $\frac{4}{11} \div \frac{6}{13} = \frac{4}{11} \times n$
9. $\frac{12}{13} \div \frac{14}{33} = n \times \frac{33}{14}$

Find the quotients.

10. $\frac{7}{9} \div \frac{2}{3}$
11. $\frac{5}{8} \div \frac{5}{16}$
12. $\frac{3}{4} \div \frac{7}{24}$
13. $\frac{7}{6} \div \frac{14}{15}$
14. $\frac{3}{7} \div \frac{5}{28}$
15. $7 \div \frac{2}{3}$
16. $\frac{20}{27} \div \frac{5}{9}$
17. $\frac{3}{10} \div \frac{4}{15}$
18. $\frac{9}{8} \div 3$
19. $\frac{15}{18} \div \frac{5}{9}$
20. $\frac{21}{32} \div \frac{7}{8}$
21. $\frac{18}{7} \div \frac{9}{14}$
22. $\frac{5}{21} \div \frac{6}{27}$
23. $\frac{9}{16} \div \frac{15}{24}$
24. $\frac{3}{10} \div \frac{12}{25}$
25. $\frac{11}{12} \div \frac{33}{42}$

Reflected answers, Set 31: 1. 7, 2. 11, 3. 13, 10. $1\frac{1}{6}$, 11. 2, 12. $2\frac{4}{7}$, 13. $1\frac{1}{4}$

Set 32 *For use with page 207*

Find the quotients.

1. $2\frac{3}{4} \div \frac{2}{5}$
2. $3\frac{1}{6} \div 1\frac{1}{2}$
3. $\frac{7}{8} \div 4\frac{1}{4}$
4. $7\frac{1}{2} \div \frac{5}{6}$
5. $1\frac{3}{5} \div \frac{3}{10}$
6. $3\frac{1}{7} \div 1\frac{1}{3}$
7. $4\frac{1}{2} \div 1\frac{7}{8}$
8. $2\frac{1}{6} \div 3\frac{2}{3}$
9. $4\frac{1}{5} \div 3\frac{1}{3}$
10. $2\frac{5}{9} \div 3\frac{1}{3}$
11. $3\frac{3}{5} \div 2\frac{1}{10}$
12. $4\frac{1}{3} \div 3\frac{2}{9}$

Simplify each expression.

13. $\dfrac{\frac{4}{5}}{\frac{7}{8}}$
14. $\dfrac{\frac{2}{3}}{\frac{3}{10}}$
15. $\dfrac{\frac{2}{7}}{\frac{9}{8}}$
16. $\dfrac{\frac{5}{7}}{\frac{14}{20}}$

Find the quotients. Check your work.

17. $49 \div 6$
18. $37 \div 4$
19. $123 \div 4$
20. $239 \div 7$
21. $371 \div 5$
22. $184 \div 6$
23. $431 \div 7$
24. $165 \div 9$
25. $2314 \div 6$
26. $849 \div 7$
27. $3642 \div 9$
28. $4733 \div 4$

Reflected answers, Set 32: 1. $6\frac{7}{8}$, 2. $2\frac{1}{9}$, 3. $\frac{7}{34}$, 4. 9

Set 33 — *For use with page 211*

1. Mike kept a record of his last three math test scores. They were 78, 83, and 86. What was his average for the three tests?

2. Mr. Haley bought $183\frac{1}{3}$ kilograms of wheat and $465\frac{3}{5}$ kilograms of rye. How many kilograms of grain did he buy all together?

3. Mike weighs $45\frac{2}{3}$ kg, John weighs $66\frac{1}{4}$ kg, and Tom weighs $73\frac{1}{2}$ kg. How much do the three boys weigh together?

4. The temperatures in Vancouver for the last 4 days have been 22 °C, 21 °C, 25 °C, and 23 °C. What is the average temperature for the last four days?

5. Sam's beagle weighs $12\frac{3}{4}$ kg. Bill has a collie that weighs $22\frac{2}{5}$ kg. How much more does Bill's collie weigh than Sam's beagle?

Reflected answers, Set 33: **1.** $82\frac{1}{3}$, **2.** $648\frac{14}{15}$

Set 34 — *For use with page 231*

Find the sums and differences.

1. 0.36 + 0.48	**2.** 3.72 + 0.59	**3.** 0.73 − 0.62	**4.** 4.51 − 1.81	**5.** 2.88 + 4.76
6. 32.7 + 4.89	**7.** 21.06 − 13.95	**8.** 5.631 + 4.782	**9.** 12.347 + 13.89	**10.** 15.04 − 7.65
11. 403.6 + 702.9	**12.** 0.032 − 0.018	**13.** 46.28 + 39.57	**14.** 54.07 − 25.95	**15.** 66.9 + 72.43
16. 72.36 − 7.88	**17.** 82.031 + 46.799	**18.** 326.7 − 211.9	**19.** 52.6 − 14.91	**20.** 93.66 + 43.77
21. 5.461 + 7.957	**22.** 6.042 − 3.789	**23.** 354.22 + 467.95	**24.** 264.32 − 95.71	**25.** 48.72 − 3.98

Reflected answers, Set 34: **1.** 0.84 **2.** 4.31 **3.** 0.11 **4.** 2.70 **5.** 7.64

Set 35 *For use with page 235*

Round each decimal to the nearest hundredth.

1. 0.476	**2.** 0.506	**3.** 0.876	**4.** 0.732	**5.** 0.322

Round each decimal to the nearest thousandth.

6. 0.62532	**7.** 0.53291	**8.** 0.88385	**9.** 0.73884	**10.** 0.48741

Round each decimal to the nearest ten thousandth.

11. 0.56675	**12.** 0.40316	**13.** 0.95437	**14.** 0.21382	**15.** 0.32609

Round each decimal to the nearest hundred thousandth.

16. 0.013711	**17.** 0.723488	**18.** 0.503267	**19.** 0.365229	**20.** 0.630105

Round each decimal to the nearest millionth.

21. 0.3651322	**22.** 0.7040321	**23.** 0.4379264	**24.** 0.9846278	**25.** 0.6437885

Reflected answers, Set 35: 1. 0.48, 2. 0.51, 3. 0.88, 4. 0.73, 5. 0.32

Set 36 *For use with page 237*

Find the products.

1. 10 × 0.58	**9.** 100 × 43.7	**17.** 10 × 0.003	**25.** 1000 × 0.0003
2. 100 × 0.3615	**10.** 10 × 0.087	**18.** 10 × 0.4	**26.** 10 × 345.62
3. 100 × 8.211	**11.** 10 × 3.2	**19.** 100 × 0.32	**27.** 100 × 426.703
4. 10 × 0.473	**12.** 100 × 56.03	**20.** 100 × 4.006	**28.** 1000 × 0.05612
5. 10 × 3.76	**13.** 100 × 50.736	**21.** 100 × 6.02	**29.** 1000 × 0.7
6. 100 × 21.53	**14.** 10 × 463.2	**22.** 100 × 0.0501	**30.** 100 × 0.4
7. 100 × 32.762	**15.** 100 × 0.0064	**23.** 10 × 3.01	**31.** 1000 × 2.006
8. 100 × 0.0541	**16.** 10 × 3.05	**24.** 1000 × 0.0652	**32.** 1000 × 0.07

Reflected answers, Set 36: 1. 5.8, 9. 4370, 17. 0.03, 25. 0.3

Set 37 *For use with page 239*

Find the products.

1. 0.6 × 0.5	**6.** 0.32 × 0.04	**11.** 0.7 × 0.93	**16.** 0.74 × 0.22
2. 0.7 × 0.8	**7.** 0.76 × 0.21	**12.** 0.21 × 0.56	**17.** 24.2 × 0.7
3. 0.4 × 0.72	**8.** 0.05 × 0.45	**13.** 0.33 × 0.61	**18.** 0.031 × 0.6
4. 0.65 × 0.3	**9.** 73 × 0.9	**14.** 0.24 × 0.7	**19.** 0.42 × 3.61
5. 0.42 × 0.8	**10.** 0.82 × 0.06	**15.** 0.31 × 0.48	**20.** 1.45 × 0.32

21. 4.9 × 3.2	**22.** 6.8 × 0.54	**23.** 27.2 × 0.5	**24.** 3.84 × 0.7
25. 74.2 × 0.61	**26.** 2.43 × 0.711	**27.** 61.42 × 8	**28.** 4.26 × 0.21

Reflected answers, Set 37: **1.** 0.3, **6.** 0.0128, **11.** 0.651, **16.** 0.1628, **21.** 15.68, **22.** 3.672, **23.** 13.6, **24.** 2.688

Set 38 *For use with page 243*

Estimate each product. Then find the product and compare it with your estimate.

1. 34.3 × 6	**4.** 0.427 × 4	**7.** 2.384 × 5	**10.** 0.643 × 6
2. 48.6 × 5	**5.** 0.596 × 7	**8.** 32.2 × 7	**11.** 8.26 × 9
3. 6.53 × 71	**6.** 7.64 × 3	**9.** 24.7 × 3	**12.** 83.5 × 4

Find the quotients. Check your work.

13. 5)160.5	**14.** 7)32.41	**15.** 4)3.060	**16.** 2)73.4
17. 9)42.408	**18.** 12)57.6	**19.** 10)3.260	**20.** 6)249.6
21. 8)449.68	**22.** 30)16.20	**23.** 5)222.55	**24.** 40)14.440
25. 20)96.20	**26.** 70)2275.0	**27.** 80)450.40	**28.** 60)1698.0

Reflected answers, Set 38: **1.** 205.8, **4.** 1.708, **7.** 11.92, **10.** 3.858, **13.** 32.1, **14.** 4.63, **15.** 0.765, **16.** 36.7

Set 39 *For use with page 245*

Find the quotients. Check your work.

1. $0.7\overline{)22.4}$	**2.** $0.5\overline{)3.355}$	**3.** $0.02\overline{)0.068}$	**4.** $3.4\overline{)27.2}$
5. $0.21\overline{)1.0962}$	**6.** $6.3\overline{)3.276}$	**7.** $0.33\overline{)1.518}$	**8.** $0.62\overline{)0.1426}$
9. $5.7\overline{)410.4}$	**10.** $4.8\overline{)1.488}$	**11.** $2.5\overline{)162.5}$	**12.** $0.71\overline{)1.136}$
13. $0.06\overline{)49.32}$	**14.** $0.36\overline{)0.0180}$	**15.** $8.2\overline{)0.574}$	**16.** $0.61\overline{)0.854}$
17. $3.5\overline{)213.5}$	**18.** $0.003\overline{)0.375}$	**19.** $9.4\overline{)432.4}$	**20.** $0.26\overline{)5.486}$
21. $0.52\overline{)16.692}$	**22.** $7.3\overline{)350.4}$	**23.** $4.4\overline{)360.8}$	**24.** $0.38\overline{)119.32}$
25. $8.6\overline{)19.584}$	**26.** $4.22\overline{)25.32}$	**27.** $7.63\overline{)46.543}$	**28.** $50.4\overline{)70.46}$

Reflected answers, Set 39: 1. 32, 2. 6.71, 3. 3.4, 4. 8

Set 40 *For use with page 249*

Write a decimal for each fraction. (The denominators are factors of 10 or 100.)

1. $\frac{1}{4}$	**2.** $\frac{9}{20}$	**3.** $\frac{3}{10}$	**4.** $\frac{2}{5}$	**5.** $\frac{17}{25}$
6. $\frac{21}{50}$	**7.** $\frac{7}{10}$	**8.** $\frac{23}{25}$	**9.** $\frac{13}{20}$	**10.** $\frac{37}{50}$

Use division to find decimals for these fractions.

11. $\frac{7}{25}$	**12.** $\frac{1}{5}$	**13.** $\frac{6}{15}$	**14.** $\frac{7}{20}$	**15.** $\frac{18}{30}$
16. $\frac{9}{50}$	**17.** $\frac{3}{25}$	**18.** $\frac{6}{50}$	**19.** $\frac{9}{25}$	**20.** $\frac{1}{20}$

Find the repeating decimal for each of these.

21. $\frac{5}{12}$	**22.** $\frac{7}{18}$	**23.** $\frac{4}{6}$	**24.** $\frac{4}{9}$	**25.** $\frac{1}{12}$
26. $\frac{3}{11}$	**27.** $\frac{5}{18}$	**28.** $\frac{7}{11}$	**29.** $\frac{7}{12}$	**30.** $\frac{2}{6}$

Reflected answers, Set 40: 1. 0.25, 2. 0.45, 3. 0.3, 4. 0.4, 5. 0.68

Set 41 *For use with page 251*

Give each quotient rounded to the nearest hundredth.

1. $6\overline{)2.088}$ **2.** $4\overline{)2.93}$ **3.** $7\overline{)4.836}$ **4.** $5\overline{)3.6421}$

5. $1.6\overline{)0.825}$ **6.** $2.4\overline{)0.4732}$ **7.** $3.6\overline{)4.271}$ **8.** $4.3\overline{)0.6125}$

9. $0.26\overline{)3.436}$ **10.** $0.78\overline{)5.9821}$ **11.** $5.2\overline{)3.7076}$ **12.** $0.34\overline{)7.3478}$

13. $0.89\overline{)10.7065}$ **14.** $6.4\overline{)4.609}$ **15.** $2.1\overline{)1.654}$ **16.** $0.46\overline{)10.8149}$

Give each quotient expressed as a mixed-decimal numeral in hundredths.

17. $7\overline{)1.72}$ **18.** $4\overline{)1.47}$ **19.** $2\overline{)12.71}$ **20.** $6\overline{)0.73}$

21. $9\overline{)2.36}$ **22.** $3\overline{)2.56}$ **23.** $8\overline{)3.72}$ **24.** $5\overline{)3.68}$

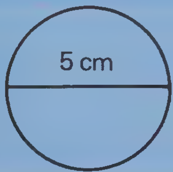
25. $0.4\overline{)0.217}$ **26.** $0.02\overline{)0.4273}$ **27.** $0.3\overline{)4.955}$ **28.** $0.6\overline{)1.047}$

Reflected answers, Set 41: 1. 0.35, 2. 0.73, 3. 0.69, 4. 0.73

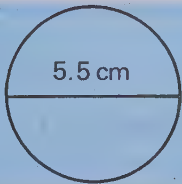
Set 42 *For use with page 263*

Find the circumference of each circle with the given diameter.
$\pi = 3.14$ (to the nearest hundredth)

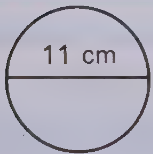
1.



2.



3.



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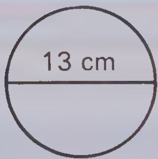


5. $d = 13$ cm
6. $d = 12.5$ cm
7. $d = 28.6$ cm
8. $d = 54.2$ cm

9. $d = 15$ cm
10. $d = 23$ cm
11. $d = 14.6$ cm
12. $d = 36.4$ cm

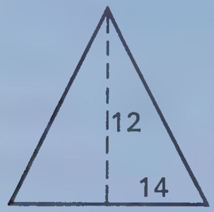
13. $d = 32.2$ cm
14. $d = 19$ cm
15. $d = 44.1$ cm
16. $d = 61$ cm

Reflected answers, Set 42: 1. 15.7 2. 17.27

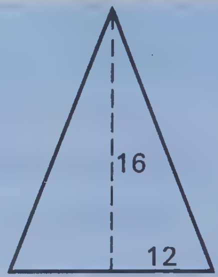
Set 43 *For use with page 267*

Find the area of each triangle.

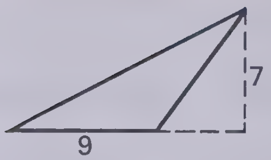
1.



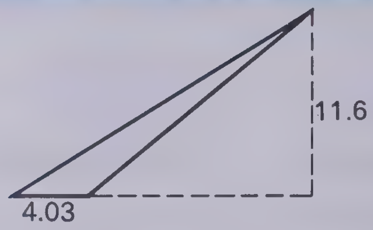
2.



3.



4.



Given the base and height, find the area of each triangle.

5. $b = 16$, $h = 5$
6. $b = 32$, $h = 7$
7. $b = 15$, $h = 42$
8. $b = 9$, $h = 16$
9. $b = 48$, $h = 7$
10. $b = 56$, $h = 11$
11. $b = 10.8$, $h = 12.4$
12. $b = 32.6$, $h = 9$
13. $b = 14\frac{1}{2}$, $h = 7$
14. $b = 8$, $h = 10\frac{1}{3}$
15. $b = 17.5$, $h = 12.6$
16. $b = 44.3$, $h = 52.2$

Reflected answers, Set 43: 1. 84, 2. 96

Set 44 *For use with page 271*

Find the area of each circle with the radius given. (Use $\pi = 3.14$.)

1. $r = 4$
2. $r = 1$
3. $r = 8$
4. $r = 6$
5. $r = 20$
6. $r = 3.4$
7. $r = 2.6$
8. $r = 9$
9. $r = 12.1$
10. $r = 32$
11. $r = 10.4$
12. $r = 42$
13. $r = 5$
14. $r = 7.3$
15. $r = 4.6$
16. $r = 12$

Reflected answers, Set 44: 1. 50.24, 2. 3.14, 3. 200.96, 4. 113.04

Set 45	*For use with page 273*

Use the volume formula, $V = \ell \times w \times h$, to find the volume of each figure.

1. $\ell = 5$ $w = 6$ $h = 6$	2. $\ell = 2$ $w = 7$ $h = 9$	3. $\ell = 5$ $w = 4$ $h = 10$	4. $\ell = 4$ $w = 4$ $h = 4$
5. $\ell = 9$ $w = 7$ $h = 12$	6. $\ell = 4$ $w = 7$ $h = 8$	7. $\ell = 10$ $w = 12$ $h = 5$	8. $\ell = 13$ $w = 6$ $h = 5$
9. $\ell = 2.1$ $w = 3.6$ $h = 4.4$	10. $\ell = 3\frac{1}{2}$ $w = 2\frac{1}{7}$ $h = 1\frac{1}{5}$	11. $\ell = 5.6$ $w = 2.08$ $h = 1.29$	12. $\ell = 5\frac{1}{3}$ $w = 2\frac{1}{4}$ $h = 6$

Reflected answers, Set 45: 1. 180, 2. 126, 3. 200, 4. 64

Set 46	*For use with page 307*

Find the missing numbers in each exercise.

1. $\frac{1}{3} = \frac{4}{\square}$	2. $\frac{\square}{5} = \frac{16}{20}$	3. $\frac{9}{8} = \frac{\square}{24}$	4. $\frac{7}{10} = \frac{21}{\square}$
5. $\frac{3}{\square} = \frac{12}{16}$	6. $\frac{\square}{7} = \frac{10}{35}$	7. $\frac{5}{3} = \frac{\square}{9}$	8. $\frac{5}{6} = \frac{20}{\square}$
9. $\frac{4}{9} = \frac{\square}{90}$	10. $\frac{12}{\square} = \frac{36}{15}$	11. $\frac{3}{10} = \frac{\square}{40}$	12. $\frac{7}{2} = \frac{35}{\square}$

Write a fractional number equation and solve it to answer the following ratio problems.

13. Ratio of tires to cars, 4:1. 12 cars. How many tires?

14. Ratio of boys to girls, 7:8. 35 boys. How many girls?

15. Ratio of oranges to cents, 5:22. 66 cents. How many oranges?

16. Ratio of dogs to cats, 12:17. 68 cats. How many dogs?

Reflected answers, Set 46: 1. 12, 2. 4, 3. 27, 4. 30, 13. 48, 15. 15

Set 47	*For use with page 317*

1. $\frac{3}{20}$ = ____ % **2.** $\frac{4}{5}$ = ____ % **3.** $\frac{1}{6}$ = ____ % **4.** $\frac{7}{25}$ = ____ %

5. $\frac{24}{25}$ = ____ % **6.** $\frac{9}{25}$ = ____ % **7.** $\frac{9}{20}$ = ____ % **8.** $\frac{49}{50}$ = ____ %

Solve the following short story problems.

9. 80% of Mr. Jones' orchard: apple trees. 620 trees in all. How many apple trees?

10. 760 students at Thomas High School. 45% are girls. How many are girls?

11. Book: 575 pages. Read 56%. Read how many pages?

12. Paint: 16 litres. Used 75% to paint house. Used how many litres?

Reflected answers, Set 47: **1.** 15, **2.** 80, **3.** $16\frac{3}{5}$, **4.** 28, **9.** 496, **11.** 322

Set 48	*For use with page 323*

Complete the following table.

Savings	Interest rate (per year)	Amount of interest in one year	Savings plus interest
$34.00	2%	**1.**	**2.**
$125.00	4%	**3.**	**4.**
$78.00	3%	**5.**	**6.**
$236.00	5%	**7.**	**8.**
$68.00	4%	**9.**	**10.**
$95.00	2%	**11.**	**12.**
$430.00	3%	**13.**	**14.**
$750.00	4%	**15.**	**16.**
$520.00	5%	**17.**	**18.**
$1230.00	2%	**19.**	**20.**

Reflected answers, Set 48: **1.** 68¢, **2.** $34.68

Books to Explore

Adler, Irving. ***The Giant Golden Book of Mathematics.*** New York, Golden Press, 1960. (Available from Whitman Golden Ltd., Cambridge, Ontario.)

Have you ever wondered how a tree grows or why a volcano is shaped as it is or what makes a card trick work? This colorful book answers these and many other questions by exploring the world of mathematics. Here are just a few of the interesting topics:

How to find π by dropping a toothpick 26
Your chance of getting two heads and a tail when you toss three coins 72

Bendick, Jeanne and Levin, Marcia O. ***Take Shapes, Lines and Letters.*** New York, McGraw-Hill Book Company, 1962. (Available from McGraw-Hill Ryerson, Scarborough, Ontario.)

This book teases your imagination with mathematics in art, music, and everyday life; with shapes and curves in nature; and with drawings, graphs, and secret codes. Ideas to explore in this book include:

The golden rectangle in art 44
A strip of paper that has only one side 72
Geometry on a doughnut 74

Boeke, Kees. ***Cosmic View: The Universe in 40 Jumps.*** New York, John Day Company, 1957. (Available from Longman Canada Ltd., Don Mills, Ontario.)

If you have a lively interest in the world in which you live, then take a graphic journey through the universe. Travel from outer space to the nucleus of an atom and take a look at everything between. A series of pictures and drawings begins with a girl shown one-tenth life-size. Each successive picture shows her with other objects in the solar system until the scale is $1:10^{26}$. Similarly, a small portion of her hand is magnified until the scale is $10^{13}:1$ in a picture that compares that part of her hand with the nucleus of a sodium atom.

Gardner, Martin. ***Perplexing Puzzles and Tantalizing Teasers.*** New York, Simon & Schuster, 1969. (Available from Musson Book Company, Don Mills, Ontario.)

This book is a collection of many different kinds of puzzles. None of them is very difficult, but some are tricky. Try your best before you look at the solutions given at the end of the book. Puzzles you may enjoy include:

Fun with palindromes 14
The maze of the Minotaur 20
Find the best words 40
More sneaky arithmetic 49

Other books to look for in your library are listed below.

Diggins, Julia E. ***String, Straightedge, and Shadow.*** New York, Viking Press, Inc., 1965. (Macmillan Company of Canada Ltd., Toronto.)

Based on historical records and legends, this book tells how men made the major discoveries of mathematics by using only three simple tools. It includes an interesting discussion on the origin of the word "mathematician."

Friskey, Margaret. ***ed.*** ***About Measurement.*** Chicago, Children's Press, 1965. (Scholars Choice Limited, Stratford, Ontario.)

Based on "A History of Mathematics," a poster published by the Educational Affairs Department of the Ford Motor Company, this book traces the history of units of length — cubit, foot, inch, fathom, yard, rod, furlong, meter, decimal inch, light-year.

Kaplan, Philip. ***Posers.*** New York, Harper & Row, 1963. (Fitzhenry & Whiteside Ltd., Don Mills, Ontario.)

As you will discover here, posers are not riddles made to deceive, or problems which require mathematical skill, or even puzzles which tease without purpose. They are statements of fact from which a "logical conclusion can be drawn." Anyone with a "reasonably agile mind" should enjoy the eighty posers in this book.

Meadow, Charles. ***The Story of Computers.*** Irvington-on-Hudson, New York, Harvey House, 1970. (Burns & MacEachern Ltd., Don Mills, Ontario.)

Today's computers can tackle problems as intricate as guiding spaceships and predicting the weather. This book tells how they are programmed, or instructed to solve these and other problems.

Rogers, James T. ***Story of Mathematics for Young People.*** New York, Pantheon Books, 1966. (Random House of Canada Limited, Toronto.)

This history explains how man created mathematics to solve his problems. Diagrams and photographs trace the lives of great mathematicians and reveal the part that mathematics plays in all phases of life, from calculating the curves of a suspension bridge to totalling a grocery bill.

Shimek, William. ***Patterns: What Are They ?*** Minneapolis, Minnesota, Lerner Publications, 1969. (J.M. Dent & Sons (Canada) Ltd., Don Mills, Ontario.)

Number patterns have fascinated men for thousands of years. This Math Concept Book tells how mathematics helps record and explore patterns and how such a study brings a deeper understanding of mathematics and even simplifies computation.

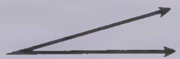
Zim, Herbert S. ***Codes and Secret Writing.*** New York, William Morrow and Company, 1948. George J. McLeod Ltd., Toronto.)

Learn to break and make up your own secret codes. This book will show you how.

addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the result is called the sum of the addends.

angle Two rays from a single point.



approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

arithmetic mean (average) The arithmetic mean of a set of numbers is the quotient resulting when the sum of the numbers in the set is divided by the number of addends.

associative (grouping) principle When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

Examples: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 2) = (3 \times 4) \times 2$

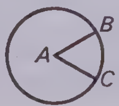
base of numeration The term "base" refers to the type of grouping involved in the system of numeration. For example, in base 6:
23 means 2 sixes and 3.
321 means 3 thirty-sixes (6×6), 2 sixes, and 1.
In base ten:
23 means 2 tens and 3.
321 means 3 hundreds (10×10), 2 tens, and 1.

base of a polygon Any of the sides of a polygon may be referred to as a base. The term is normally used with reference to the height from a vertex (or parallel side) to the given base. *See* height of a triangle.

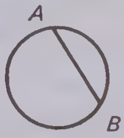
bisect To divide in half or find the midpoint.

centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

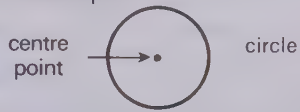
central angle In the figure below, angle *BAC* illustrates a central angle with respect to a given circle with centre *A*.



chord A line segment that has its end points on a given circle.



circle The set of all points in a plane which are a specified distance from a given point called the centre or centre point.



circumference The distance around a circle.

clock arithmetic A mathematical system using only the twelve numbers of a clock face. It is also called modular arithmetic or remainder arithmetic.

common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

common multiple A number is a common multiple of two numbers if it is a multiple of each of the numbers.

commutative (order) principle When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Examples: $4 + 5 = 5 + 4$
$3 \times 6 = 6 \times 3$

composite number Any whole number greater than 1 that is not prime.

cone Generally thought of in terms of a right circular cone, illustrated below.

congruent figures Geometric figures that have the same size and shape.

congruent triangles

coordinate axes Two number lines intersecting at right angles at 0.

coordinates Number pair used in graphing.

cube A rectangular prism (box) such that all faces are squares.

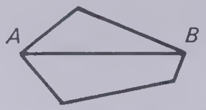
cylinder Generally thought of as a right circular cylinder illustrated below.

decimal Any base-ten numeral that uses place value to represent a number.

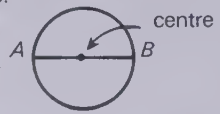
degree A unit of angle measure.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment *AB*.



diameter A chord that passes through the centre point of the circle.



distributive (multiplication-addition) principle This principle is sometimes described in terms of "breaking apart" a number before multiplying.

Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$

dividend In the problem $33 \div 7$, 33 is called the dividend.

Example:
$$\begin{array}{r} 4 \\ 7\overline{)33} \\ \underline{28} \\ 5 \end{array}$$
← dividend; divisor: 7; 5 → remainder

division An operation related to multiplication as illustrated:

$$3 \times 4 = 12 \begin{cases} \nearrow 12 \div 4 = 3 \\ \searrow 12 \div 3 = 4 \end{cases}$$

divisor In the problem $33 \div 7$, 7 is called the divisor.

empty set A set that has no objects in it.

equality (equals; or $=$) A mathematical relation of being exactly the same.

equally likely outcomes Outcomes that have the same chance of occurring.

equation A mathematical sentence involving the use of the equality symbol.

Examples: $5 + 4 = 9$; $7 + \square = 8$; $n + 3 = 7$.

equivalent fractions Two fractions are equivalent when it can be shown that they each can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:

$\frac{3}{4}$ $\frac{6}{8}$ $\longrightarrow 4 \times 6 \longrightarrow 24$

$\longrightarrow 3 \times 8 \longrightarrow 24$

equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, . . .).

exponent In the symbol 10^3, the "3" is the exponent. It indicates that 10 is used as a factor three times. Thus: $10^3 = 10 \times 10 \times 10 = 1000$

$5^4 = 5 \times 5 \times 5 \times 5 = 625$

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor *See* multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

fraction A symbol for a fractional number such as $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$, and so on.

fractional number The one number we think about for each set of equivalent fractions.

function The set of number pairs (input, output) generated by a function rule applied to a given set of numbers (input numbers).

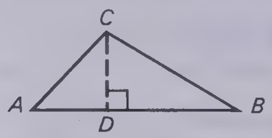
graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than ($>$) One of the two basic inequality relations.

Examples: $8 > 5$, $28 > 25$, $80 > 50$.

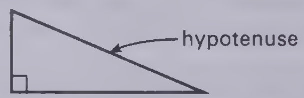
greatest common factor The largest, or greatest, number that is a factor of each of two numbers.

height of a triangle The height of a triangle from any vertex is the perpendicular distance from that vertex to the opposite side (usually called the base). In the figure, the length of $\overline{CD}$ is the height of the triangle from vertex *C* to base $\overline{AB}$.



hexagon A six-sided polygon.

hypotenuse The side opposite the right angle in a right triangle.



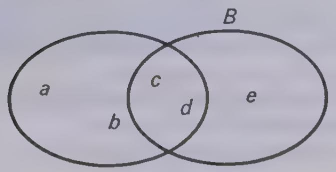
improper fraction A fraction in which the numerator is greater than or equal to the denominator.

Examples: $\frac{8}{5}, \frac{6}{6}, \frac{12}{3}, \frac{7}{7}$

inequality ($<, \neq, >$) A relation indicating that the two numbers are not the same.

inscribed circle A circle is said to be inscribed in a polygon if the circle lies within the polygon and each side of the polygon is tangent to the circle.

intersection of sets The intersection of two sets is the set of elements common to both of the sets. If A and B are sets, $A \cap B$ denotes the intersection of the sets.

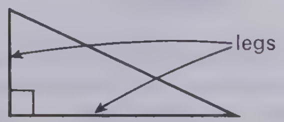


$A \cap B = \{c, d\}$

least common denominator The least common multiple of two denominators. The least common denominator for $\frac{3}{4}$ and $\frac{5}{6}$ is 12.

least common multiple The smallest non-zero number that is a multiple of each of two given numbers. The least common multiple of 4 and 6 is 12.

legs of a right triangle The two sides of a right triangle other than the hypotenuse.



length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

less than (<) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

metre The basic unit of length in the Metric System.

midpoint A point that divides a line segment into two parts of the same size.

mixed-decimal numeral Numerals such as $0.7\frac{1}{2}$ and $0.33\frac{1}{3}$

mixed numeral A symbol such as $2\frac{1}{2}$ and $5\frac{1}{4}$.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number.

Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the result is called the product.

negative number A number which will add to a positive number to give a sum of zero.

For example: $5 + {}^{-}5 = 0$

$19 + {}^{-}19 = 0$

number line A line with a subset of its points matched with a subset of the real numbers. We say that the rational number line has "holes" in it because some points are not matched with rational numbers. The real number line is said to be "complete" because each point is matched with some real number.

The number line

The whole number line

The rational number line

number pair Any pair of numbers. Each pair of numbers can be matched with a unique point in the co-ordinate plane.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not even.

one principle Any number multiplied by 1 is that same number. Sometimes called the identity principle for multiplication.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set and each element of the second set is matched with exactly one element of the first set.

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with its opposite sides parallel.

parentheses A pair of curved symbols ( ), used to indicate grouping or order of performing operations. Examples: $(5 \times 4) - 2 = 18$

$5 \times (4 - 2) = 10$

pentagon A five-sided polygon.

percent (%) Per 100; for each 100; $\frac{1}{100}$.

perimeter The sum of the lengths of the sides of a given polygon.

period In arithmetic, each set of three digits indicated by spaces when writing a numeral is called a period.

Example: 3 4 2   6 7 4   2 0 8

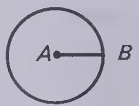
millions' period   thousands' period   units' period

perpendicular lines Two lines that intersect in right angles are perpendicular to each other.

pi (π) The ratio of the circumference to the diameter of a circle; approximately 3.14.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. In the numeral 3257 the 5 stands for 50; in the numeral 36 289 the 6 stands for 6000.

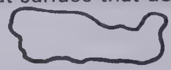
polygon A closed geometric figure made up of line segments. A regular polygon has congruent sides and congruent angles.
prime number A number greater than 1 whose only factors are itself and 1.
probability The probability that an event will occur in a set of equally likely outcomes is the number of ways the event can occur divided by the total number of possible outcomes. For example, the probability that a 3 or a 4 will turn up in a single toss of a die is $\frac{2}{6}$ since there are 2 ways the event can occur and there are 6 possible equally likely outcomes.
product The result of the multiplication operation. In $6 \times 7 = 42$, 42 is the product of 6 and 7.
protractor An instrument used for measuring angles.
pyramid A space figure which has a polygonal base and triangular lateral faces.
quadrilateral A four-sided polygon.
quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.
radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ratio A pair of numbers used in making certain comparisons. The ratio of 3 to 4 is written 3 : 4 or $\frac{3}{4}$.
ray The heavy part of the line shows a ray.

reciprocal Two numbers are reciprocals of one another if their product is 1. Example: $\frac{4}{7}$ and $\frac{7}{4}$ are reciprocals of each other.
rectangle A quadrilateral that has four right angles.
rhombus A parallelogram with 4 congruent sides.
right angle An angle that has the measure of 90 degrees.
right triangle A triangle that has one right angle.
Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.
rotation A motion in which a given figure is turned about a fixed point.
scale drawing A drawing constructed so the ratio of all the dimensions in the drawing to those of the actual object is the same.
segment Two points on a line and all the points on that line that are between the two points.
sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group or collection of objects.
simple closed curve Can be thought of as a loop of string on a flat surface that does not cross itself.



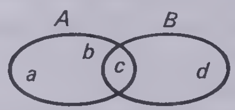
skew lines Two lines that are not in the same plane.
solution The number or numbers which result from solving an equation or a given problem.
square A quadrilateral that has four right angles and four sides that are the same length.
subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{cases} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{cases}$$

sum The result obtained by adding any set of numbers.
symmetric figure A plane figure which can be divided into two congruent halves by a line.
tangent A line is tangent to a circle if the two figures are in one plane and have exactly one point in common.
tessellation A repeated pattern of regions that can cover a plane.

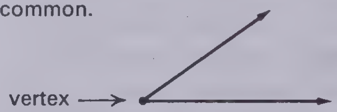
translation A motion in which each point of a figure is moved the same distance and the same direction.
trapezoid A quadrilateral with at least one pair of parallel sides.
triangle A three-sided polygon.
triangular pyramid A 4-sided space figure that has triangular regions for all faces.

union of sets If A and B are sets, then $A \cup B$ (the union of A and B) is the set consisting of all elements that belong to at least one of the two sets.



$A \cup B = \{a, b, c, d\}$

unit An amount or quantity adopted as a standard of measurement.
vertex The point that the two rays of an angle have in common.



volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.
whole number Any number in the set $\{\ldots, {}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3, \ldots\}$
zero principle Any number added to zero is that same number. (Also called the identity principle for addition.)

Tables of Measures

LENGTH	
10 millimetres (mm) = 1 centimetre (cm)	1000 millimetres = 1 metre
10 centimetres = 1 decimetre (dm)	100 centimetres = 1 metre
10 decimetres = 1 metre (m)	10 decimetres = 1 metre
1000 metres = 1 kilometre (km)	1 / 1000 kilometres = 1 metre

TIME	
60 seconds (s) = 1 minute (min)	52 weeks = 1 year
60 minutes = 1 hour (h)	12 months (mo) = 1 year
24 hours = 1 day	365 days = 1 year
7 days = 1 week (wk)	366 days = 1 leap year

CAPACITY
10 millilitres (ml) = 1 centilitre (cl)
10 centilitres = 1 decilitre (dl)
10 decilitres = 1 litre (*l*)
1000 litres = 1 kilolitre (kl)

WEIGHT
1000 grams (g) = 1 kilogram (kg)
1000 kilograms = 1 tonne (t)

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